012 – Solving Quadratic Equations by Factoring

The following are examples of quadratic equations.

$$x^2 = 25 \qquad 2x^2 + 11x + 5 = 0 \qquad x(x-1) = 0$$

The equation $ax^2 + bx + c = 0$ is quadratic equation written in **standard form** ($a \neq 0$).

Most quadratic equations have two solutions (or roots). For example, $x^2 = 25$ has solutions $x = 5$ and $x = -5$. One method of solving quadratic equations is the **factoring method**. It is based on the following rule.

The $A \cdot B = 0$ rule If $A \cdot B = 0$, then $A = 0$ or $B = 0$.	←	If the product of two numbers is 0 then at least one of the numbers must be equal to 0.

The Factoring Method for Solving a Quadratic Equation

1. Put the quadratic equation in standard form $ax^2 + bx + c = 0$.
2. Factor $ax^2 + bx + c$ (if possible).
3. Set each factor equal to zero and solve.

EXAMPLE 1: Solve for x. **A)** $x^2 - 10x + 21 = 0$ **B)** $10x^2 + 30x = 0$ **C)** $9x^2 - 25 = 0$

SOLUTIONS:

A) $x^2 - 10x + 21 = (x-3)(x-7) = 0$

The quadratic equation is in standard form. So we start by factoring (reverse FOIL).

$x - 3 = 0$ $\quad$ $x - 7 = 0$

$x = 3$ $\quad$ $x = 7$

Set each factor equal to zero and solve. The two solutions are $x = 3$ and $x = 7$.

Check: $x = 3$; $(3)^2 - 10(3) + 21 = 9 - 30 + 21 = 0$.

$x = 7$; $(7)^2 - 10(7) + 21 = 49 - 70 + 21 = 0$.

We can check our solutions by substituting into the original equation.

B) $10x^2 + 30x = 10x(x+3) = 0$

The quadratic equation is in standard form. So we start by factoring (greatest common factor).

$10x = 0$ $\quad$ $x + 3 = 0$

$x = 0$ $\quad$ $x = -3$

Set each factor equal to zero and solve. The two solutions are $x = 0$ and $x = -3$.

C) $9x^2 - 25 = (3x+5)(3x-5) = 0$

The quadratic equation is in standard form. So we start by factoring (difference of two squares).

$3x + 5 = 0$ $\quad$ $3x - 5 = 0$

$3x = -5$ $\quad$ $3x = 5$

$x = -\frac{5}{3}$ $\quad$ $x = \frac{5}{3}$

Set each factor equal to zero and solve. The two solutions are $x = -\frac{5}{3}$ and $x = \frac{5}{3}$. ■

EXAMPLE 2: Solve for x. **A)** $2x^2 + 3x = 35$ **B)** $10x^2 = 160$

SOLUTIONS:

A) $2x^2 + 3x = 35$

Put in standard form. Subtract 35 from both sides of the equation.

$2x^2 + 3x - 35 = (2x-7)(x+5) = 0$

Factor.

$2x - 7 = 0$ $2x = 7$ $x = \frac{7}{2}$	$x + 5 = 0$ $x = -5$	Set each factor equal to zero and solve. The two solutions are $x = \frac{7}{2}$ and $x = -5$.
B) $10x^2 = 160$		Divide both sides by 10 to simplify.
$x^2 = 16$		Put in standard form. Subtract 16 from both sides of the equation.
C) $x^2 - 16 = (x + 4)(x - 4) = 0$		Factor.
$x + 4 = 0$ $x = -4$	$x - 4 = 0$ $x = 4$	Set each factor equal to zero and solve. The two solutions are $x = -4$ and $x = 4$. ■

012 – EXERCISES

For 1 – 12, solve for x.			
1.	$x^2 + 12x + 35 = 0$	**2.**	$x^2 - 8x + 15 = 0$
3.	$x^2 - x = 0$	**4.**	$5x^2 + 10x = 0$
5.	$4x^2 - 49 = 0$	**6.**	$9x^2 - 16 = 0$
7.	$2x^2 - 17x + 21 = 0$	**8.**	$3x^2 + 13x - 10 = 0$
9.	$7x^2 - 33x - 10 = 0$	**10.**	$14x^2 + 29x - 15 = 0$
11.	$3x^2 - 28x - 55 = 0$	**12.**	$2x^2 - 13x + 6 = 0$

For 13 – 22, put in standard form and solve for x.			
13.	$12x^2 = 27$	**14.**	$10x^2 = 490$
15.	$50x^2 = 32$	**16.**	$x^2 = -6x$
17.	$15x^2 = 10x$	**18.**	$x^2 - 3x = 28$
19.	$2x^2 + 15 = 5x$	**20.**	$5x^2 - 53x = 22$
21.	$3x^2 + 17x = 6$	**22.**	$7x^2 - 10 = 33x$

012 – WORKSHEET: Solving Quadratic Equations By Factoring

For 1 – 12, solve for x.			
1.	$x^2 + 10x + 21 = 0$	**2.**	$x^2 - 7x + 10 = 0$
3.	$x^2 - 4x = 0$	**4.**	$7x^2 + 14x = 0$
5.	$25x^2 - 9 = 0$	**6.**	$4x^2 - 81 = 0$
7.	$3x^2 - 13x + 14 = 0$	**8.**	$2x^2 - x - 15 = 0$
9.	$5x^2 - 3x - 14 = 0$	**10.**	$14x^2 + 11x - 15 = 0$
11.	$5x^2 - 52x - 33 = 0$	**12.**	$3x^2 - 46x + 15 = 0$

For 13 – 22, put in standard form and solve for x.			
13.	$5x^2 = 45$	**14.**	$10x^2 = 250$
15.	$5x^2 = -15x$	**16.**	$21x^2 = 35x$
17.	$x^2 + 4x = 21$	**18.**	$5x^2 - 6x = 8$
19.	$3x^2 - 13x = -14$	**20.**	$2x^2 - 17x = 55$
21.	$7x^2 + 41x = 6$	**22.**	$5x^2 - 6 = -13x$