Chapter 1 Equations and Inequalities

1.1 Linear Equations in One Variable

1.1 Practice Problems

1. a. Both sides of the equation $\frac{x}{3} - 7 = 5$ are defined for all real numbers, so the domain is $(-\infty, \infty)$.

b. The left side of the equation $\frac{2}{2-x} = 4$ is not defined if $x = 2$. The right side of the equation is defined for all real numbers, so the domain is $(-\infty, 2) \cup (2, \infty)$.

c. The left side of the equation $\sqrt{x-1} = 0$ is not defined if $x < 1$. The right side of the equation is defined for all real numbers, so the domain is $[1, \infty)$.

2. $\frac{2}{3} - \frac{3}{2}x = \frac{1}{6} - \frac{7}{3}x$

To clear the fractions, multiply both sides of the equation by the LCD, 6.

$$\begin{aligned} 4 - 9x &= 1 - 14x \\ 4 - 9x + 14x &= 1 - 14x + 14x \\ 4 + 5x &= 1 \\ 4 + 5x - 4 &= 1 - 4 \\ 5x &= -3 \\ \frac{5x}{5} &= \frac{-3}{5} \\ x &= -\frac{3}{5} \end{aligned}$$

Solution set: $\left\{-\frac{3}{5}\right\}$

3.
$$\begin{aligned} 3x - [2x - 6(x+1)] &= 7x - 1 \\ 3x - (2x - 6x - 6) &= 7x - 1 \\ 3x - (-4x - 6) &= 7x - 1 \\ 3x + 4x + 6 &= 7x - 1 \\ 7x + 6 &= 7x - 1 \\ 7x + 6 - 7x &= 7x - 1 - 7x \\ 6 &= -1 \end{aligned}$$

Since $6 = -1$ is false, no number satisfies this equation. Thus, the equation is inconsistent, and the solution set is $\varnothing$.

4.
$$\begin{aligned} 2(3x-6) + 5 &= 12 - (19 - 6x) \\ 6x - 12 + 5 &= 12 - 19 + 6x \\ 6x - 7 &= -7 + 6x \\ 6x - 7 - 6x &= -7 + 6x - 6x \\ -7 &= -7 \\ -7 + 7 &= -7 + 7 \\ 0 &= 0 \end{aligned}$$

The equation 0 = 0 is always true. Therefore, the original equation is an identity, and the solution set is $(-\infty, \infty)$.

5.
$$\begin{aligned} F &= \frac{9}{5}C + 32 \\ 50 &= \frac{9}{5}C + 32 \\ 50 - 32 &= \frac{9}{5}C + 32 - 32 \\ 18 &= \frac{9}{5}C \\ 18 \cdot \frac{5}{9} &= \frac{5}{9} \cdot \frac{9}{5}C \\ 10 &= C \end{aligned}$$

Thus, 50ºF converts to 10ºC.

6. $P = 2l + 2w$

Subtract $2l$ from both sides.

$P - 2l = 2w$

Now, divide both sides by 2.

$\frac{P - 2l}{2} = w$

7. Let w = the width of the rectangle.
Then $2w + 5$ = the length of the rectangle.
$P = 2l + 2w$, so we have

$$\begin{aligned} 28 &= 2(2w + 5) + 2w \\ 28 &= 4w + 10 + 2w \\ 28 &= 6w + 10 \\ 18 &= 6w \\ 3 &= w \end{aligned}$$

The width of the rectangle is 3 m and the length is 2(3) + 5 = 11 m

8. Let x = the amount invested in stocks. Then 15,000 − x = the amount invested in bonds.

$$\begin{aligned} x &= 3(15{,}000 - x) \\ x &= 45{,}000 - 3x \\ 4x &= 45{,}000 \\ x &= 11{,}250 \end{aligned}$$

Tyrick invested \$11,250 in stocks and \$15,000 − \$11,250 = \$3,750 in bonds**.**

9. Let x = the amount of capital. Then $\frac{x}{5}=$ the amount invested at 5%, $\frac{x}{6}=$ the amount invested at 8%, and $x-\left(\frac{x}{5}+\frac{x}{6}\right)=\frac{19x}{30}=$ the amount invested at 10%.

Principal	Rate	Time	Interest
$\frac{x}{5}$	0.05	1	$0.05\left(\frac{x}{5}\right)$
$\frac{x}{6}$	0.08	1	$0.08\left(\frac{x}{6}\right)$
$\frac{19x}{30}$	0.1	1	$0.1\left(\frac{19x}{30}\right)$

The total interest is $130, so

$$0.05\left(\frac{x}{5}\right)+0.08\left(\frac{x}{6}\right)+0.1\left(\frac{19x}{30}\right)=130$$

Multiply by the LCD, 30.

$$\begin{aligned}0.3x+0.4x+1.9x&=3900\\2.6x&=3900\\x&=1500\end{aligned}$$

The total capital is $1500.

10. Let x = the length of the bridge.
Then $x + 130$ = the distance the train travels.
$rt = d$, so

$$25(21)=x+130 \Rightarrow 525=x+130 \Rightarrow 395=x$$

The bridge is 395 m long.

11. Following the reasoning in example 10, we have $x + 2x = 3x$ is the maximum extended length (in feet) of the cord.

$$\begin{aligned}3x+7+10&=120\\3x+17&=120\\3x+17-17&=120-17\\3x&=103\\\frac{3x}{3}&=\frac{103}{3}\Rightarrow x\approx 34.3\end{aligned}$$

The cord should be no longer than 34.3 feet.

1.1 Basic Concepts and Skills

1. The domain of the variable in an equation is the set of all real number for which both sides of the equation are <u>defined</u>.

2. Standard form for a linear equation in x is of the form <u>$ax + b = 0$</u>.

3. Two equations with the same solution sets are called <u>equivalent</u>.

4. A conditional equation is one that is not true for <u>some</u> values of the variables.

5. False. The interest $I = (100)(0.05)(3)$.

6. False. Since the rate is given in feet per second, the time must also be converted to seconds. 15 minutes = 15(60) = 900 seconds Therefore, $d = 60(900)$ feet.

7. a. Substitute 0 for x in the equation $x-2=5x+6$:

$$0-2=5(0)+6 \Rightarrow -2\neq 6$$

So, 0 is not a solution of the equation.

b. Substitute –2 for x in the equation $x-2=5x+6$:

$$-2-2=5(-2)+6 \Rightarrow -4=-10+6 \Rightarrow -4=-4$$

So, –2 is a solution of the equation.

8. a. Substitute –1 for x in the equation $8x+3=14x-1$:

$$8(-1)+3=14(-1)-1 \Rightarrow -8+3=-14-1 \Rightarrow -5\neq -15$$

So, –1 is not a solution of the equation.

b. Substitute $2/3$ for x in the equation $8x+3=14x-1$:

$$8\left(\frac{2}{3}\right)+3=14\left(\frac{2}{3}\right)-1 \Rightarrow \frac{16}{3}+3=\frac{28}{3}-1 \Rightarrow \frac{25}{3}=\frac{25}{3}$$

So, $2/3$ is a solution of the equation.

9. a. Substitute 4 for x in the equation $\frac{2}{x}=\frac{1}{3}+\frac{1}{x+2}$:

$$\frac{2}{4}=\frac{1}{3}+\frac{1}{4+2} \Rightarrow \frac{1}{2}=\frac{1}{3}+\frac{1}{6} \Rightarrow \frac{1}{2}=\frac{1}{2}$$

So, 4 is a solution of the equation.

b. Substitute 1 for x in the equation $\frac{2}{x}=\frac{1}{3}+\frac{1}{x+2}$:

$$\frac{2}{1}=\frac{1}{3}+\frac{1}{1+2} \Rightarrow 2=\frac{1}{3}+\frac{1}{3} \Rightarrow 2\neq \frac{2}{3}$$

So, 1 is not a solution of the equation.

10. a. Substitute $1/2$ for x in the equation $(x-3)(2x+1)=0$:

$$\left(\frac{1}{2}-3\right)\left(2\cdot\frac{1}{2}+1\right)=0 \Rightarrow \left(-\frac{5}{2}\right)(2)=0 \Rightarrow -5\neq 0$$

So, $1/2$ is not a solution of the equation.

b. Substitute 3 for x in the equation $(x-3)(2x+1)=0$:

$(3-3)(2\cdot 3+1)=0 \Rightarrow (0)(7)=0 \Rightarrow 0=0$

So, 3 is a solution of the equation.

11. a. The equation $2x+3x=5x$ is an identity, so every real number is a solution of the equation. Thus 157 is a solution of the equation. This can be checked by substituting 157 for x in the equation:

$2(157)+3(157)=5(157) \Rightarrow$
$314+471=785 \Rightarrow 785=785$

b. The equation $2x+3x=5x$ is an identity, so every real number is a solution of the equation. Thus -2046 is a solution of the equation. This can be checked by substituting -2046 for x in the equation:

$$2(-2046)+3(-2046)=5(-2046)$$
$$-4092-6138=-10,230$$
$$-10,230=-10,230$$

12. Both sides of the equation $(2-x)-4x=7-3(x+4)$ are defined for all real numbers, so the domain is $(-\infty,\infty)$.

13. The left side of the equation $\frac{y}{y-1}=\frac{3}{y+2}$ is not defined if $y=1$, and the right side of the equation is not defined if $y=-2$. The domain is $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$.

14. The left side of the equation $\frac{1}{y}=2+\sqrt{y}$ is not defined if $y=0$. The right side of the equation is not defined if $y<0$, so the domain is $(0,\infty)$.

15. The left side of the equation $\frac{3x}{(x-3)(x-4)}=2x+9$ is not defined if $x=3$ or $x=4$. The right side is defined for all real numbers. So, the domain is $(-\infty,3)\cup(3,4)\cup(4,\infty)$.

16. The left side of the equation $\frac{1}{\sqrt{x}}=x^2-1$ is not defined if $x\le 0$. The right side of the equation is defined for all real numbers. So the domain is $(0,\infty)$.

17. Substitute 0 for x in $2x+3=5x+1$. Because $3\ne 1$, the equation is not an identity.

18. When the like terms on the right side of the equation $3x+4=6x+2-(3x-2)$ are collected, the equation becomes $3x+4=3x+4$, which is an identity.

19. When the terms on the left side of the equation $\frac{1}{x}+\frac{1}{2}=\frac{2+x}{2x}$ are collected, the equation becomes $\frac{2+x}{2x}=\frac{2+x}{2x}$, which is an identity.

20. The right side of the equation $\frac{1}{x+3}=\frac{1}{x}+\frac{1}{3}$ is not defined for $x=0$, while the left side is defined for $x=0$. Therefore, the equation is not an identity.

In exercises 21–46, solve the equations using the procedures listed on page 79 in your text: eliminate fractions, simplify, isolate the variable term, combine terms, isolate the variable term, and check the solution.

21.
$$3x+5=14$$
$$3x+5-5=14-5$$
$$3x=9$$
$$\frac{3x}{3}=\frac{9}{3}$$
$$x=3$$
Solution set: $\{3\}$

22.
$$2x-17=7$$
$$2x-17+17=7+17$$
$$2x=24$$
$$\frac{2x}{2}=\frac{24}{2}$$
$$x=12$$
Solution set: $\{12\}$

23.
$$-10x+12=32$$
$$-10x+12-12=32-12$$
$$-10x=20$$
$$\frac{-10x}{-10}=\frac{20}{-10}$$
$$x=-2$$
Solution set: $\{-2\}$

24.
$$-2x+5=6$$
$$-2x+5-5=6-5$$
$$-2x=1$$
$$\frac{-2x}{-2}=\frac{1}{-2}\Rightarrow x=-\frac{1}{2}$$
Solution set: $\left\{-\frac{1}{2}\right\}$

25.
$$\begin{aligned} 3-y&=-4\\ 3-y-3&=-4-3\\ -y&=-7\Rightarrow y=7 \end{aligned}$$
Solution set: $\{7\}$

26.
$$\begin{aligned} 2-7y&=23\\ 2-7y-2&=23-2\\ -7y&=21\\ \frac{-7y}{-7}&=\frac{21}{-7}\Rightarrow y=-3 \end{aligned}$$
Solution set: $\{-3\}$

27.
$$\begin{aligned} 7x+7&=2(x+1)\\ 7x+7&=2x+2\\ 7x+7-7&=2x+2-7\\ 7x&=2x-5\\ 7x-2x&=2x-5-2x\\ 5x&=-5\\ \frac{5x}{5}&=\frac{-5}{5}\Rightarrow x=-1 \end{aligned}$$
Solution set: $\{-1\}$

28.
$$\begin{aligned} 3(x+2)&=4-x\\ 3x+6&=4-x\\ 3x+6+x&=4-x+x\\ 4x+6&=4\\ 4x+6-6&=4-6\\ 4x&=-2\\ \frac{4x}{4}&=\frac{-2}{4}\Rightarrow x=-\frac{1}{2} \end{aligned}$$
Solution set: $\left\{-\frac{1}{2}\right\}$

29.
$$\begin{aligned} 3(2-y)+5y&=3y\\ 6-3y+5y&=3y\\ 6+2y&=3y\\ 6+2y-2y&=3y-2y\Rightarrow 6=y \end{aligned}$$
Solution set: $\{6\}$

30.
$$\begin{aligned} 9y-3(y-1)&=6+y\\ 9y-3y+3&=6+y\\ 6y+3&=6+y\\ 6y+3-y&=6+y-y\\ 5y+3&=6\\ 5y+3-3&=6-3\\ 5y&=3\\ \frac{5y}{5}&=\frac{3}{5}\Rightarrow y=\frac{3}{5} \end{aligned}$$
Solution set: $\left\{\frac{3}{5}\right\}$

31. $4y-3y+7-y=2-(7-y)$
Distribute -1 to clear the parentheses.
$$\begin{aligned} 7&=2-7+y\\ 7&=-5+y\\ 7+5&=-5+y+5\Rightarrow 12=y \end{aligned}$$
Solution set: $\{12\}$

32.
$$\begin{aligned} 3(y-1)&=6y-4+2y-4y\\ 3y-3&=4y-4\\ 3y-3+3&=4y-4+3\\ 3y&=4y-1\\ 3y-4y&=4y-1-4y\\ -y&=-1\Rightarrow y=1 \end{aligned}$$
Solution set: $\{1\}$

33. $3(x-2)+2(3-x)=1$
$$3x-6+6-2x=1\Rightarrow x=1$$
Solution set: $\{1\}$

34. $2x-3-(3x-1)=6$
Distribute -1 to clear the parentheses.
$$\begin{aligned} 2x-3-3x+1&=6\\ -x-2&=6\\ -x-2+2&=6+2\\ -x&=8\Rightarrow x=-8 \end{aligned}$$
Solution set: $\{-8\}$

35.
$$\begin{aligned} 2x+3(x-4)&=7x+10\\ 2x+3x-12&=7x+10\\ 5x-12&=7x+10\\ 5x-12+12&=7x+10+12\\ 5x&=7x+22\\ 5x-7x&=7x+22-7x\\ -2x&=22\\ \frac{-2x}{-2}&=\frac{22}{-2}\Rightarrow x=-11 \end{aligned}$$
Solution set: $\{-11\}$

36.
$$\begin{aligned} 3(2-3x)-4x&=3x-10\\ 6-9x-4x&=3x-10\\ 6-13x&=3x-10\\ 6-13x-6&=3x-10-6\\ -13x&=3x-16\\ -13x-3x&=3x-16-3x\\ -16x&=-16\\ \frac{-16x}{-16}&=\frac{-16}{-16}\Rightarrow x=1 \end{aligned}$$
Solution set: $\{1\}$

37. $4[x+2(3-x)]=2x+1$
Distribute 2 to clear the inner parentheses.
$$4[x+6-2x]=2x+1$$
Combine like terms within the brackets.
$$4[6-x]=2x+1$$
Distribute 4 to clear the brackets.
$$\begin{aligned} 24-4x&=2x+1\\ 24-4x-24&=2x+1-24\\ -4x&=2x-23\\ -4x-2x&=-23\\ -6x&=-23\\ \frac{-6x}{-6}&=\frac{-23}{-6}\Rightarrow x=\frac{23}{6} \end{aligned}$$
Solution set: $\left\{\frac{23}{6}\right\}$

38. $3-[x-3(x+2)]=4$

Distribute -3 to clear the parentheses.

$3-[x-3x-6]=4$

Combine like terms in the brackets.

$3-[-2x-6]=4$

Distribute -1 to clear the brackets.

$$\begin{aligned}3+2x+6&=4\\2x+9&=4\\2x+9-9&=4-9\\2x&=-5\\\frac{2x}{2}&=\frac{-5}{2}\Rightarrow x=-\frac{5}{2}\end{aligned}$$

Solution set: $\left\{-\frac{5}{2}\right\}$

39. $3(4y-3)=4[y-(4y-3)]$

Distribute 3 on the left side and -1 on the right side to clear parentheses.

$12y-9=4[y-4y+3]$

Combine like terms in the brackets.

$12y-9=4[-3y+3]$

Distribute 4 to clear the brackets.

$$\begin{aligned}12y-9&=-12y+12\\12y-9+9&=-12y+12+9\\12y&=-12y+21\\12y+12y&=-12y+21+12y\\24y&=21\\\frac{24y}{24}&=\frac{21}{24}\Rightarrow y=\frac{21}{24}=\frac{7}{8}\end{aligned}$$

Solution set: $\left\{\frac{7}{8}\right\}$

40. $5-(6y+9)+2y=2(y+1)$

Distribute -1 on the left and 2 on the right to clear the parentheses.

$$\begin{aligned}5-6y-9+2y&=2y+2\\-4-4y&=2y+2\\-4-4y+4&=2y+2+4\\-4y&=2y+6\\-4y-2y&=2y+6-2y\\-6y&=6\\\frac{-6y}{-6}&=\frac{6}{-6}\\y&=-1\end{aligned}$$

Solution set: $\{-1\}$

41. $2x-3(2-x)=(x-3)+2x+1$

Distribute -3 on the left to clear the parentheses.

$$\begin{aligned}2x-6+3x&=x-3+2x+1\\5x-6&=3x-2\\5x-6+6&=3x-2+6\\5x&=3x+4\\5x-3x&=3x+4-3x\\2x&=4\\\frac{2x}{2}&=\frac{4}{2}\Rightarrow x=2\end{aligned}$$

Solution set: $\{2\}$

42. $5(x-3)-6(x-4)=-5$

Distribute 5 to clear the first set of parentheses. Distribute -6 to clear the second set of parentheses.

$$\begin{aligned}5x-15-6x+24&=-5\\-x+9&=-5\\-x+9-9&=-5-9\\-x&=-14\Rightarrow x=14\end{aligned}$$

Solution set: $\{14\}$

43. $\frac{2x+1}{9}-\frac{x+4}{6}=1$

To clear the fractions, multiply both sides of the equation by the least common denominator, 36.

$$\begin{aligned}36\left(\frac{2x+1}{9}-\frac{x+4}{6}\right)&=36(1)\\4(2x+1)-6(x+4)&=36\\8x+4-6x-24&=36\\2x-20&=36\\2x-20+20&=36+20\\2x&=56\\\frac{2x}{2}&=\frac{56}{2}\Rightarrow x=28\end{aligned}$$

Solution set: $\{28\}$

44. $\frac{2-3x}{7}+\frac{x-1}{3}=\frac{3x}{7}$

To clear the fractions, multiply both sides of the equation by the least common denominator, 21.

$$\begin{aligned}21\left(\frac{2-3x}{7}+\frac{x-1}{3}\right)&=21\left(\frac{3x}{7}\right)\\3(2-3x)+7(x-1)&=3(3x)\\6-9x+7x-7&=9x\\-1-2x&=9x\\-1-2x+2x&=9x+2x\\-1&=11x\\\frac{-1}{11}&=\frac{11x}{11}\Rightarrow x=-\frac{1}{11}\end{aligned}$$

Solution set: $\left\{-\frac{1}{11}\right\}$

45. $$\frac{1-x}{4}+\frac{5x+1}{2}=3-\frac{2(x+1)}{8}$$
To clear the fractions, multiply both sides by the least common denominator, 8.
$$8\left(\frac{1-x}{4}+\frac{5x+1}{2}\right)=8\left(3-\frac{2(x+1)}{8}\right)$$
Distribute the 8 on both sides.
$$8\left(\frac{1-x}{4}\right)+8\left(\frac{5x+1}{2}\right)=8(3)-8\left(\frac{2(x+1)}{8}\right)$$
$$2(1-x)+4(5x+1)=8(3)-2(x+1)$$
Simplify by collecting like terms and combining constants.
$$2-2x+20x+4=24-2x-2$$
$$18x+6=22-2x$$
$$18x+6+2x=22-2x+2x$$
$$20x+6=22$$
$$20x+6-6=22-6$$
$$20x=16$$
$$\frac{20x}{20}=\frac{16}{20}$$
$$x=\frac{16}{20}=\frac{4}{5}$$
Solution set: $\left\{\frac{4}{5}\right\}$

46. $$\frac{x+4}{3}+2x-\frac{1}{2}=\frac{3x+2}{6}$$
To clear the fractions, multiply both sides of the equation by the least common denominator, 6.
$$6\left(\frac{x+4}{3}+2x-\frac{1}{2}\right)=6\left(\frac{3x+2}{6}\right)$$
Distribute the 6 on both sides.
$$6\left(\frac{x+4}{3}\right)+6(2x)-6\left(\frac{1}{2}\right)=6\left(\frac{3x+2}{6}\right)$$
$$2(x+4)+12x-3=3x+2$$
Simplify by collecting like terms and combining constants.
$$2x+8+12x-3=3x+2$$
$$14x+5=3x+2$$
$$14x+5-3x=3x+2-3x$$
$$11x+5=2$$
$$11x+5-5=2-5$$
$$11x=-3$$
$$\frac{11x}{11}=\frac{-3}{11}\Rightarrow x=-\frac{3}{11}$$
Solution set: $\left\{-\frac{3}{11}\right\}$

47. To solve $d=rt$ for r, divide both sides of the equation by t. $r=\frac{d}{t}$.

48. To solve $F=ma$ for a, divide both sides of the equation by m. $a=\frac{F}{m}$.

49. To solve $C=2\pi r$ for r, divide both sides of the equation by 2π. $r=\frac{C}{2\pi}$.

50. To solve $A=2\pi rx+\pi r^2$ for x, subtract πr^2 from both sides.
$$A-\pi r^2=2\pi rx+\pi r^2-\pi r^2$$
$$A-\pi r^2=2\pi rx$$
Divide both sides by $2\pi r$.
$$\frac{A-\pi r^2}{2\pi r}=\frac{2\pi rx}{2\pi r}$$
$$\frac{A-\pi r^2}{2\pi r}=x$$

51. To solve $I=\frac{E}{R}$ for R, multiply both sides by R.
$$RI=R\left(\frac{E}{R}\right)\Rightarrow RI=E$$
Divide both sides by I.
$$\frac{RI}{I}=\frac{E}{I}\Rightarrow R=\frac{E}{I}$$

52. To solve $A=P(1+rt)$ for t, distribute P.
$$A=P+Prt$$
Subtract P from both sides.
$$A-P=P+Prt-P$$
$$A-P=Prt$$
Divide both sides by Pr.
$$\frac{A-P}{Pr}=\frac{Prt}{Pr}$$
$$\frac{A-P}{Pr}=t$$

53. To solve $A=\frac{(a+b)h}{2}$ for h, multiply both sides by 2.
$$2A=(a+b)h$$
Divide both sides by $(a+b)$.
$$\frac{2A}{a+b}=\frac{(a+b)h}{a+b}\Rightarrow\frac{2A}{a+b}=h$$

54. To solve $T = a + (n-1)d$ for d, subtract a from both sides.

$$T - a = a + (n-1)d - a$$
$$T - a = (n-1)d$$

Divide both sides by $(n-1)$.

$$\frac{T-a}{n-1} = \frac{(n-1)d}{n-1} \Rightarrow \frac{T-a}{n-1} = d$$

55. To solve $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ for u, clear the fractions by multiplying both sides by the least common denominator, fuv.

$$fuv\left(\frac{1}{f}\right) = fuv\left(\frac{1}{u} + \frac{1}{v}\right)$$
$$fuv\left(\frac{1}{f}\right) = fuv\left(\frac{1}{u}\right) + fuv\left(\frac{1}{v}\right)$$

Simplify.

$$uv = fv + fu$$

Subtract fu from both sides.

$$uv - fu = fv + fu - fu$$
$$uv - fu = fv$$

Factor the left side.

$$u(v-f) = fv$$

Divide both sides by $v - f$.

$$\frac{u(v-f)}{v-f} = \frac{fv}{v-f} \Rightarrow u = \frac{fv}{v-f}$$

56. To solve $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R_2, clear the fractions by multiplying both sides by the least common denominator, $R \bullet R_1 \bullet R_2$.

$$R \cdot R_1 \cdot R_2\left(\frac{1}{R}\right) = R \cdot R_1 \cdot R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
$$R \cdot R_1 \cdot R_2\left(\frac{1}{R}\right) = R \cdot R_1 \cdot R_2\left(\frac{1}{R_1}\right) + R \bullet R_1 \bullet R_2\left(\frac{1}{R_2}\right)$$

Simplify.

$$R_1R_2 = RR_2 + RR_1$$

Subtract RR_2 from both sides.

$$R_1R_2 - RR_2 = RR_2 + RR_1 - RR_2$$
$$R_1R_2 - RR_2 = RR_1$$

Factor the left side.

$$R_2(R_1 - R) = RR_1$$

Divide both sides by $(R_1 - R)$.

$$\frac{R_2(R_1-R)}{R_1-R} = \frac{RR_1}{R_1-R} \Rightarrow R_2 = \frac{RR_1}{R_1-R}$$

57. To solve $y = mx + b$ for m, subtract b from both sides.

$$y - b = mx + b - b \Rightarrow y - b = mx$$

Divide both sides by x.

$$\frac{y-b}{x} = \frac{mx}{x} \Rightarrow \frac{y-b}{x} = m$$

58. To solve $ax + by = c$ for y, subtract ax from both sides.

$$ax + by - ax = c - ax \Rightarrow by = c - ax$$

Divide both sides by b.

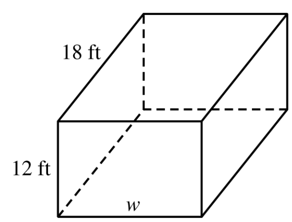
$$\frac{by}{b} = \frac{c-ax}{b} \Rightarrow y = \frac{c-ax}{b}$$

59. $0.065x$ **60.** $\frac{5}{6}x$

61. $\$22,000 - x$ **62.** $\frac{229.50 - 72}{x}$

1.1 Applying the Concepts

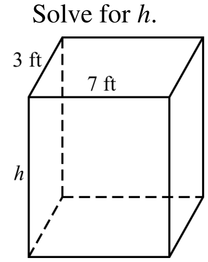
63. The formula for volume is $V = lwh$. Substitute 2808 for V, 18 for l, and 12 for h. Solve for w.



$$2808 = 18 \cdot 12 \cdot w$$
$$2808 = 216w$$
$$\frac{2808}{216} = \frac{216w}{216}$$
$$13 = w$$

The width of the pool is 13 ft.

64. The formula for volume is $V = lwh$. Substitute 168 for V, 7 for l, and 3 for w. Solve for h.



$$168 = 7 \cdot 3 \cdot h$$
$$168 = 21h$$
$$\frac{168}{21} = \frac{21h}{21}$$
$$8 = h$$

The hole must be 8 ft deep.

65. Let w = the width of the rectangle. Then $2w - 5$ = the length of the rectangle.

$$2w + 2(2w-5) = 80$$
$$2w + 4w - 10 = 80$$
$$6w - 10 = 80$$
$$6w = 90$$
$$w = 15,\ 2w - 5 = 25$$

The width of the rectangle is 15 ft and its length is 25 feet.

66. Let l = the length of the rectangle.

Then $3+\frac{1}{2}l =$ the width of the rectangle.

$$2l+2\left(3+\frac{1}{2}l\right)=36$$
$$2l+6+l=36$$
$$3l+6=36$$
$$3l=30$$
$$l=10,\ 3+\frac{1}{2}l=8$$

The length of the rectangle is 10 ft and its width is 8 ft.

67. The formula for circumference of a circle is $C=2\pi r$. Substitute 114π for C. Solve for r.

$$114\pi=2\pi r \Rightarrow \frac{114\pi}{2\pi}=\frac{2\pi r}{2\pi}\Rightarrow 57=r$$

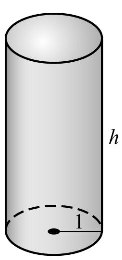
The radius is 57 cm.

68. The formula for perimeter of a rectangle is $P=2l+2w$. Substitute 28 for P and 5 for w. Solve for l.

$$28=2l+2(5)$$
$$28=2l+10$$
$$28-10=2l+10-10$$
$$18=2l$$
$$\frac{18}{2}=\frac{2l}{2}$$
$$9=l$$

The length is 9 m.

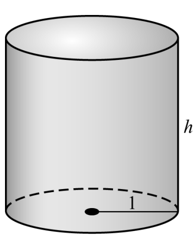
69. The formula for surface area of a cylinder is $S=2\pi rh+2\pi r^2$. Substitute 6π for S and 1 for r. Solve for h.



$$6\pi=2\pi(1)h+2\pi(1^2)$$
$$6\pi=2\pi h+2\pi$$
$$6\pi-2\pi=2\pi h+2\pi-2\pi$$
$$4\pi=2\pi h$$
$$\frac{4\pi}{2\pi}=\frac{2\pi h}{2\pi}\Rightarrow 2=h$$

The height is 2 m.

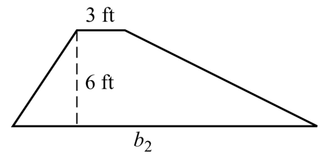
70. The formula for volume of a cylinder is $V=\pi r^2 h$. Substitute 148π for V and 2 for r. Solve for h.



$$148\pi=\pi\cdot 2^2\cdot h$$
$$148\pi=4\pi h$$
$$\frac{148\pi}{4\pi}=\frac{4\pi h}{4\pi}$$
$$37=h$$

The height of the can is 37 cm.

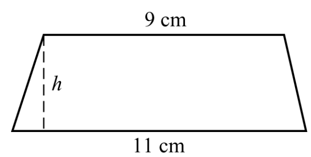
71. The formula for area of a trapezoid is $A=\frac{1}{2}h\left(b_1+b_2\right)$. Substitute 66 for A, 6 for h, and 3 for b_1. Solve for b_2.



$$66=\frac{1}{2}\cdot 6\left(3+b_2\right)$$
$$66=3\left(3+b_2\right)$$
$$66=9+3b_2$$
$$66-9=9+3b_2-9$$
$$57=3b_2$$
$$\frac{57}{3}=\frac{3b_2}{3}$$
$$19=b_2$$

The length of the second base is 19 ft.

72. The formula for area of a trapezoid is $A=\frac{1}{2}h\left(b_1+b_2\right)$. Substitute 35 for A, 9 for b_1, and 11 for b_2. Solve for h.



$$35=\frac{1}{2}h(9+11)$$
$$35=\frac{1}{2}h(20)$$
$$35=10h$$
$$\frac{35}{10}=\frac{10h}{10}\Rightarrow 3.5=h$$

The height of the trapezoid is 3.5 cm.

73. Let $x =$ the cost of the less expensive land. Then x + 23,000 = the cost of the more expensive land. Together they cost $147,000, so

$$x+(x+23{,}000)=147{,}000$$
$$2x+23{,}000=147{,}000$$
$$2x=124{,}000\Rightarrow x=62{,}000$$

The less expensive piece of land costs $62,000 and the more expensive piece of land costs $62,000 + $23,000 = $85,000.

74. Let x = the amount the assistant manager earns. Then x + 450 = the amount the manager earns. Together they earn $3700, so

$$x+(x+450)=3700$$
$$2x+450=3700$$
$$2x=3250\Rightarrow x=1625$$

The assistant manager earns $1625, and the manager earns $1625 + $450 = $2075.

75. Let x = the lottery ticket sales in July. Then $1.10x$ = the lottery ticket sales in August. A total of 1113 tickets were sold, so

$$x + 1.10x = 1113$$
$$2.10x = 1113 \Rightarrow x = 530$$

530 tickets were sold in July, and 1.10(530) = 583 tickets were sold in August.

76. Let x = Jan's commission in March. Then $15 + 0.5x$ = Jan's commission in February. She earned a total of $633, so

$$x + (15 + 0.5x) = 633$$
$$1.5x + 15 = 633 \Rightarrow 1.5x = 618 \Rightarrow x = 412$$

Jan's commission was $412 in March and 15 + 0.5(412) = $221 in February.

77. Let x = the amount the younger son receives. Then $4x$ = the amount the older son receives. Together they receive $225,000, so

$$x + 4x = 225{,}000 \Rightarrow 5x = 225{,}000 \Rightarrow$$
$$x = 45{,}000$$

The younger son will received $45,000, and the older son will receive 4($45,000) = $180,000.

78. Let x = the amount Kevin kept for himself. Then $x/2$ = the amount he gave his daughter, and $x/4$ = the amount he gave his dad. He won $735,000, so

$$x + \frac{x}{2} + \frac{x}{4} = 735{,}000$$
$$4\left(x + \frac{x}{2} + \frac{x}{4}\right) = 4(735{,}000)$$
$$4x + 2x + x = 2{,}940{,}000$$
$$7x = 2{,}940{,}000 \Rightarrow x = 420{,}000$$

Kevin kept $420,000 for himself. He gave $\$420{,}000/2 = \$210{,}000$ to his daughter and $\$420{,}000/4 = \$105{,}000$ to his dad.

79. a. Let x = the number of points needed to average 75.

$$\frac{87 + 59 + 73 + x}{4} = 75$$
$$219 + x = 300$$
$$x = 81$$

You need to score 81 in order to average 75.

b.

$$\frac{87 + 59 + 73 + 2x}{5} = 75$$
$$219 + 2x = 375$$
$$2x = 156$$
$$x = 78$$

You need to score 78 in order to average 75 if the final carries double weight.

80. Let x = the amount invested in real estate. Then $4200 - x$ = the amount invested in a savings and loan.

Investment	Principal	Rate	Time	Interest
Real estate	x	0.15	1	$0.15x$
Savings	$4200 - x$	0.08	1	$0.08(4200 - x)$

The total income was $448, so

$$0.15x + 0.08(4200 - x) = 448$$
$$0.15x + 336 - 0.08x = 448$$
$$0.07x + 336 = 448$$
$$0.07x = 112 \Rightarrow x = 1600$$

So, the real estate agent invested $1600 in real estate and 4200 – 1600 = $2600 in a savings and loan.

81. Let x = the amount invested in a tax shelter. Then $7000 - x$ = the amount invested in a bank.

Investment	Principal	Rate	Time	Interest
Tax shelter	x	0.09	1	$0.09x$
Bank	$7000 - x$	0.06	1	$0.06(7000 - x)$

The total interest was $540, so

$$0.09x + 0.06(7000 - x) = 540$$
$$0.09x + 420 - 0.06x = 540$$
$$0.03x + 420 = 540$$
$$0.03x = 120 \Rightarrow x = 4000$$

Mr. Mostafa invested $4000 in a tax shelter and 7000 – 4000 = $3000 in a bank.

82. Let x = the amount invested at 6%. Then $4900 - x$ = the amount invested at 8%

Principal	Rate	Time	Interest
x	0.06	1	$0.06x$
$4900 - x$	0.08	1	$0.08(4900 - x)$

The amount of interest for each investment is equal, so

$$0.06x = 0.08(4900 - x)$$
$$0.06x = 392 - 0.08x$$
$$0.14x = 392 \Rightarrow x = 2800$$

Ms. Jordan invested $2800 at 6% and $2100 at 8%. The amount of interest she earned on each investment is $168, so she earned $336 in all.

83. Let x = the amount to be invested at 8%.

Principal	Rate	Time	Interest
5000	0.05	1	250
x	0.08	1	$0.08x$
$5000 + x$	0.06	1	$0.06(5000 + x)$

The amount of interest for the total investment is the sum of the interest earned on the individual investments, so

$$\begin{aligned} 0.06(5000+x) &= 250+0.08x \\ 300+0.06x &= 250+0.08x \\ 50+0.06x &= 0.08x \\ 50 = 0.02x &\Rightarrow 2500 = x \end{aligned}$$

So, \$2500 must be invested at 8%.

84. Let x = the selling price. Then $x - 480$ = the profit. So $x-480 = 0.2x \Rightarrow -480 = -0.8x \Rightarrow 600 = x$. The selling price is \$600.

85. There is a profit of \$2 on each shaving set. They want to earn \$40,000 + \$30,000 = \$70,000. Let x = the number of shaving sets to be sold. Then $2x$ = the amount of profit for x shaving sets. So, $2x = 70,000 \Rightarrow x = 35,000$ They must sell 35,000 shaving sets.

86. Let t = the time each traveled.

Then $\frac{100}{t}$ = Angelina's rate and $\frac{150}{t}$ = Harry's rate.

	Rate	Time	Distance
Angelina	$\frac{100}{t}$	t	100
Harry	$\frac{150}{t}$	t	150

Harry's rate is 15 meters per minute faster than Angelina's, so we have

$$\begin{aligned} \frac{100}{t}+15 &= \frac{150}{t} \\ 100+15t &= 150 \\ 15t &= 50 \\ t &= \frac{10}{3} \text{ min} \end{aligned}$$

So, Angelina jogged at $\frac{100}{10/3} = 30$ meters per minute. Harry biked at $\frac{150}{10/3} = 45$ meters per minute.

87. Let x = the time the second car travels. Then $1 + x$ = the time the first car travels. So,

	Rate	Time	Distance
First car	50	$1 + x$	$50(1+x)$
Second car	70	x	$70x$

The distances are equal, so

$$\begin{aligned} 50(1+x) &= 70x \\ 50+50x &= 70x \\ 50 = 20x &\Rightarrow 2.5 = x \end{aligned}$$

So, it will take the second car 2.5 hours to overtake the first car.

88. Let x = the time the planes travel. So,

	Rate	Time	Distance
First plane	470	x	$470x$
Second plane	430	x	$430x$

The planes are 2250 km apart, so

$$470x+430x = 2250 \Rightarrow 900x = 2250 \Rightarrow x = 2.5$$

So, the planes will be 2250 km apart at 2.5 hours.

89. At 20 miles per hour, it will take Lucas two minutes to bike the remaining 2/3 of a mile. $\left(\frac{20\text{ mi}}{1\text{ hr}} = \frac{20\text{ mi}}{60\text{ min}} = \frac{1\text{ mi}}{3\text{ min}}\right)$ So his brother will have to bike 1 mile in 2 minutes:

$$\frac{1\text{ mi}}{2\text{ min}} = \frac{30\text{ mi}}{60\text{ min}} = \frac{30\text{ mi}}{1\text{ hr}}$$

90. Driving at 40 miles per hour, it will take Karen's husband $45/40$ hours or 1 hour and 7.5 minutes to get to the airport. Driving at 60 miles per hour, it will take Karen 45 minutes to get to the airport. Her husband has already driven for 15 minutes, so it will take him an additional 52.5 minutes to get to the airport. Karen will get there before he does.

91. Let x = the rate the slower car travels. Then $x + 5$ = the rate the faster car travels. So,

	Rate	Time	Distance
First car	x	3	$3x$
Second car	$x + 5$	3	$3(x+5)$

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The cars are 405 miles apart, so

$$3x+3(x+5)=405$$
$$3x+3x+15=405$$
$$6x+15=405 \Rightarrow 6x=390 \Rightarrow x=65$$

One car is traveling at 65 miles per hour, and the other car is traveling at 70 miles per hour.

92. Let x = the distance to Aya's friend's house.

	Rate	Distance	Time
go	16	x	$\frac{x}{16}$
return	80	x	$\frac{x}{80}$

She traveled for a total of 3 hours, so

$$\frac{x}{16}+\frac{x}{80}=3$$
$$80\left(\frac{x}{16}+\frac{x}{80}\right)=80(3)$$
$$5x+x=240 \Rightarrow 6x=240 \Rightarrow x=40$$

So, her friend lives 40 km away.

93. Substitute 170 for P into the formula $P=200-0.02q$. Solve for q.

$$170=200-0.02q$$
$$170-200=200-0.02q-200$$
$$-30=-0.02q$$
$$\frac{-30}{-0.02}=\frac{-0.02q}{-0.02} \Rightarrow 1500=q$$

Note that the solution must fall between 100 and 2000 cameras. 1500 cameras must be ordered.

94. Substitute 37,000 for Q, 1500 for L, and 3200 for I into the formula $Q=\frac{A-I}{L}$. Solve for A.

$$37,000=\frac{A-3200}{1500}$$
$$1500(37,000)=1500\left(\frac{A-3200}{1500}\right)$$
$$55,500,000=A-3200$$
$$55,500,000+3200=A-3200+3200$$
$$55,503,200=A$$

The current assets are \$55,503,200.

95. Let x = one number. Then $3x$ = the other number.

$$x+3x=28 \Rightarrow 4x=28 \Rightarrow x=7,\ 3x=3(7)=21$$

The numbers are 7 and 21.

96. Let x = the first even integer. Then $x + 2$ = the second even integer, and $x + 4$ = the third even integer.

$$x+(x+2)+(x+4)=42$$
$$3x+6=42$$
$$3x=36 \Rightarrow x=12$$
$$x+2=14,\ x+4=16$$

The numbers are 12, 14, and 16.

97. Let x = one number. Then $2x$ is the second number.

$$2x-x=14$$
$$x=14$$

The numbers are 14 and 28.

98. Let x = one number. Then $x + 5$ = the second number. Note that the second number is the larger number.

$$x+2(x+5)=49$$
$$x+2x+10=49$$
$$3x+10=49$$
$$3x=39$$
$$x=13$$

The numbers are 13 and 18.

1.1 Beyond the Basics

99. a. The solution set of $x^2=x$ is $\{0,1\}$, while the solution set of $x=1$ is $\{1\}$. Therefore, the equations are not equivalent.

b. The solution set of $x^2=9$ is $\{-3,3\}$, while the solution set of $x=3$ is $\{3\}$. Therefore, the equations are not equivalent.

c. The solution set of $x^2-1=x-1$ is $\{0, 1\}$, while the solution set of $x=0$ is $\{0\}$. Therefore, the equations are not equivalent.

d. The equation $\frac{x}{x-2}=\frac{2}{x-2}$ is an inconsistent equation, so is solution set is $\varnothing$. The solution set of $x=2$ is $\{2\}$. Therefore, the equations are not equivalent.

100. First, solve $7x+2=16$. Subtracting 2 from both sides, we have $7x=14$. Then divide both sides by 7; we obtain $x=2$. Now substitute 2 for x in $3x-1=k$. This becomes $3(2)-1=k$, so $k=5$.

101. Let x = the average speed for the second half of the trip.

	Rate	Distance	Time
1st half	75	D	$\frac{D}{75}$
2nd half	x	D	$\frac{D}{x}$
Entire trip	60	$2D$	$\frac{2D}{60}$

$$\frac{D}{75}+\frac{D}{x}=\frac{2D}{60}$$
$$300x\left(\frac{D}{75}+\frac{D}{x}\right)=300x\left(\frac{2D}{60}\right)$$
$$4Dx+300D=5x(2D)$$
$$4Dx+300D=10Dx$$
$$300D=6Dx\Rightarrow 50=x$$

The average speed for the second half of the drive is 50 mph.

102. First we need to compute how much time it will take for Davinder and Mikhael to meet. Let t = the time it will take for them to meet. So,

	Rate	Time	Distancee
Davinder	3.7	t	$3.7t$
Mikhail	4.3	t	$4.3t$

$$3.7t+4.3t=2\Rightarrow 8t=2\Rightarrow t=0.25$$

They will be walking for 0.25 hour until they meet.

The dog starts with Davinder. Let $t_{d1}=$ the amount of time it takes for the dog to meet Mikhail. So,

	Rate	Time	Distance
dog	6	t_{d1}	$6t_{d1}$
Mikhail	4.3	t_{d1}	$4.3t_{d1}$

$$6t_{d1}+4.3t_{d1}=2$$
$$10.3t_{d1}=2$$
$$t=0.19$$

The dog meets Mikhail for the first time when they have walked for 0.19 hour. The dog will have traveled 1.14 mi.
While the dog has been running towards Mikhail, Davinder has continued to walk. During the 0.19 hour, he walked 0.70 mi, so now he and the dog are 1.14 – 0.70 = 0.44 mi apart. Let $t_{d2}=$ the time it takes the dog to meet Davinder.

	Rate	Time	Distance
dog	6	t_{d2}	$6t_{d2}$
Davinder	3.7	t_{d2}	$3.7t_{d2}$

$$6t_{d2}+3.7t_{d2}=0.44$$
$$9.7t_{d2}=0.44$$
$$t=0.05$$

So, the dog meets Davinder when they have walked for another 0.05 hour. The dog will have traveled 0.3 mi. They have now walked for 0.19 + 0.05 = 0.24 hr. Since Davinder and Mikhail don't meet until they have walked for 0.25 hours, the dog must walk for 0.01 hr more. In that time, the dog will travel 0.06 mi. So in total the dog will travel 1.14 + 0.3 + 0.06 = 1.5 mi.

103. Let x = the number of liters of water in the original mixture. Then $5x$ = the number of liters of alcohol in the original mixture, and $6x$ = the total number of liters in the original mixture. $x + 5$ = the number of liters of water in the new mixture. Then $6x + 5$ = the total number of liters in the new mixture. Since the ratio of alcohol to water in the new mixture is 5:2, then the amount of alcohol in the new mixture is 5/7 of the total mixture or $\frac{5}{7}(6x+5)$.

There was no alcohol added, so the amount of alcohol in the original mixture equals the amount of alcohol in the new mixture. This gives

$$\frac{5}{7}(6x+5)=5x$$
$$5(6x+5)=35x$$
$$30x+25=35x\Rightarrow 25=5x\Rightarrow 5=x$$

So, there were 5 liters of water in the original mixture and 25 liters of alcohol.

104. Let x = the amount of each alloy. There are 13 parts in the first alloy and 8 parts in the second alloy. We can use the following table to organize the information:

	Total	Zinc	Copper
Alloy 1	x	$\frac{5x}{13}$	$\frac{8x}{13}$
Alloy 2	x	$\frac{5x}{8}$	$\frac{3x}{8}$
Total	$2x$	$\frac{5x}{13}+\frac{5x}{8}$	$\frac{8x}{13}+\frac{3x}{8}$

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The amount of zinc in the new mixture is $\frac{5x}{13}+\frac{5x}{8}=\frac{105x}{104}$, and the amount of copper in the new mixture is $\frac{8x}{13}+\frac{3x}{8}=\frac{103x}{104}$.
So, the ratio of zinc to copper in the new mixture is $\frac{105x}{104}:\frac{103x}{104}$ or 105:103.

105. Let x = Democratus' age now. Then $x/6$ = the number of years as a boy, $x/8$ = the number of years as a youth, and $x/2$ = the number of years as a man. He has spent 15 years as a mature adult. So,

$$\begin{aligned}\frac{x}{6}+\frac{x}{8}+\frac{x}{2}+15&=x\\24\left(\frac{x}{6}+\frac{x}{8}+\frac{x}{2}+15\right)&=24x\\4x+3x+12x+360&=24x\\19x+360&=24x\\360=5x&\Rightarrow 72=x\end{aligned}$$

Democratus is 72 years old.

106. Let x = the man's age now. When the woman is x years old, the man will be $119-x$ years old. So the difference in their ages is $(119-x)-x=119-2x$ years. So the woman's age now is $x-(119-2x)=3x-119$. When the man was $3x-119$ years old, she was $\frac{3x-119}{2}$ years old. Since the difference in their ages is $119-2x$, we have

$$\begin{aligned}(3x-119)-\frac{3x-119}{2}&=119-2x\\6x-238-3x+119&=238-4x\\3x-119&=238-4x\\7x-119&=238\\7x=357&\Rightarrow x=51\end{aligned}$$

So the man is now 51 years old.
Check by verifying the facts in the problem. When she is 51 years old, he will be 119 – 51 = 68 years old. The difference in their ages is 68 – 51 = 17 years. So she is 51 – 17 = 34 years old now. When he was 34 years old, she was 17 years old, which is 1/2 of 34.

107. There are 180 minutes from 3 p.m. to 6 p.m. So, the number of minutes before 6 p.m. plus 50 minutes plus 4 × the number of minutes before 6 p.m. equals 180 minutes. Let x = the number of minutes before 6 p.m. So,
$x+50+4x=180\Rightarrow 5x+50=180\Rightarrow$
$5x=130\Rightarrow x=26$
So it is 26 minutes before 6 p.m. or 5:34 p.m. Check this by verifying that 26 + 50 = 76 minutes before 6 p.m. is the same time as 4(26) = 104 minutes after 3 p.m. Seventy-six minutes before 6 p.m. is 4:44 p.m., while 104 minutes after 3 p.m. is also 4:44 p.m.

108. Let x = the number of minutes pipe B is open. Pipe A is open for 18 minutes, so it fills 18/24 or 3/4 of the tank. Pipe B fills x/32 of the tank.
So, $\frac{3}{4}+\frac{x}{32}=1\Rightarrow 32\left(\frac{3}{4}+\frac{x}{32}\right)=32(1)\Rightarrow$
$24+x=32\Rightarrow x=8$
Pipe B should be turned off after 8 minutes.

109. a. Because of the head wind, the plane flies at 140 mph from Atlanta to Washington and 160 mph from Washington to Atlanta. Let x = the distance the plane flew before turning back. So,

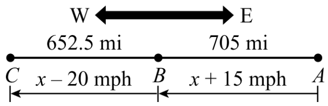
	Rate	Distance	Time
to	140	x	$x/140$
from	160	x	$x/160$

$$\begin{aligned}\frac{x}{140}+\frac{x}{160}&=1.5\\160x+140x&=1.5(140)(160)\\300x=33,600&\Rightarrow x=112\end{aligned}$$

The plane flew 112 miles before turning back.

b. The plane traveled 224 miles in 1.5 hours, so the average speed is $\frac{224}{1.5}=149.33$ mph.

110. Let x = the airspeed of the plane. Because of the wind, the actual speed of the plane between airports A and B is $x + 15$. The actual speed of the plane between airports B and C is $x - 20$.



	Rate	Distance	Time
A to B	$x + 15$	705	$\frac{705}{x+15}$
B to C	$x - 20$	652.5	$\frac{652.5}{x-20}$

The times are the same, so we have

$$\begin{aligned} \frac{705}{x+15} &= \frac{652.5}{x-20} \\ 705(x-20) &= 652.5(x+15) \\ 705x - 14{,}100 &= 652.5x + 9787.5 \\ 52.5x - 14{,}100 &= 9787.5 \\ 52.5x &= 23887.5 \\ x &= 455 \end{aligned}$$

The airspeed of the plane is 455 mph.

1.1 Critical Thinking/Discussion/Writing

111. If x represents the amount the pawn shop owner paid for the first watch and the owner made a profit of 10%, then $1.1x = 499$, so $x = 453.64$. If y represents the amount the pawn shop owner paid for the second watch and the owner lost 10%, then $0.9y = 499$, so $y = 554.44$. Together the two watches cost \$453.64 + \$554.44 = \$1008.08. But the pawn shop owner sold the two watches for \$998, so there was a loss. The amount of loss is $(1008.08 - 998)/1008.08 = 10.08/1008.08 \approx$ 0.01 = 1%. The answer is (C).

112. Let x represent the amount of gasoline used in July. Then $0.8x$ represents the amount of gasoline used in August. Let y represent the price of gasoline in July. Then $1.2y$ represents the cost of gasoline in August. The cost of gasoline used in July is xy (amount × price), and the cost of gasoline used in August is $0.8x \times 1.2y = 0.96xy$. So the cost of gasoline used in August is 96% of the cost of gasoline used in July, which is a decrease of 4%. The answer is (D).

1.1 Maintaining Skills

113. $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

114. $\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$

115. $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

116. $\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$

117. $\frac{2+3\sqrt{8}}{2} = \frac{2+3\cdot 2\sqrt{2}}{2} = \frac{2}{2} + \frac{3\cdot 2\sqrt{2}}{2} = 1 + 3\sqrt{2}$

118. $\frac{3+\sqrt{18}}{3} = \frac{3+3\sqrt{2}}{3} = \frac{3}{3} + \frac{3\sqrt{2}}{3} = 1 + \sqrt{2}$

119. $\frac{15-\sqrt{75}}{5} = \frac{15-5\sqrt{3}}{5} = \frac{15}{5} - \frac{5\sqrt{3}}{5} = 3 - \sqrt{3}$

120. $$\begin{aligned} \frac{35-14\sqrt{12}}{7} &= \frac{35-2\cdot 14\sqrt{3}}{7} \\ &= \frac{35}{7} - \frac{2\cdot 14\sqrt{3}}{7} = 5 - 4\sqrt{3} \end{aligned}$$

121. $x^2 + x = x(x+1)$

122. $2x^2 - 4x = 2x(x-2)$

123. $x^2 - 4 = (x-2)(x+2)$

124. $x^2 - 25 = (x-5)(x+5)$

125. $x^2 + 4x + 4 = (x+2)^2$

126. $x^2 - 6x + 9 = (x-3)^2$

127. $x^2 - 8x + 7 = (x-1)(x-7)$

128. $x^2 + 2x - 15 = (x-3)(x+5)$

129. $6x^2 - x - 1 = (3x+1)(2x-1)$

130. $14x^2 + 17x - 6 = (7x-2)(2x+3)$

131. $$\begin{aligned} -5x^2 + 3x + 2 &= (5x+2)(-x+1) \\ &= (5x+2)(1-x) \end{aligned}$$

132. $$\begin{aligned} -12x^2 + 9x + 3 &= -3\left(4x^2 - 3x - 1\right) \\ &= -3(4x+1)(x-1) \end{aligned}$$

1.2 Quadratic Equations

1.2 Practice Problems

1. $x^2 + 25x = -84$
$x^2 + 25x + 84 = 0$
$(x+4)(x+21) = 0$
$x + 4 = 0 \Rightarrow x = -4$ | $x + 21 = 0 \Rightarrow x = -21$
Solution set: $\{-21, -4\}$

2. $2m^2 = 5m$
$2m^2 - 5m = 0$
$m(2m-5) = 0$
$m = 0$ | $2m - 5 = 0$, $2m = 5$, $m = \frac{5}{2}$
Solution set: $\left\{0, \frac{5}{2}\right\}$

3. $x^2 - 6x = -9$
$x^2 - 6x + 9 = 0$
$(x-3)^2 = 0$
$x - 3 = 0$
$x = 3$
Solution set: $\{3\}$

4. $(x+2)^2 = 5$
$x + 2 = \pm\sqrt{5}$
$x = -2 \pm \sqrt{5}$
Solution set: $\left\{-2-\sqrt{5}, -2+\sqrt{5}\right\}$

5. $x^2 - 6x + 7 = 0$
$x^2 - 6x = -7$
$x^2 - 6x + 9 = -7 + 9$
$(x-3)^2 = 2$
$x - 3 = \pm\sqrt{2}$
$x = 3 \pm \sqrt{2}$
Solution set: $\left\{3-\sqrt{2}, 3+\sqrt{2}\right\}$

6. $4x^2 - 24x + 25 = 0$
$4x^2 - 24x = -25$
$x^2 - 6x = -\frac{25}{4}$
$x^2 - 6x + 9 = -\frac{25}{4} + 9$
$(x-3)^2 = \frac{11}{4}$
$x - 3 = \pm\frac{\sqrt{11}}{2} \Rightarrow x = 3 \pm \frac{\sqrt{11}}{2}$
Solution set: $\left\{3 - \frac{\sqrt{11}}{2}, 3 + \frac{\sqrt{11}}{2}\right\}$

7. $6x^2 - x - 2 = 0$
$a = 6, b = -1, c = -2$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-2)}}{2(6)}$$
$$= \frac{1 \pm \sqrt{49}}{12} = \frac{1 \pm 7}{12} = \frac{-6}{12} = -\frac{1}{2} \text{ or } \frac{8}{12} = \frac{2}{3}$$
Solution set: $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$

8. Let x = the frontage of the building.
Then $5x$ = the depth of the building and $5x - 45$ = the depth of the rear portion.
$x(5x - 45) = 2100$
$5x^2 - 45x = 2100$
$5x^2 - 45x - 2100 = 0$
$a = 5, b = -45, c = -2100$
$$x = \frac{-(-45) \pm \sqrt{(-45)^2 - 4(5)(-2100)}}{2(5)} = \frac{45 \pm \sqrt{44,025}}{10} \approx -16.48 \text{ or } 25.48$$
Reject the negative solution.
$5x = 5 \cdot 25.482 = 127.41$
The building is approximately 25.48 ft by 127.41 ft.

9. $\Phi = \frac{\text{length}}{\text{width}} \Rightarrow \frac{1+\sqrt{5}}{2} = \frac{x}{36}$
$$x = 36\left(\frac{1+\sqrt{5}}{2}\right) = 18 + 18\sqrt{5} \approx 58.25 \text{ ft}$$

1.2 Basic Concepts and Skills

1. Any equation of the form $ax^2 + bx + c = 0$ with $a \neq 0$, is called a <u>quadratic</u> equation.

2. If $P(x)$, $D(x)$, and $Q(x)$ are polynomials, and $P(x) = D(x)Q(x)$, then the solutions of $P(x) = 0$ are the solutions of $Q(x) = 0$ together with the solutions of <u>$D(x)$</u> = 0.

3. From the Square Root Property, we know that if $u^2 = 5$, then <u>$u = \pm\sqrt{5}$</u>.

4. If you complete the square in the quadratic equation $ax^2 + bx + c = 0$, you get the quadratic formula for the solutions: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

5. True

6. False. We form a perfect square by adding $\left(\dfrac{k}{2}\right)^2$.

7. $(-6)^2 + 4(-6) - 12 = 36 - 24 - 12 = 0$
–6 is a solution of the equation.

8. $9^2 - 8(9) - 9 = 81 - 72 - 9 = 0$
9 is a solution of the equation.

9. $3\left(\dfrac{2}{3}\right)^2 + 7\left(\dfrac{2}{3}\right) - 6 = 3\left(\dfrac{4}{9}\right) + \dfrac{14}{3} - 6 = \dfrac{4}{3} + \dfrac{14}{3} - 6 = 0$
2/3 is a solution of the equation.

10. $2\left(-\dfrac{1}{2}\right)^2 - 5\left(-\dfrac{1}{2}\right) - 3 = 2\left(\dfrac{1}{4}\right) + \dfrac{5}{2} - 3 = \dfrac{1}{2} + \dfrac{5}{2} - 3 = 0$
–1/2 is a solution of the equation.

11. $\left(2-\sqrt{3}\right)^2 - 4\left(2-\sqrt{3}\right) + 1 = \left(4 - 4\sqrt{3} + 3\right) - 8 + 4\sqrt{3} + 1 = 0$
$2-\sqrt{3}$ is a solution of the equation.

12. $\left(3+2\sqrt{2}\right)^2 - 6\left(3+2\sqrt{2}\right) + 1 = 17 + 12\sqrt{2} - 18 - 12\sqrt{2} + 1 = 0$
$3+2\sqrt{2}$ is a solution of the equation.

13. $4\left(2+\sqrt{3}\right)^2 - 8\left(2+\sqrt{3}\right) + 13$
$= 4\left(7+4\sqrt{3}\right) - 8\left(2+\sqrt{3}\right) + 13$
$= 28 + 16\sqrt{3} - 16 - 8\sqrt{3} + 13 = 25 + 8\sqrt{3} \neq 0$
$2+\sqrt{3}$ is not a solution of the equation.

14. $\left(5-\sqrt{2}\right)^2 - 6\left(5-\sqrt{2}\right) + 13 = 27 - 10\sqrt{2} - 30 + 6\sqrt{2} + 13 = 10 - 4\sqrt{2} \neq 5$
$5-\sqrt{2}$ is not a solution of the equation.

15. $k(1)^2 + 1 - 3 = 0 \Rightarrow k - 2 = 0 \Rightarrow k = 2$

16. $k\left(\sqrt{7}\right)^2 + \sqrt{7} - 3 = 0 \Rightarrow 7k + \sqrt{7} - 3 = 0 \Rightarrow 7k = 3 - \sqrt{7} \Rightarrow k = \dfrac{3-\sqrt{7}}{7}$

17. $x^2 - 5x = 0 \Rightarrow x(x-5) = 0 \Rightarrow x = 0 \text{ or } x - 5 = 0 \Rightarrow x = 0 \text{ or } x = 5$

18. $x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
$x - 4 = 0 \text{ or } x - 1 = 0 \Rightarrow x = 4 \text{ or } x = 1$

19. $x^2 + 5x = 14$
$x^2 + 5x - 14 = 0$
$(x+7)(x-2) = 0$
$x + 7 = 0 \text{ or } x - 2 = 0$
$x = -7 \text{ or } x = 2$

20. $x^2 - 11x = 12$
$x^2 - 11x - 12 = 0$
$(x-12)(x+1) = 0$
$x - 12 = 0 \text{ or } x + 1 = 0$
$x = 12 \text{ or } x = -1$

21. $x^2 = 5x + 6$
$x^2 - 5x - 6 = 0$
$(x-6)(x+1) = 0$
$x - 6 = 0 \text{ or } x + 1 = 0$
$x = 6 \text{ or } x = -1$

22. $x = x^2 - 12$
$0 = x^2 - x - 12$
$0 = (x-4)(x+3)$
$x - 4 = 0 \text{ or } x + 3 = 0$
$x = 4 \text{ or } x = -3$

23. $3x^2 = 48 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

24. $2x^2 = 50 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

25. $x^2 + 1 = 5 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

26. $2x^2 - 1 = 17 \Rightarrow 2x^2 = 18 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

27. $(x-1)^2 = 16$
$x - 1 = -4 \text{ or } x - 1 = 4 \Rightarrow x = -3 \text{ or } x = 5$

28. $(2x-3)^2 = 25 \Rightarrow$
$2x - 3 = -5 \text{ or } 2x - 3 = 5 \Rightarrow x = -1 \text{ or } x = 4$

29. To complete the square, find $1/2$ of the coefficient of the x-term, $4/2 = 2$, and then square the answer. $2^2 = 4$.

30. To complete the square, find $1/2$ of the coefficient of the y-term, $10/2=5$, and then square the answer. $5^2=25$.

31. To complete the square, find $1/2$ of the coefficient of the x-term, $6/2=3$, and then square the answer. $3^2=9$.

32. To complete the square, find $1/2$ of the coefficient of the y-term, $8/2=4$, and then square the answer. $4^2=16$.

33. To complete the square, find $1/2$ of the coefficient of the x-term and then square the answer. $\left(\frac{7}{2}\right)^2=\frac{49}{4}$.

34. To complete the square, find $1/2$ of the coefficient of the x-term and then square the answer. $\left(\frac{3}{2}\right)^2=\frac{9}{4}$.

35. To complete the square, find $1/2$ of the coefficient of the x-term, $\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$ and then square the answer. $\left(\frac{1}{6}\right)^2=\frac{1}{36}$.

36. To complete the square, find $1/2$ of the coefficient of the x-term, $\frac{1}{2}\cdot\frac{3}{2}=\frac{3}{4}$ and then square the answer. $(3/4)^2=9/16$.

37. To complete the square, find $1/2$ of the coefficient of the x-term and then square the answer. $(a/2)^2=a^2/4$.

38. To complete the square, find 1/2 of the coefficient of the x-term,
$\frac{1}{2}\cdot\frac{2a}{3}=\frac{a}{3},$ and then square the answer.
$\left(\frac{a}{3}\right)^2=\frac{a^2}{9}$.

39. $x^2+2x-5=0\Rightarrow x^2+2x=5$
Now, complete the square.
$x^2+2x+1=5+1\Rightarrow(x+1)^2=6\Rightarrow$
$x+1=\pm\sqrt{6}\Rightarrow x=-1\pm\sqrt{6}$

40. $x^2+6x=-7$
Now, complete the square.
$x^2+6x+9=-7+9$
$(x+3)^2=2\Rightarrow x+3=\pm\sqrt{2}\Rightarrow x=-3\pm\sqrt{2}$

41. $x^2-3x-1=0$
$x^2-3x=1$
Now, complete the square.
$x^2-3x+\frac{9}{4}=1+\frac{9}{4}$
$\left(x-\frac{3}{2}\right)^2=\frac{13}{4}$
$x-\frac{3}{2}=\pm\frac{\sqrt{13}}{2}$
$x=\frac{3}{2}\pm\frac{\sqrt{13}}{2}=\frac{3\pm\sqrt{13}}{2}$

42. $x^2-x-3=0$
$x^2-x=3$
Now, complete the square.
$x^2-x+\frac{1}{4}=3+\frac{1}{4}\Rightarrow\left(x-\frac{1}{2}\right)^2=\frac{13}{4}$
$x-\frac{1}{2}=\pm\sqrt{\frac{13}{4}}=\pm\frac{\sqrt{13}}{2}\Rightarrow x=\frac{1}{2}\pm\frac{\sqrt{13}}{2}$

43. $2r^2+3r=9$
$r^2+\frac{3}{2}r=\frac{9}{2}$
$r^2+\frac{3}{2}r+\frac{9}{16}=\frac{9}{2}+\frac{9}{16}$
$\left(r+\frac{3}{4}\right)^2=\frac{81}{16}$
$r+\frac{3}{4}=\pm\frac{9}{4}\Rightarrow r=\frac{-3\pm9}{4}=\frac{3}{2}$ or -3

44. $3k^2-5k+1=0$
$3k^2-5k=-1$
$k^2-\frac{5}{3}k=-\frac{1}{3}$
Now, complete the square.
$k^2-\frac{5}{3}k+\frac{25}{36}=-\frac{1}{3}+\frac{25}{36}$
$\left(k-\frac{5}{6}\right)^2=\frac{13}{36}$
$k-\frac{5}{6}=\pm\sqrt{\frac{13}{36}}=\pm\frac{\sqrt{13}}{6}$
$k=\frac{5}{6}\pm\frac{\sqrt{13}}{6}=\frac{5\pm\sqrt{13}}{6}$

In exercises 45−50, use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

45. $x^2 + 2x - 4 = 0 \Rightarrow a = 1, b = 2, c = -4$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)} = \frac{-2 \pm \sqrt{4+16}}{2} = \frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$$

46. $m^2 + 3m + 2 = 0 \Rightarrow a = 1, b = 3, c = 2$

$$m = \frac{-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)} = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2}$$

$$m = \frac{-3+1}{2} = \frac{-2}{2} = -1 \text{ or } m = \frac{-3-1}{2} = \frac{-4}{2} = -2$$

47. $6x^2 = 7x + 5 \Rightarrow 6x^2 - 7x - 5 = 0 \Rightarrow a = 6, b = -7, c = -5$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-5)}}{2(6)} = \frac{7 \pm \sqrt{49+120}}{12} = \frac{7 \pm \sqrt{169}}{12} = \frac{7 \pm 13}{12}$$

$$x = \frac{7+13}{12} = \frac{20}{12} = \frac{5}{3} \text{ or } x = \frac{7-13}{12} = \frac{-6}{12} = -\frac{1}{2}$$

48. $t^2 - 7 = 4t \Rightarrow t^2 - 4t - 7 = 0 \Rightarrow a = 1, b = -4, c = -7$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)} = \frac{4 \pm \sqrt{16+28}}{2} = \frac{4 \pm \sqrt{44}}{2} = \frac{4 \pm 2\sqrt{11}}{2} = 2 \pm \sqrt{11}$$

49. $3z^2 - 2z = 7 \Rightarrow 3z^2 - 2z - 7 = 0 \Rightarrow a = 3, b = -2, c = -7$

$$z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-7)}}{2(3)} = \frac{2 \pm \sqrt{4+84}}{6} = \frac{2 \pm \sqrt{88}}{6} = \frac{2 \pm 2\sqrt{22}}{6} = \frac{1 \pm \sqrt{22}}{3}$$

50. $6y^2 + 11y = 10 \Rightarrow 6y^2 + 11y - 10 = 0 \Rightarrow a = 6, b = 11, c = -10$

$$y = \frac{-11 \pm \sqrt{11^2 - 4(6)(-10)}}{2(6)} = \frac{-11 \pm \sqrt{121+240}}{12} = \frac{-11 \pm \sqrt{361}}{12} = \frac{-11 \pm 19}{12}$$

$$y = \frac{8}{12} = \frac{2}{3} \text{ or } y = \frac{-30}{12} = -\frac{5}{2}$$

51.
$$2x^2 + 5x - 3 = 0$$
$$(2x-1)(x+3) = 0$$
$$2x - 1 = 0 \text{ or } x + 3 = 0$$
$$x = \frac{1}{2} \text{ or } x = -3$$

52.
$$2x^2 - 9x + 10 = 0$$
$$(2x-5)(x-2) = 0$$
$$2x - 5 = 0 \text{ or } x - 2 = 0$$
$$x = \frac{5}{2} \text{ or } x = 2$$

53.
$$(3x-2)^2 - 16 = 0$$
$$(3x-2)^2 = 16$$
$$3x - 2 = \pm 4$$
$$3x - 2 = -4 \quad \text{or} \quad 3x - 2 = 4$$
$$3x = -2 \qquad 3x = 6$$
$$x = -\frac{2}{3} \quad \text{or} \quad x = 2$$

54.
$$(4x+1)^2 - 25 = 0$$
$$(4x+1)^2 = 25$$
$$4x + 1 = \pm 5$$
$$4x + 1 = -5 \quad \text{or} \quad 4x + 1 = 5$$
$$4x = -6 \qquad 4x = 4$$
$$x = -\frac{3}{2} \quad \text{or} \quad x = 1$$

55.
$$5x^2 - 6x = 4x^2 + 6x - 3$$
$$x^2 - 12x = -3$$

Now, complete the square.

$$x^2 - 12x + 36 = -3 + 36$$
$$(x-6)^2 = 33$$
$$x - 6 = \pm\sqrt{33} \Rightarrow x = 6 \pm \sqrt{33}$$

56. $$x^2+7x-5=x-x^2$$
$$2x^2+6x-5=0$$
$$2x^2+6x=5$$
$$x^2+3x=\frac{5}{2}$$
Now, complete the square.
$$x^2+3x+\frac{9}{4}=\frac{5}{2}+\frac{9}{4}$$
$$\left(x+\frac{3}{2}\right)^2=\frac{19}{4}$$
$$x+\frac{3}{2}=\pm\sqrt{\frac{19}{4}}=\pm\frac{\sqrt{19}}{2}$$
$$x=-\frac{3}{2}\pm\frac{\sqrt{19}}{2}=\frac{-3\pm\sqrt{19}}{2}$$

57. $3p^2+8p+4=0\Rightarrow a=3,b=8,c=4$
$$p=\frac{-8\pm\sqrt{8^2-4(3)(4)}}{2(3)}$$
$$=\frac{-8\pm\sqrt{64-48}}{6}=\frac{-8\pm\sqrt{16}}{6}=\frac{-8\pm 4}{6}$$
$$p=\frac{-4}{6}=-\frac{2}{3}\text{ or }p=\frac{-12}{6}=-2$$

58. $x^2=5(x-1)\Rightarrow x^2=5x-5\Rightarrow$
$x^2-5x+5=0\Rightarrow a=1,b=-5,c=5$
$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(5)}}{2(1)}$$
$$=\frac{5\pm\sqrt{25-20}}{2}=\frac{5\pm\sqrt{5}}{2}$$

59. $$3y^2+5y+2=0$$
$$(3y+2)(y+1)=0$$
$$3y+2=0\text{ or }y+1=0$$
$$y=-\frac{2}{3}\text{ or }y=-1$$

60. $$6x^2+11x+4=0$$
$$(3x+4)(2x+1)=0$$
$$3x+4=0\text{ or }2x+1=0$$
$$x=-\frac{4}{3}\text{ or }x=-\frac{1}{2}$$

61. $$5x^2+12x+4=0$$
$$(5x+2)(x+2)=0$$
$$5x+2=0\text{ or }x+2=0$$
$$x=-\frac{2}{5}\text{ or }x=-2$$

62. $$3x^2-2x-5=0$$
$$(3x-5)(x+1)=0$$
$$3x-5=0\text{ or }x+1=0$$
$$x=\frac{5}{3}\text{ or }x=-1$$

63. $$5y^2+10y+4=2y^2+3y+1$$
$$3y^2+7y=-3$$
$$y^2+\frac{7}{3}y=-1$$
Now, complete the square.
$$y^2+\frac{7}{3}y+\frac{49}{36}=-1+\frac{49}{36}$$
$$\left(y+\frac{7}{6}\right)^2=\frac{13}{36}$$
$$y+\frac{7}{6}=\pm\sqrt{\frac{13}{36}}=\pm\frac{\sqrt{13}}{6}$$
$$y=-\frac{7}{6}\pm\frac{\sqrt{13}}{6}=\frac{-7\pm\sqrt{13}}{6}$$

64. $$3x^2-1=5x^2-3x-5$$
$$4=2x^2-3x$$
$$2=x^2-\frac{3}{2}x$$
Now, complete the square.
$$2+\frac{9}{16}=x^2-\frac{3}{2}x+\frac{9}{16}$$
$$\frac{41}{16}=\left(x-\frac{3}{4}\right)^2$$
$$\pm\sqrt{\frac{41}{16}}=x-\frac{3}{4}\Rightarrow\pm\frac{\sqrt{41}}{4}=x-\frac{3}{4}$$
$$x=\frac{3}{4}\pm\frac{\sqrt{41}}{4}=\frac{3\pm\sqrt{41}}{4}$$

65. $$2x^2+x=15$$
$$2x^2+x-15=0$$
$$(2x-5)(x+3)=0$$
$$2x-5=0\text{ or }x+3=0\Rightarrow x=\frac{5}{2}\text{ or }x=-3$$

66. $$6x^2=1-x$$
$$6x^2+x-1=0$$
$$(3x-1)(2x+1)=0$$
$$3x-1=0\text{ or }2x+1=0$$
$$x=\frac{1}{3}\text{ or }x=-\frac{1}{2}$$

67.
$$12x^2 - 10x = 12$$
$$12x^2 - 10x - 12 = 0$$
$$2(6x^2 - 5x - 6) = 0$$
$$6x^2 - 5x - 6 = 0$$
$$(3x+2)(2x-3) = 0$$
$$3x+2 = 0 \text{ or } 2x-3 = 0$$
$$x = -\frac{2}{3} \text{ or } x = \frac{3}{2}$$

68.
$$-x^2 + 10x + 1200 = 0$$
$$x^2 - 10x - 1200 = 0$$
$$(x-40)(x+30) = 0$$
$$x - 40 = 0 \text{ or } x + 30 = 0$$
$$x = 40 \text{ or } x = -30$$

69. $(x+13)(x+5) = -2 \Rightarrow x^2 + 18x + 65 = -2 \Rightarrow$
$x^2 + 18x + 67 = 0 \Rightarrow a = 1, b = 18, c = 67$
$$x = \frac{-18 \pm \sqrt{18^2 - 4(1)(67)}}{2(1)}$$
$$= \frac{-18 \pm \sqrt{324 - 268}}{2} = \frac{-18 \pm \sqrt{56}}{2}$$
$$= \frac{-18 \pm 2\sqrt{14}}{2} = -9 \pm \sqrt{14}$$

70. $3(x^2 + 1) = 2x^2 + 4x + 1 \Rightarrow$
$3x^2 + 3 = 2x^2 + 4x + 1 \Rightarrow x^2 - 4x + 2 = 0 \Rightarrow$
$a = 1, b = -4, c = 2$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$
$$= 2 \pm \sqrt{2}$$

71.
$$18x^2 - 45x = -7$$
$$18x^2 - 45x + 7 = 0$$
$$(3x-7)(6x-1) = 0$$
$$3x - 7 = 0 \text{ or } 6x - 1 = 0 \Rightarrow x = \frac{7}{3} \text{ or } x = \frac{1}{6}$$

72.
$$18x^2 + 57x + 45 = 0$$
$$3(6x^2 + 19x + 15) = 0$$
$$6x^2 + 19x + 15 = 0$$
$$(3x+5)(2x+3) = 0$$
$$3x + 5 = 0 \text{ or } 2x + 3 = 0$$
$$x = -\frac{5}{3} \text{ or } x = -\frac{3}{2}$$

73. $2t^2 - 5 = 0 \Rightarrow a = 2, b = 0, c = -5$
$$t = \frac{0 \pm \sqrt{0^2 - 4(2)(-5)}}{2(2)} = \frac{\pm\sqrt{40}}{4}$$
$$= \pm\frac{2\sqrt{10}}{4} = \pm\frac{\sqrt{10}}{2}$$

74. $3k^2 - 48 = 0 \Rightarrow a = 3, b = 0, c = -48$
$$t = \frac{0 \pm \sqrt{0^2 - 4(3)(-48)}}{2(3)} = \frac{\pm\sqrt{576}}{6} = \pm\frac{24}{6} = \pm 4$$

75.
$$4x^2 - 10x - 750 = 0$$
$$2(2x^2 - 5x - 375) = 0$$
$$2x^2 - 5x - 375 = 0$$
$$(2x+25)(x-15) = 0$$
$$2x + 25 = 0 \text{ or } x - 15 = 0$$
$$x = -\frac{25}{2} \text{ or } x = 15$$

76.
$$12x^2 + 43x + 36 = 0$$
$$(3x+4)(4x+9) = 0$$
$$3x + 4 = 0 \text{ or } 4x + 9 = 0$$
$$x = -\frac{4}{3} \text{ or } x = -\frac{9}{4}$$

77. $\Phi = \frac{\text{length}}{\text{width}} \Rightarrow \frac{1+\sqrt{5}}{2} = \frac{x}{14.72}$
$$x = 14.72\left(\frac{1+\sqrt{5}}{2}\right) \approx 23.82 \text{ in.}$$

78. $\Phi = \frac{\text{length}}{\text{width}} \Rightarrow \frac{1+\sqrt{5}}{2} = \frac{x}{18.63}$
$$x = 18.63\left(\frac{1+\sqrt{5}}{2}\right) \approx 30.14 \text{ ft}$$

79. $\Phi = \frac{\text{length}}{\text{width}} \Rightarrow \frac{1+\sqrt{5}}{2} = \frac{8.46}{x}$
$$x = 8.46\left(\frac{2}{1+\sqrt{5}}\right) \approx 5.23 \text{ cm}$$

80. $\Phi = \frac{\text{length}}{\text{width}} \Rightarrow \frac{1+\sqrt{5}}{2} = \frac{4.68}{x}$
$$x = 4.68\left(\frac{2}{1+\sqrt{5}}\right) \approx 2.89 \text{ m}$$

1.2 Applying the Concepts

81. Let x = the width of the plot. Then $3x$ = the length of the plot. So, $3x^2 = 10{,}800 \Rightarrow$ $x^2 = 3600 \Rightarrow x = 60$.
The plot is 60 ft by 180 ft.

82. Let x = the length of the side of the square. Then $x + 4$ = the length of the diagonal of the square. Using the Pythagorean theorem, we have

$x^2 + x^2 = (x+4)^2 \Rightarrow 2x^2 = x^2 + 8x + 16 \Rightarrow$

$x^2 - 8x - 16 = 0$. So, $a = 1, b = -8, c = -16$

and $x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-16)}}{2(1)} \Rightarrow$

$x = \dfrac{8 \pm \sqrt{64+64}}{2} \Rightarrow x = \dfrac{8 \pm \sqrt{128}}{2} \Rightarrow$

$x = \dfrac{8 \pm 8\sqrt{2}}{2} = 4 \pm 4\sqrt{2}$.

The length cannot be a negative number, so eliminate the negative root. The length of the square is $4 + 4\sqrt{2}$ in.

83. Let x = the first integer. Then $28 - x =$ the second integer. So, $x(28 - x) = 147 \Rightarrow$

$28x - x^2 = 147 \Rightarrow 0 = x^2 - 28x + 147 \Rightarrow$

$0 = (x-7)(x-21) \Rightarrow x = 7$ or $x = 21$.

The sum of the two numbers is 28. So, the numbers are 7 and 21.

84. Let x = the integer. Then $2x^2 + x = 55 \Rightarrow$

$2x^2 + x - 55 = 0 \Rightarrow (2x+11)(x-5) = 0 \Rightarrow$

$x = -\dfrac{11}{2}$ or $x = 5$.

The problem calls for an integer, so we eliminate $-11/2$ as a solution. Check that 5 is the solution by verifying that $2(5^2) + 5 = 55$. The integer is 5.

85. Let x = one number. Then $57 - x$ = the other number.

$$x(57 - x) = 782$$
$$-x^2 + 57x = 782$$
$$x^2 - 57x = -782$$
$$x^2 - 57x + 782 = 0$$
$$(x-23)(x-34) = 0$$
$$x - 23 = 0 \text{ or } x - 34 = 0$$
$$x = 23 \text{ or } x = 34$$

86. Let w = the width of the rectangle. Since perimeter = 2 × width + 2 × length, we have $82 = 2w + 2l \Rightarrow 82 - 2w = 2l \Rightarrow l = 41 - w.$ The area is given by length × width, so

$$(41 - w)w = 400$$
$$41w - w^2 = 400$$
$$0 = w^2 - 41w + 400$$
$$0 = (w-25)(w-16)$$
$$w - 25 = 0 \text{ or } w - 16 = 0$$
$$w = 25 \qquad w = 16$$

The dimensions of the rectangle are 25 ft by 16 ft.

87. Let x = the width of the rectangle. Then $x + 5 =$ the length of the rectangle. So, $x(x+5) = 500$

$\Rightarrow x^2 + 5x = 500 \Rightarrow x^2 + 5x - 500 = 0 \Rightarrow$

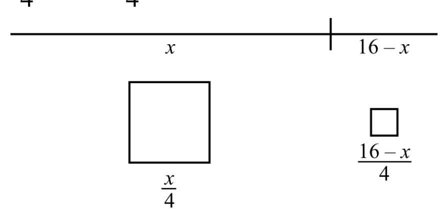
$(x+25)(x-20) = 0 \Rightarrow x = -25$ or $x = 20$.

Since the length cannot be negative, we reject that solution. $x = 20 \Rightarrow x + 5 = 25$. So the rectangle is 25 cm by 20 cm.

88. Let $3x$ = the length of the rectangle. Then $2x$ = the width of the rectangle, and $(3x)(2x) = 216$.

So $6x^2 = 216 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$. Length cannot be negative, so we reject –6 as a solution. The dimensions of the rectangle are $3x = 18$ cm and $2x = 12$ cm.

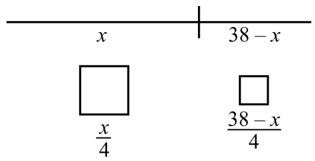
89. Let x = one piece of the wire. Then $16 - x$ = the other piece of the wire. Each piece is bent into a square, so the sides of the squares are $\dfrac{x}{4}$ and $\dfrac{16-x}{4}$, respectively.



$$\left(\frac{x}{4}\right)^2 + \left(\frac{16-x}{4}\right)^2 = 10$$
$$\frac{x^2}{16} + \frac{256 - 32x + x^2}{16} = 10$$
$$x^2 + 256 - 32x + x^2 = 160$$
$$2x^2 - 32x + 96 = 0$$
$$x^2 - 16x + 48 = 0$$
$$(x-12)(x-4) = 0 \Rightarrow x = 12 \text{ or } x = 4$$

If one piece of the wire is 12, then the other piece is 16 – 12 = 4. So the pieces are 12 in. and 4 in.

90. Let x = one piece of the wire. Then $38 - x$ = the other piece of the wire. Each piece is bent into a square, so the sides of the squares are $\frac{x}{4}$ and $\frac{38-x}{4}$, respectively.

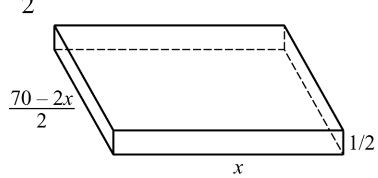


$$3\left(\frac{x}{4}\right)^2 - \left(\frac{38-x}{4}\right)^2 = 95.75$$
$$3\left(\frac{x^2}{16}\right) - \frac{1444 - 76x + x^2}{16} = 95.75$$
$$3x^2 - 1444 + 76x - x^2 = 1532$$
$$2x^2 + 76x - 2976 = 0$$
$$x^2 + 38x - 1488 = 0$$
$$(x+62)(x-24) = 0$$
$$x = -62 \text{ or } x = 24$$

The answer cannot be negative, so we reject –62. If one piece of the wire is 24, then the other piece is 38 – 24 = 14. So the pieces are 24 in. and 14 in.

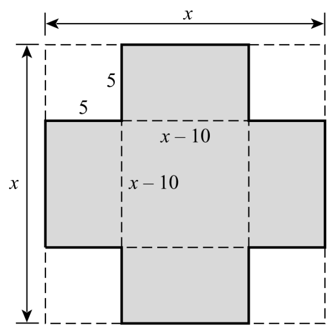
91. Let r = the radius of the can. Then
$$32\pi = 2\pi r(6) + 2\pi r^2$$
$$32\pi = 12\pi r + 2\pi r^2$$
Divide both sides by 2π
$$16 = 6r + r^2$$
$$0 = r^2 + 6r - 16$$
$$0 = (r-2)(r+8)$$
$$r = 2 \text{ or } r = -8$$
The answer cannot be negative, so we reject –8. The radius of the can is 2 inches.

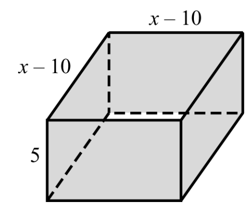
92. Let x = the length of the patio. Then $\frac{70-2x}{2}$ = the width of the patio.



The height is 6 inches, while the length and width are given in feet, so the height must be converted to $1/2$ feet.
$$x\left(\frac{70-2x}{2}\right)\left(\frac{1}{2}\right) = 138$$
$$\frac{70x - 2x^2}{4} = 138$$
$$70x - 2x^2 = 552$$
Divide both sides by 2.
$$35x - x^2 = 276$$
$$0 = x^2 - 35x + 276$$
$$0 = (x-12)(x-23)$$
$$x = 12 \text{ or } x = 23$$
The dimensions of the patio are 12 ft by 23 ft.

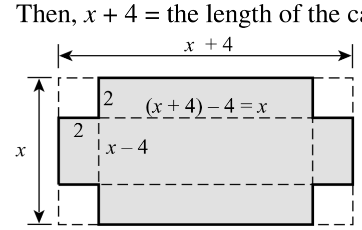
93. Let x = the length of the piece of tin. Then $x - 10$ = the length of the box.





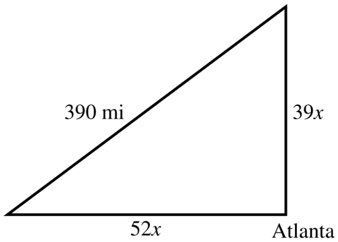
$$(x-10)(x-10)(5) = 480$$
$$5(x^2 - 20x + 100) = 480$$
$$x^2 - 20x + 100 = 96$$
$$x^2 - 20x + 4 = 0$$
Solve using the quadratic formula with $a = 1$, $b = -20$, and $c = 4$.
$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(4)}}{2(1)}$$
$$= \frac{20 \pm \sqrt{400 - 16}}{2} = \frac{20 \pm \sqrt{384}}{2}$$
$$\approx \frac{20 + 19.6}{2} \text{ or } \frac{20 - 19.6}{2} \approx 19.8 \text{ or } 0.18$$
The length cannot be 0.18 inches, so we reject that solution. The tin is 19.8 in. by 19.8 in.

94. Let x = the width of cardboard. Then, $x + 4$ = the length of the cardboard.



$$x(x-4)(2) = 64$$
$$2(x^2 - 4x) = 64$$
$$x^2 - 4x = 32$$
$$x^2 - 4x - 32 = 0$$
$$(x-8)(x+4) = 0 \Rightarrow x = 8 \text{ or } x = -4$$
The length cannot be –4 inches, so we reject that solution. Since $x = 8$, then $x - 4 = 4$. The box is 4 in. wide by 8 in. long by 2 in. deep.

95. Let x = the time the buses travel. So the distance the first bus travels = $52x$ mi, and the distance the second bus travels = $39x$ mi.

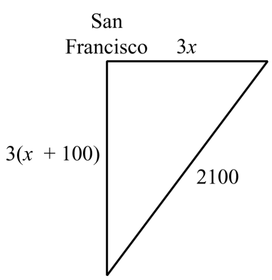


Using the Pythagorean theorem, we have

$$(52x)^2 + (39x)^2 = 390^2$$
$$2704x^2 + 1521x^2 = 152,100$$
$$4225x^2 = 152,100$$
$$x^2 = 36 \Rightarrow x = \pm 6$$

Time cannot be negative, so we reject –6. The buses will be 390 miles apart after 6 hours.

96.



Let x = the speed of the plane traveling east. Then $x + 100$ = the speed of the plane traveling south. They each traveled for three hours, so they traveled $3x$ and $3(x + 100)$ km, respectively. Using the Pythagorean theorem, we have

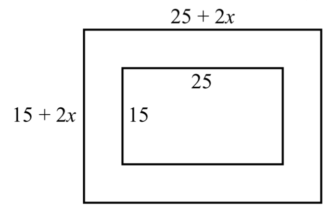
$$(3x)^2 + (3(x+100))^2 = 2100^2$$
$$9x^2 + (3x+300)^2 = 4,410,000$$
$$9x^2 + 9x^2 + 1800x + 90,000 = 4,410,000$$
$$18x^2 + 1800x + 90,000 = 4,410,000$$
$$18x^2 + 1800x - 4,320,000 = 0$$
$$x^2 + 100x - 240,000 = 0$$

Solve using quadratic formula with $a = 1$, $b = 100$, and $c = -240,000$.

$$x = \frac{-100 \pm \sqrt{100^2 - 4(1)(-240,000)}}{2(1)}$$
$$= \frac{-100 \pm \sqrt{10,000 + 960,000}}{2}$$
$$= \frac{-100 \pm \sqrt{970,000}}{2}$$
$$\approx \frac{-100 + 984.88}{2} \text{ or } \frac{-100 - 984.88}{2}$$
$$\approx 442.44 \text{ or } -542.44$$

We reject the negative answer, so the plane flying east traveled at 442.44 km/h, and the plane flying south traveled at 542.44 km/h.

97. Let x = the width of the border. Then length of the garden with the border is $25 + 2x$, and the width of the garden with the border is $15 + 2x$. The area of the border = the area of the garden with the border – the area of the garden.



$$A_b = 624 = (25 + 2x)(15 + 2x) - (15)(25)$$
$$= 375 + 80x + 4x^2 - 375$$
$$= 80x + 4x^2$$

$$624 = 80x + 4x^2$$
$$0 = 4x^2 + 80x - 624$$
$$0 = x^2 + 20x - 156$$
$$0 = (x + 26)(x - 6)$$
$$x = -26 \text{ or } x = 6$$

We reject the negative solution. The width of the border is 6 feet.

98. $-16t^2 + 5000 = 1000 \Rightarrow -16t^2 = -4000 \Rightarrow$
$t^2 = 250 \Rightarrow t = \pm\sqrt{250} \Rightarrow t \approx \pm 15.8$
Reject the negative solution. The diver is in free fall for 15.8 seconds.

99. a. $h = -16(2^2) + 112(2) = 160$ feet

b. $96 = -16t^2 + 112t \Rightarrow 16t^2 - 112t + 96 = 0$
$\Rightarrow t^2 - 7t + 6 = 0 \Rightarrow (t-1)(t-6) = 0 \Rightarrow$
$t = 1$ or $t = 6$
The ball will be at a height of 96 feet at 1 second and at 6 seconds.

c. $0 = -16t^2 + 112t \Rightarrow 16t^2 - 112t = 0 \Rightarrow$
$t^2 - 7t = 0 \Rightarrow t(t-7) = 0 \Rightarrow t = 0$ or
$t = 7$
The ball will return to the ground at 7 seconds.

100. a. $v = \frac{1087\sqrt{20 + 273}}{16.25} \approx 1145.01$ ft/sec

b.
$$1150 = \frac{1087\sqrt{T+273}}{16.25}$$
$$18687.5 = 1087\sqrt{T+273}$$
$$\frac{18687.5}{1087} = \sqrt{T+273}$$
$$\left(\frac{18687.5}{1087}\right)^2 = T + 273$$
$$\left(\frac{18687.5}{1087}\right)^2 - 273 = T$$
$$22.56°\text{C} \approx T$$

101. a.
$$-16t^2 + 96t + 480 = 592$$
$$-16t^2 + 96t - 112 = 0$$
$a = -16$, $b = 96$, $c = -112$
$$x = \frac{-96 \pm \sqrt{96^2 - 4(-16)(-112)}}{2(-16)}$$
$$\approx 1.59 \text{ or } 4.41$$
The projectile will be at a height of 592 ft at about 1.59 sec and 4.41 sec.

b.
$$-16t^2 + 96t + 480 = 0$$
$a = -16$, $b = 96$, $c = 480$
$$x = \frac{-96 \pm \sqrt{96^2 - 4(-16)(480)}}{2(-16)}$$
≈ -3.24 (reject this) or 9.24

The projectile will crash on the ground after approximately 9.24 seconds.

102. a.
$$-16t^2 + 112t + 480 = 592$$
$$-16t^2 + 112t - 112 = 0$$
$a = -16$, $b = 112$, $c = -112$
$$x = \frac{-112 \pm \sqrt{112^2 - 4(-16)(-112)}}{2(-16)}$$
$$\approx 1.21 \text{ or } 5.79$$
The projectile will be at a height of 592 ft at about 1.21 sec and 5.79 sec.

b.
$$-16t^2 + 112t + 480 = 0$$
$a = -16$, $b = 112$, $c = 480$
$$x = \frac{-112 \pm \sqrt{112^2 - 4(-16)(480)}}{2(-16)}$$
≈ -3 (reject this) or 10

The projectile will crash on the ground after 10 seconds.

103. a.
$$-16t^2 + 256t + 480 = 592$$
$$-16t^2 + 256t - 112 = 0$$
$a = -16$, $b = 256$, $c = -112$
$$x = \frac{-256 \pm \sqrt{256^2 - 4(-16)(-112)}}{2(-16)}$$
$$\approx 0.45 \text{ or } 15.55$$
The projectile will be at a height of 592 ft at about 0.45 sec and 15.55 sec.

b.
$$-16t^2 + 256t + 480 = 0$$
$a = -16$, $b = 256$, $c = 480$
$$x = \frac{-256 \pm \sqrt{256^2 - 4(-16)(480)}}{2(-16)}$$
≈ -1.70 (reject this) or 17.70

The projectile will crash on the ground after 17.70 seconds.

104. a.
$$-16t^2 + 64t + 480 = 592$$
$$-16t^2 + 64t - 112 = 0$$
$a = -16$, $b = 64$, $c = -112$
$$x = \frac{-64 \pm \sqrt{64^2 - 4(-16)(-112)}}{2(-16)}$$
$$= \frac{-64 \pm \sqrt{-3072}}{-32}$$
Since the solution is nonreal complex, the projectile will never reach 592 feet.

b.
$$-16t^2 + 64t + 480 = 0$$
$a = -16$, $b = 64$, $c = 480$
$$x = \frac{-64 \pm \sqrt{64^2 - 4(-16)(480)}}{2(-16)}$$
≈ -3.83 (reject this) or 7.83

The projectile will crash on the ground after 7.83 seconds.

1.2 Beyond the Basics

In exercises 105−110, the equation has equal roots if it can be written in the form $(ax+b)^2 = 0$ or $(ax-b)^2 = 0$. Then we equate the coefficients.

105. $x^2 - kx + 3 = (x-b)^2 = x^2 - 2bx + b^2$

So, $b^2 = 3 \Rightarrow b = \pm\sqrt{3}$.

Then, $k = 2b = 2\left(\pm\sqrt{3}\right) = \pm 2\sqrt{3}$.

106. $x^2+3kx+8=(x+b)^2=x^2+2bx+b^2$

So, $b^2=8\Rightarrow b=\pm\sqrt{8}=\pm2\sqrt{2}$

Then, $k=2b=2\left(\pm2\sqrt{2}\right)=\pm4\sqrt{2}.$

107. $2x^2+kx+k=2(x-b)^2=2x^2-4bx+2b^2$

So, $k=2b^2$. Then, $k=-4b\Rightarrow 2b^2=-4b\Rightarrow$

$b^2+2b=0\Rightarrow b(b+2)=0\Rightarrow b=0$ or $b=-2.$

Therefore, $k=2(0)^2=0$ or $k=2(-2)^2=8.$

108. $kx^2+2x+6=k(x+b)^2=kx^2+2kbx+kb^2$

So, $2=2kb\Rightarrow k=\frac{1}{b}$, and

$6=kb^2\Rightarrow 6=\frac{1}{b}\cdot b^2\Rightarrow 6=b.$

Therefore, $k=\frac{1}{6}.$

109. $x^2+k^2=2(k+1)x\Rightarrow$

$x^2-2(k+1)x+k^2=(x-b)^2=x^2-2bx+b^2$

So, $k^2=b^2\Rightarrow k=\pm b.$

$2(k+1)=2b\Rightarrow k+1=b$

If $k = b$, we have $b + 1 = b$, which is false, so we disregard this solution. If $k = -b$, we have

$-b+1=b\Rightarrow 1=2b\Rightarrow b=\frac{1}{2}.$ Therefore,

$k=\frac{1}{2}.$

110. $kx^2+(k+3)x+4=k(x+b)^2$

$=kx^2+2kbx+kb^2$

So, $kb^2=4\Rightarrow k=\frac{4}{b^2}$, and

$k+3=2kb\Rightarrow \frac{4}{b^2}+3=2b\left(\frac{4}{b^2}\right)\Rightarrow$

$4+3b^2=8b\Rightarrow 3b^2-8b+4=0\Rightarrow$

$(3b-2)(b-2)=0\Rightarrow b=\frac{2}{3}$ or $b=2$

$b=\frac{2}{3}\Rightarrow k=\frac{4}{\left(\frac{2}{3}\right)^2}=9$

$b=2\Rightarrow k=\frac{4}{2^2}=1$

Thus, $k = 1$ or $k = 9$.

111. Since r and s are roots of the equation, then

$$ar^2+br+c=0=as^2+bs+c\Rightarrow$$
$$ar^2+br+c-c=as^2+bs+c-c$$
$$ar^2+br=as^2+bs$$
$$ar^2-as^2=bs-br$$
$$a(r^2-s^2)=b(s-r)$$
$$a(r^2-s^2)=-b(r-s)$$
$$\frac{r^2-s^2}{r-s}=-\frac{b}{a}\Rightarrow r+s=-\frac{b}{a}$$

To find $r\cdot s$, first divide both sides of $ar^2+br+c=0$ by a. We have $r^2+\frac{b}{a}r+\frac{c}{a}=0$. Now use the results from the first part of the problem and substitute $-(r+s)$ for $\frac{b}{a}$. This gives

$$r^2-(r+s)r+\frac{c}{a}=0$$
$$r^2-r^2-rs+\frac{c}{a}=0$$
$$\frac{c}{a}=rs$$

112. Use the results from exercise 111.

a. $r+s=-\frac{5}{3}, rs=-\frac{5}{3}$

b. $r+s=\frac{7}{3}, rs=-\frac{1}{3}$

c. $r+s=-\frac{-3}{\sqrt{3}}=\frac{3\sqrt{3}}{3}=\sqrt{3}$

$rs=-\frac{4}{\sqrt{3}}=-\frac{4\sqrt{3}}{3}$

d. $r+s=-\frac{-\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}}{1+\sqrt{2}}\cdot\frac{1-\sqrt{2}}{1-\sqrt{2}}$

$=\frac{\sqrt{2}-2}{-1}=2-\sqrt{2}$

$rs=\frac{-5}{1+\sqrt{2}}=\frac{-5}{1+\sqrt{2}}\cdot\frac{1-\sqrt{2}}{1-\sqrt{2}}=\frac{-5+5\sqrt{2}}{-1}$

$=5-5\sqrt{2}$

Use the results from exercise 111 to solve exercises 113 and 114.

113. $-\frac{k-3}{2}=\frac{3k-5}{2}\Rightarrow -k+3=3k-5\Rightarrow$

$-4k=-8\Rightarrow k=2$

114. $-\frac{2k-3}{5}=\frac{-2k+3}{5}\Rightarrow -2k+3=-2k+3$

This is an identity, so the solution is all real numbers, or $(-\infty, \infty)$.

115. From exercise 111, we have

$r+s=-\frac{b}{a}\Rightarrow b=-a(r+s)$ and

$rs=\frac{c}{a}\Rightarrow c=ars$. Substitute these values into the equation:

$$\begin{aligned}ax^2+bx+c &= ax^2-a(r+s)x+ars\\ &= a(x^2-(r+s)x+rs)\\ &= a(x^2-rx-sx+rs)\\ &= a[x(x-r)-s(x-r)]\\ &= a(x-r)(x-s)\end{aligned}$$

116. a. Use the quadratic equation to solve $4x^2+4x-5=0$.

$$x=\frac{-4\pm\sqrt{16+80}}{8}=\frac{-4\pm\sqrt{96}}{8}=\frac{-4\pm4\sqrt{6}}{8}=-\frac{1}{2}\pm\frac{\sqrt{6}}{2}$$

So, we have

$$\begin{aligned}&4x^2+4x-5\\ &=4\left[x-\left(-\frac{1}{2}+\frac{\sqrt{6}}{2}\right)\right]\left[x-\left(-\frac{1}{2}-\frac{\sqrt{6}}{2}\right)\right]\\ &=2\left[x-\left(-\frac{1}{2}+\frac{\sqrt{6}}{2}\right)\right]\cdot2\left[x-\left(-\frac{1}{2}-\frac{\sqrt{6}}{2}\right)\right]\\ &=\left(2x+1-\sqrt{6}\right)\left(2x+1+\sqrt{6}\right)\end{aligned}$$

b. Use the quadratic equation to solve $25x^2+40x+11=0$.

$$x=\frac{-40\pm\sqrt{40^2-4(25)(11)}}{2(25)}=\frac{-40\pm\sqrt{500}}{50}=\frac{-40\pm10\sqrt{5}}{50}=-\frac{4}{5}\pm\frac{\sqrt{5}}{5}$$

So, we have $25x^2+40x+11=$

$$25\left[x-\left(-\frac{4}{5}+\frac{\sqrt{5}}{5}\right)\right]\left[x-\left(-\frac{4}{5}-\frac{\sqrt{5}}{5}\right)\right]=$$

$$5\left[x-\left(-\frac{4}{5}+\frac{\sqrt{5}}{5}\right)\right]\cdot5\left[x-\left(-\frac{4}{5}-\frac{\sqrt{5}}{5}\right)\right]=$$

$$\left(5x+4-\sqrt{5}\right)\left(5x+4+\sqrt{5}\right)$$

c. Use the quadratic equation to solve $25x^2-30x+14=0$.

$$x=\frac{-(-30)\pm\sqrt{(-30)^2-4(25)(4)}}{2(25)}=\frac{30\pm\sqrt{500}}{50}=\frac{30\pm10\sqrt{5}}{50}=\frac{3}{5}\pm\frac{\sqrt{5}}{5}$$

So, we have $25x^2-30x+4=$

$$\begin{aligned}&25\left[x-\left(\frac{3}{5}+\frac{\sqrt{5}}{5}\right)\right]\left[x-\left(\frac{3}{5}-\frac{\sqrt{5}}{5}\right)\right]\\ &=5\left[x-\left(\frac{3}{5}+\frac{\sqrt{5}}{5}\right)\right]\cdot5\left[x-\left(\frac{3}{5}-\frac{\sqrt{5}}{5}\right)\right]\\ &=\left(5x-3-\sqrt{5}\right)\left(5x-3+\sqrt{5}\right)\end{aligned}$$

d. Use the quadratic equation to solve $72x^2+95x-1000=0$.

$$\begin{aligned}x&=\frac{-95\pm\sqrt{95^2-4(72)(-1000)}}{2(72)}\\ &=\frac{-95\pm\sqrt{297,025}}{144}=\frac{-95\pm545}{144}\\ &=\frac{450}{144}\text{ or }-\frac{640}{144}=\frac{25}{8}\text{ or }-\frac{40}{9}\end{aligned}$$

So, we have

$$\begin{aligned}&72x^2+95x-1000\\ &=72\left(x-\frac{25}{8}\right)\left[x-\left(-\frac{40}{9}\right)\right]\\ &=72\left(x-\frac{25}{8}\right)\left(x+\frac{40}{9}\right)\\ &=8\left(x-\frac{25}{8}\right)\cdot9\left(x+\frac{40}{9}\right)\\ &=(8x-25)(9x+40)\end{aligned}$$

117. a. $(x-(-3))(x-4)=0\Rightarrow(x+3)(x-4)=0\Rightarrow$
$x^2-x-12=0$

b. $(x-5)(x-5)=0\Rightarrow x^2-10x+25=0$

c. $(x-(3+\sqrt{2}))(x-(3-\sqrt{2}))=0\Rightarrow$
$x^2-3x-x\sqrt{2}-3x+x\sqrt{2}+7=0\Rightarrow$
$x^2-6x+7=0$

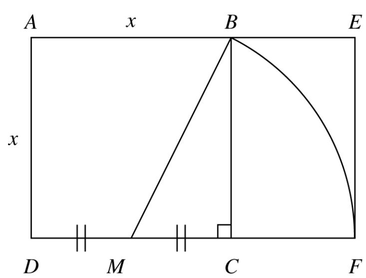
118. a. $a = 3, b = -4y, c = 5 - y^2$

$$x = \frac{-(-4y) \pm \sqrt{(-4y)^2 - 4(3)(5 - y^2)}}{2(3)}$$
$$= \frac{4y \pm \sqrt{16y^2 - 60 + 12y^2}}{6}$$
$$= \frac{4y \pm \sqrt{28y^2 - 60}}{6} = \frac{4y \pm 2\sqrt{7y^2 - 15}}{6}$$
$$= \frac{2y \pm \sqrt{7y^2 - 15}}{3}$$

b. $a = -1, b = -4x, c = 3x^2 + 5$

$$y = \frac{-(-4x) \pm \sqrt{(-4x)^2 - 4(-1)(3x^2 + 5)}}{2(-1)}$$
$$= \frac{4x \pm \sqrt{16x^2 + 12x^2 + 20}}{-2}$$
$$= \frac{4x \pm \sqrt{28x^2 + 20}}{-2} = \frac{4x \pm 2\sqrt{7x^2 + 5}}{-2}$$
$$= -2x \pm \sqrt{7x^2 + 5}$$

119. To show that a rectangle is a golden rectangle, we need to show that the ratio of the longer side to the shorter side equals the golden ratio, Φ, or $\frac{1 + \sqrt{5}}{2}$.



Note that $DM = CM = \frac{x}{2}$. Use the Pythagorean theorem to find the length of MB:

$$MB^2 = CM^2 + BC^2 = \left(\frac{x}{2}\right)^2 + x^2 = \frac{x^2}{4} + x^2$$
$$= \frac{5}{4}x^2 \Rightarrow MB = \frac{\sqrt{5}}{2}x.$$

Then

$$DF = DM + MF = \frac{x}{2} + \frac{\sqrt{5}}{2}x = \frac{1 + \sqrt{5}}{2}x, \text{ and}$$

$$\frac{DF}{AD} = \frac{\left(\frac{1 + \sqrt{5}}{2}\right)x}{x} = \frac{1 + \sqrt{5}}{2} = \Phi.$$

So, $AEFD$ is a golden rectangle.

120. By the quadratic formula, we know that $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$. If $\frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{2a}{-b + \sqrt{b^2 - 4ac}}$ then

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{2a}{-b - \sqrt{b^2 - 4ac}} \Rightarrow$$
$$\left(-b + \sqrt{b^2 - 4ac}\right)\left(-b - \sqrt{b^2 - 4ac}\right) = 4a^2 \Rightarrow$$
$$b^2 - (b^2 - 4ac) = 4a^2 \Rightarrow 4ac = 4a^2 \Rightarrow c = a$$

The answer is (ii).

121. Using the results from exercises 111 and 115, if r, s, and t are three distinct roots, we know that $rs = \frac{c}{a}, st = \frac{c}{a}$, and $rt = \frac{c}{a}$. Then $rs = st \Rightarrow r = s$, and $rs = rt \Rightarrow s = t$. So, $r = s = t$, which contradicts our assumption. Therefore, there cannot be three distinct roots of a quadratic equation.

122. Solve the equation using the quadratic formula. $b = -(p + 1) = -p - 1$. So,

$$x = \frac{-(-p - 1) \pm \sqrt{(-p - 1)^2 - 4(1)p}}{2(1)}$$
$$= \frac{(p + 1) \pm \sqrt{p^2 + 2p + 1 - 4p}}{2}$$
$$= \frac{(p + 1) \pm \sqrt{p^2 - 2p + 1}}{2}$$
$$= \frac{(p + 1) \pm (p - 1)}{2}$$
$$= \frac{2p}{2} \text{ or } \frac{2}{2} = p \text{ or } 1$$

Because p is an integer greater than 1, there are two unequal real roots. The answer is (iv).

123. Amar's solutions of 8 and 2 give the equation $(x - 8)(x - 2) = x^2 - 10x + 16 = 0$. Akbar's solutions of –9 and –1 give the equation $(x + 9)(x + 1) = x^2 + 10x + 9 = 0$. We know that Akbar misread the coefficient of the x term, and that Amar misread the constant term, so the correct equation must be $x^2 - 10x + 9 = 0$. Anthony correctly solved the equation:

$$x^2 - 10x + 9 = 0 \Rightarrow (x - 9)(x - 1) = 0 \Rightarrow$$
$$x = 9 \text{ or } x = 1$$

124. C. Solving $x^2+4x-5=0$ gives $(x+5)(x-1)=0 \Rightarrow x=-5,\ 1.$ Solving $|x+2|=3$ gives $x+2=3 \Rightarrow x=1$ or $x+2=-3 \Rightarrow x=-5.$ Since the equations have the same solution set, the equations are equivalent.

125. B. If $k > 4$, then we have

$$\begin{aligned} x^2+4x&=-k \\ x^2+4x+4&=-k+4 \\ (x+2)^2&=-k+4 \\ x+2&=\pm\sqrt{-k+4} \end{aligned}$$

However, if $k > 4$, then $-k + 4$ is negative and $\sqrt{-k+4}$ is not real. Therefore, $k \le 4$.

1.2 Critical Thinking/Discussion/Writing

126. $(x-a)(x-b)=k^2 \Rightarrow$
$x^2-(a+b)x+ab-k^2=0$

$$\begin{aligned} &\left[-(a+b)\right]^2-4(1)\left(ab-k^2\right) \\ &=a^2+2ab+b^2-4ab+4k^2 \\ &=a^2-2ab+b^2+4k^2 \\ &=(a-b)^2+4k^2>0 \end{aligned}$$

Thus, the solutions of the equation are real.

127. $ax(1-x)=1 \Rightarrow ax-ax^2=1 \Rightarrow$
$-ax^2+ax-1=0$
Examining the discriminant, we have
$a^2-4(-a)(-1)=a^2-4a$
$a^2-4a<0 \Rightarrow ax(1-x)=1$ has no real solutions for $0 < a < 4$.

1.2 Maintaining Skills

128. $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)=3^2-\left(\sqrt{2}\right)^2=9-2=7$

129. $\left(1-2\sqrt{7}\right)\left(1+2\sqrt{7}\right)=1^2-\left(2\sqrt{7}\right)^2=1-4\cdot 7$
$=1-28=-27$

130. $\left(2+\sqrt{5}\right)^2=2^2+2\cdot 2\cdot\sqrt{5}+\left(\sqrt{5}\right)^2$
$=4+4\sqrt{5}+5=9+4\sqrt{5}$

131. $\left(\sqrt{2}-\sqrt{3}\right)^2=\left(\sqrt{2}\right)^2-2\cdot\sqrt{2}\cdot\sqrt{3}+\left(\sqrt{3}\right)^2$
$=2-2\sqrt{6}+3=5-2\sqrt{6}$

132. $\dfrac{5+5\sqrt{20}}{5}=1+\sqrt{20}=1+\sqrt{4\cdot 5}=1+2\sqrt{5}$

133. $\dfrac{6+\sqrt{27}}{3}=2+\dfrac{\sqrt{9\cdot 3}}{3}=2+\dfrac{3\sqrt{3}}{3}=2+\sqrt{3}$

134. $\dfrac{16-\sqrt{100}}{2}=\dfrac{16-10}{2}=\dfrac{6}{2}=3$

135. $\dfrac{18-\sqrt{108}}{12}=\dfrac{18-\sqrt{36\cdot 3}}{12}=\dfrac{18-6\sqrt{3}}{12}=\dfrac{3-\sqrt{3}}{2}$

136. $(3x+2)+(x-7)=4x-5$

137. $(5x-9)+(6-x)=4x-3$

138. $(9x+4)-(2x+12)=9x+4-2x-12$
$=7x-8$

139. $(6x-5)-(3x+4)=6x-5-3x-4=3x-9$

140. $(3x+2)(x-9)=3x^2-27x+2x-18$
$=3x^2-25x-18$

141. $(2x-5)(3x+4)=6x^2+8x-15x-20$
$=6x^2-7x-20$

142. $(x-3)(x+3)=x^2-9$

143. $(5x+2)(5x-2)=(5x)^2-2^2=25x^2-4$

Solve each equation in exercises 144–147 using the quadratic formula, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

144. $x^2+4x+1=0$
$a=1,\ b=4,\ c=1$

$$\begin{aligned} x&=\frac{-4\pm\sqrt{4^2-4\cdot 1\cdot 1}}{2\cdot 1}=\frac{-4\pm\sqrt{12}}{2} \\ &=\frac{-4\pm 2\sqrt{3}}{2}=-2\pm\sqrt{3} \end{aligned}$$

145. $x^2+5x+5=0$
$a=1,\ b=5,\ c=5$

$$x=\frac{-5\pm\sqrt{5^2-4\cdot 1\cdot 5}}{2\cdot 1}=\frac{-5\pm\sqrt{5}}{2}$$

146. $5x^2+8x+2=0$
$a=5,\ b=8,\ c=2$

$$\begin{aligned} x&=\frac{-8\pm\sqrt{8^2-4\cdot 5\cdot 2}}{2\cdot 5}=\frac{-8\pm\sqrt{24}}{10} \\ &=\frac{-8\pm 2\sqrt{6}}{10}=\frac{-4\pm\sqrt{6}}{5} \end{aligned}$$

147. $-2x^2+5x-1=0$

$a=-2,\ b=5,\ c=-1$

$$x=\frac{-5\pm\sqrt{5^2-4(-2)(-1)}}{2(-2)}=\frac{-5\pm\sqrt{17}}{-4}$$

$$=\frac{5\pm\sqrt{17}}{4}$$

1.3 Complex Numbers: Quadratic Equations with Complex Solutions

1.3 Practice Problems

1. a. $-1+2i$

real part: −1, imaginary part: 2

b. $-\frac{1}{3}-6i$

real part: $-\frac{1}{3}$, imaginary part: −6

c. $8=8+0i$

real part: 8, imaginary part: 0

2. Let $z=(1-2a)+3i$ and let $w=5-(2b-5)i$. Then

$\operatorname{Re}(z)=\operatorname{Re}(w)$ and $\operatorname{Im}(z)=\operatorname{Im}(w)$

$1-2a=5$, $-2a=4$, $a=-2$

$3=-(2b-5)$, $3=-2b+5$, $-2=-2b$, $1=b$

3. a. $(1-4i)+(3+2i)=4-2i$

b. $(4+3i)-(5-i)=-1+4i$

c. $(3-\sqrt{-9})-(5-\sqrt{-64})=(3-3i)-(5-8i)=-2+5i$

4. a. $(2-6i)(1+4i)=2+8i-6i-24i^2=2+2i+24=26+2i$

b. $-3i(7-5i)=-21i+15i^2=-15-21i$

5. a. $(-3+\sqrt{-4})^2=(-3+2i)^2=(-3)^2+2(-3)(2i)+(2i)^2=9-12i+4i^2=9-12i-4=5-12i$

b. $(5+\sqrt{-2})(4+\sqrt{-8})=(5+i\sqrt{2})(4+2i\sqrt{2})=20+10\sqrt{2}i+4\sqrt{2}i+4i^2=20+14\sqrt{2}i-4=16+14\sqrt{2}i$

6. a. $z=1+6i\Rightarrow\overline{z}=1-6i$

$z\overline{z}=(1+6i)(1-6i)=1-36i^2=1+36=37$

b. $z=-2i\Rightarrow\overline{z}=2i$

$z\overline{z}=(-2i)(2i)=-4i^2=4$

7. a. $\frac{2}{1-i}=\frac{2}{1-i}\cdot\frac{1+i}{1+i}=\frac{2+2i}{1-i^2}=\frac{2+2i}{1+1}=\frac{2+2i}{2}=1+i$

b. $\frac{-3i}{4+\sqrt{-25}}=\frac{-3i}{4+5i}=\frac{-3i}{4+5i}\cdot\frac{4-5i}{4-5i}=\frac{-12i+15i^2}{16-25i^2}=\frac{-15-12i}{16+25}=\frac{-15-12i}{41}=-\frac{15}{41}-\frac{12i}{41}$

8. $Z_t=\frac{Z_1Z_2}{Z_1+Z_2}=\frac{(1+2i)(2-3i)}{(1+2i)+(2-3i)}=\frac{2-3i+4i-6i^2}{3-i}=\frac{2+i+6}{3-i}=\frac{8+i}{3-i}=\frac{8+i}{3-i}\cdot\frac{3+i}{3+i}=\frac{24+8i+3i+i^2}{9-i^2}=\frac{24+11i-1}{9+1}=\frac{23+11i}{10}=\frac{23}{10}+\frac{11}{10}i$

9. a. $4x^2+9=0$

$4x^2=-9$

$x^2=-\frac{9}{4}\Rightarrow x=\pm\sqrt{-\frac{9}{4}}=\pm\frac{3}{2}i$

Solution set: $\left\{-\frac{3}{2}i,\ \frac{3}{2}i\right\}$

b. $x^2=4x-13$

$x^2-4x+13=0$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(13)}}{2(1)}=\frac{4\pm\sqrt{-36}}{2}=\frac{4\pm6i}{2}=2\pm3i$$

Solution set: {2 − 3*i*, 2 + 3*i*)

10. a. $9x^2-6x+1=0\Rightarrow a=9,\ b=-6,\ c=1$

So, $D=(-6)^2-4(9)(1)=36-36=0$.

Therefore, there is one real root.

b. $x^2 - 5x + 3 = 0 \Rightarrow a = 1,\ b = -5,\ c = 3$

So, $D = (-5)^2 - 4(1)(3) = 25 - 12 = 13 > 0$
Therefore, there are two unequal real roots.

c. $2x^2 - 3x + 4 = 0 \Rightarrow a = 2,\ b = -3,\ c = 4$

So, $D = (-3)^2 - 4(2)(4) = 9 - 32 = -23 < 0$
Therefore, there are two nonreal complex solutions.

1.3 Basic Concepts and Skills

1. We define $i = \underline{\sqrt{-1}}$, so that $i^2 = \underline{-1}$.
2. A complex number in the form $a + bi$ is said to be in standard form.
3. For $b > 0$, $\sqrt{-b} = \underline{i\sqrt{b}}$.
4. The conjugate of $a + bi$ is $\underline{a - bi}$, and the conjugate of $a - bi$ is $\underline{a + bi}$.
5. True
6. True

In exercises 7–10, to find the real numbers x and y that make the equation true, set the real parts of the equation equal to each other and then set the imaginary parts of the equation equal to each other.

7. $2 + xi = y + 3i$, so $x = 3$ and $y = 2$.
8. $x - 2i = 7 + yi$, so $x = 7$ and $y = -2$.
9. $x - \sqrt{-16} = 2 + yi$. $\sqrt{-16} = 4i$, so the equation becomes $x - 4i = 2 + yi$.
   $x = 2$ and $y = -4$.
10. $3 + yi = x - \sqrt{-25}$. $\sqrt{-25} = 5i$, so the equation becomes $3 + yi = x - 5i$.
    $x = 3$ and $y = -5$.
11. $(5 + 2i) + (3 + i) = (5 + 3) + (2 + 1)i = 8 + 3i$
12. $(6 + i) + (1 + 2i) = (6 + 1) + (1 + 2)i = 7 + 3i$
13. $(4 - 3i) - (5 + 3i) = (4 - 5) + (-3 - 3)i$
    $= -1 - 6i$
14. $(3 - 5i) - (3 + 2i) = (3 - 3) + (-5 - 2)i = -7i$
15. $(-2 - 3i) + (-3 - 2i) = [-2 + (-3)] + (-3 - 2)i$
    $= -5 - 5i$
16. $(-5 - 3i) + (2 - i) = (-5 + 2) + (-3 - 1)i$
    $= -3 - 4i$
17. $3(5 + 2i) = 3(5) + 3(2i) = 15 + 6i$
18. $4(3 + 5i) = 4(3) + 4(5i) = 12 + 20i$
19. $-4(2 - 3i) = -4(2) - 4(-3i) = -8 + 12i$
20. $-7(3 - 4i) = -7(3) - 7(-4i) = -21 + 28i$
21. $3i(5 + i) = 3i(5) + 3i(i) = 15i + 3i^2$
    Because $i^2 = -1$, $3i^2 = -3$.
    So, $15i + 3i^2 = 15i - 3 = -3 + 15i$.
22. $2i(4 + 3i) = 2i(4) + 2i(3i) = 8i + 6i^2$**.** Because $i^2 = -1$, $6i^2 = -6$. So $8i + 6i^2 = -6 + 8i$.
23. $4i(2 - 5i) = 4i(2) + 4i(-5i) = 8i - 20i^2$.
    Because $i^2 = -1$, $-20i^2 = (-20)(-1) = 20$.
    So, $8i - 20i^2 = 8i + 20 = 20 + 8i$.
24. $-3i(5 - 2i) = -3i(5) - 3i(-2i) = -15i + 6i^2$.
    Because $i^2 = -1$, $6i^2 = -6$.
    So, $-15i + 6i^2 = -15i - 6 = -6 - 15i$.
25. $$\begin{aligned}(3 + i)(2 + 3i) &= 3\cdot 2 + 3\cdot 3i + i\cdot 2 + i\cdot 3i\\ &= 6 + 9i + 2i + 3i^2\\ &= 6 + 11i + 3(-1)\\ &= 3 + 11i\end{aligned}$$
26. $$\begin{aligned}(4 + 3i)(2 + 5i) &= 4\cdot 2 + 4\cdot 5i + 3i\cdot 2 + 3i\cdot 5i\\ &= 8 + 20i + 6i + 15i^2\\ &= 8 + 26i + 15(-1) = -7 + 26i\end{aligned}$$
27. $$\begin{aligned}&(2 - 3i)(2 + 3i)\\ &= 2\cdot 2 + 2\cdot 3i + (-3i)\cdot 2 + (-3i)\cdot 3i\\ &= 4 + 6i - 6i - 9i^2\\ &= 4 - 9(-1) = 4 + 9 = 13\end{aligned}$$
28. $$\begin{aligned}&(4 - 3i)(4 + 3i)\\ &= 4\cdot 4 + 4\cdot 3i + (-3i)\cdot 4 + (-3i)\cdot 3i\\ &= 16 + 12i - 12i - 9i^2\\ &= 16 - 9(-1) = 16 + 9 = 25\end{aligned}$$
29. $$\begin{aligned}&(3 + 4i)(4 - 3i)\\ &= 3\cdot 4 + 3\cdot(-3i) + 4i\cdot 4 + (4i)\cdot(-3i)\\ &= 12 - 9i + 16i - 12i^2\\ &= 12 + 7i - 12(-1)\\ &= 12 + 7i + 12 = 24 + 7i\end{aligned}$$

30. $(-2+3i)(-3+10i)$
$= (-2)\cdot(-3)+(-2)\cdot(10i) + 3i\cdot(-3)+(3i)\cdot(10i)$
$= 6-20i-9i+30i^2$
$= 6-29i+30(-1)$
$= 6-29i-30 = -24-29i$

31. $\left(\sqrt{3}-12i\right)^2$
$= \left(\sqrt{3}-12i\right)\left(\sqrt{3}-12i\right)$
$= \sqrt{3}\cdot\sqrt{3}+\sqrt{3}\cdot(-12i) - 12i\cdot\sqrt{3}+(-12i)\cdot(-12i)$
$= 3-12\sqrt{3}i-12\sqrt{3}i+144i^2$
$= 3-24\sqrt{3}i+144(-1) = 3-24\sqrt{3}i-144$
$= -141-24\sqrt{3}i$

32. $\left(-\sqrt{5}-13i\right)^2$
$= \left(-\sqrt{5}-13i\right)\left(-\sqrt{5}-13i\right)$
$= \left(-\sqrt{5}\right)\cdot\left(-\sqrt{5}\right)+\left(-\sqrt{5}\right)\cdot(-13i) - 13i\cdot\left(-\sqrt{5}\right)+(-13i)\cdot(-13i)$
$= 5+13\sqrt{5}i+13\sqrt{5}i+169i^2$
$= 5+26\sqrt{5}i+169(-1) = 5+26\sqrt{5}i-169$
$= -164+26\sqrt{5}i$

33. $\left(2-\sqrt{-16}\right)(3+5i)$
$= (2-4i)(3+5i)$
$= 2\cdot 3+2\cdot 5i-4i\cdot 3-4i\cdot 5i$
$= 6+10i-12i-20i^2$
$= 6-2i-20(-1)$
$= 6-2i+20$
$= 26-2i$

34. $(5-2i)\left(3+\sqrt{-25}\right)$
$= (5-2i)(3+5i)$
$= 5\cdot 3+5\cdot 5i-2i\cdot 3-2i\cdot 5i$
$= 15+25i-6i-10i^2$
$= 15+19i-10(-1)$
$= 15+19i+10 = 25+19i$

35. If $z = 2-3i$ then $\bar{z} = 2+3i$, and
$z\bar{z} = (2-3i)(2+3i) = 4-9i^2 = 4+9 = 13$.

36. If $z = 4+5i$ then $\bar{z} = 4-5i$, and
$z\bar{z} = (4+5i)(4-5i) = 16-25i^2 = 16+25 = 41$.

37. If $z = \frac{1}{2}-2i$ then $\bar{z} = \frac{1}{2}+2i$, and
$z\bar{z} = \left(\frac{1}{2}-2i\right)\left(\frac{1}{2}+2i\right) = \frac{1}{4}-4i^2 = \frac{1}{4}+4 = \frac{17}{4}$

38. If $z = \frac{2}{3}+\frac{1}{2}i$ then $\bar{z} = \frac{2}{3}-\frac{1}{2}i$, and
$z\bar{z} = \left(\frac{2}{3}-\frac{1}{2}i\right)\left(\frac{2}{3}+\frac{1}{2}i\right) = \frac{4}{9}-\frac{1}{4}i^2 = \frac{4}{9}+\frac{1}{4} = \frac{25}{36}$

39. If $z = \sqrt{2}-3i$ then $\bar{z} = \sqrt{2}+3i$, and
$z\bar{z} = \left(\sqrt{2}-3i\right)\left(\sqrt{2}+3i\right) = 2-9i^2 = 2+9 = 11$

40. If $z = \sqrt{5}+\sqrt{3}i$ then $\bar{z} = \sqrt{5}-\sqrt{3}i$, and
$z\bar{z} = \left(\sqrt{5}+\sqrt{3}i\right)\left(\sqrt{5}-\sqrt{3}i\right) = 5-3i^2 = 5+3 = 8$

41. The denominator is $-i$, so its conjugate is i. Multiply the numerator and denominator by i.
$\frac{5}{-i} = \frac{5i}{-i\cdot i} = \frac{5i}{1} = 5i$

42. The denominator is $-3i$, so its conjugate is $3i$. Multiply the numerator and denominator by $3i$.
$\frac{2}{-3i} = \frac{2(3i)}{-3i\cdot 3i} = \frac{6i}{9} = \frac{2}{3}i$

43. The denominator is $1+i$, so its conjugate is $1-i$. Multiply the numerator and denominator by $1-i$.
$\frac{-1}{1+i} = \frac{-1(1-i)}{(1+i)(1-i)} = \frac{-1+i}{1+1}$
$= \frac{-1+i}{2} = -\frac{1}{2}+\frac{1}{2}i$

44. The denominator is $2-i$, so its conjugate is $2+i$. Multiply the numerator and denominator by $2+i$.
$\frac{1}{2-i} = \frac{1(2+i)}{(2-i)(2+i)} = \frac{2+i}{4+1} = \frac{2+i}{5} = \frac{2}{5}+\frac{1}{5}i$

45. The denominator is $2+i$, so its conjugate is $2-i$. Multiply the numerator and denominator by $2-i$.
$\frac{5i}{2+i} = \frac{5i(2-i)}{(2+i)(2-i)} = \frac{10i-5i^2}{4+1}$
$= \frac{10i+5}{5} = 1+2i$

46. The denominator is $2-i$, so its conjugate is $2+i$. Multiply the numerator and denominator by $2+i$.
$\frac{3i}{2-i} = \frac{3i(2+i)}{(2-i)(2+i)} = \frac{6i+3i^2}{4+1}$
$= \frac{-3+6i}{5} = -\frac{3}{5}+\frac{6}{5}i$

47. The denominator is $1+i$, so its conjugate is $1-i$. Multiply the numerator and denominator by $1-i$.

$$\frac{2+3i}{1+i}=\frac{(2+3i)(1-i)}{(1+i)(1-i)}=\frac{2-2i+3i-3i^2}{1+1}$$
$$=\frac{2+i+3}{2}=\frac{5+i}{2}=\frac{5}{2}+\frac{1}{2}i$$

48. The denominator is $4+i$, so its conjugate is $4-i$. Multiply the numerator and denominator by $4-i$.

$$\frac{3+5i}{4+i}=\frac{(3+5i)(4-i)}{(4+i)(4-i)}=\frac{12-3i+20i-5i^2}{16+1}$$
$$=\frac{12+17i+5}{17}=\frac{17+17i}{17}=1+i$$

49. The denominator is $4-7i$, so its conjugate is $4+7i$. Multiply the numerator and denominator by $4+7i$.

$$\frac{2-5i}{4-7i}=\frac{(2-5i)(4+7i)}{(4-7i)(4+7i)}=\frac{8+14i-20i-35i^2}{16+49}$$
$$=\frac{8-6i+35}{65}=\frac{43-6i}{65}=\frac{43}{65}-\frac{6}{65}i$$

50. The denominator is $1-3i$, so its conjugate is $1+3i$. Multiply the numerator and denominator by $1+3i$.

$$\frac{3+5i}{1-3i}=\frac{(3+5i)(1+3i)}{(1-3i)(1+3i)}=\frac{3+9i+5i+15i^2}{1+9}.$$
$$=\frac{3+14i-15}{10}=\frac{-12+14i}{10}=-\frac{6}{5}+\frac{7}{5}i$$

51. The denominator is 1 i, so its conjugate is $1-i$. Multiply the numerator and denominator by $1-i$.

$$\frac{2+\sqrt{-4}}{1+i}=\frac{(2+2i)}{(1+i)}=\frac{(2+2i)(1-i)}{(1+i)(1-i)}$$
$$=\frac{2-2i+2i-2i^2}{1+1}=\frac{2-2i^2}{2}=\frac{2+2}{2}=2$$

52. The denominator is $3+2i$, so its conjugate is $3-2i$. Multiply the numerator and denominator by $3-2i$.

$$\frac{5-\sqrt{-9}}{3+2i}=\frac{5-3i}{3+2i}=\frac{(5-3i)(3-2i)}{(3+2i)(3-2i)}$$
$$=\frac{15-10i-9i+6i^2}{9+4}=\frac{15-19i+6i^2}{13}$$
$$=\frac{15-19i-6}{13}=\frac{9-19i}{13}=\frac{9}{13}-\frac{19}{13}i$$

53. The denominator is $2-3i$, so its conjugate is $2+3i$. Multiply the numerator and denominator by $2+3i$.

$$\frac{-2+\sqrt{-25}}{2-3i}=\frac{-2+5i}{2-3i}=\frac{(-2+5i)(2+3i)}{(2-3i)(2+3i)}$$
$$=\frac{-4-6i+10i+15i^2}{4+9}$$
$$=\frac{-4+4i-15}{13}=\frac{-19+4i}{13}$$
$$=-\frac{19}{13}+\frac{4}{13}i$$

54. The denominator simplifies to $5-3i$, so its conjugate is $5+3i$. Multiply the numerator and denominator by $5+3i$.

$$\frac{-5-\sqrt{-4}}{5-\sqrt{-9}}=\frac{-5-2i}{5-3i}=\frac{(-5-2i)(5+3i)}{(5-3i)(5+3i)}$$
$$=\frac{-25-15i-10i-6i^2}{25+9}$$
$$=\frac{-25-25i+6}{34}=\frac{-19-25i}{34}$$
$$=-\frac{19}{34}-\frac{25}{34}i$$

55. $x^2+5=1\Rightarrow x^2=-4\Rightarrow x=\pm\sqrt{-4}=\pm 2i$

56. $4x^2+9=0\Rightarrow 4x^2=-9\Rightarrow x^2=-\frac{9}{4}\Rightarrow$
$x=\pm\sqrt{-\frac{9}{4}}=\pm\frac{3}{2}i$

57.
$$z^2-2z+2=0$$
$$z^2-2z=-2$$
Now, complete the square.
$$z^2-2z+1=-2+1$$
$$(z-1)^2=-1\Rightarrow z-1=\pm i\Rightarrow z=1\pm i$$

58.
$$x^2-6x+11=0$$
$$x^2-6x=-11$$
Now, complete the square.
$$x^2-6x+9=-11+9$$
$$(x-3)^2=-2$$
$$x-3=\pm i\sqrt{2}\Rightarrow x=3\pm i\sqrt{2}$$

59.
$$2x^2-20x+49=-7$$
$$2x^2-20x=-56$$
$$x^2-10x=-28$$
Now, complete the square.
$$x^2-10x+25=-28+25\Rightarrow(x-5)^2=-3$$
$$x-5=\pm i\sqrt{3}\Rightarrow x=5\pm i\sqrt{3}$$

60. $4y^2 + 4y + 5 = 0$

$4y^2 + 4y = -5$

$y^2 + y = -\frac{5}{4}$

Now, complete the square.

$y^2 + y + \frac{1}{4} = -\frac{5}{4} + \frac{1}{4}$

$\left(y + \frac{1}{2}\right)^2 = -1 \Rightarrow$

$y + \frac{1}{2} = \pm i$

$y = -\frac{1}{2} \pm i$

61. $8(x^2 - x) = x^2 - 3 \Rightarrow 8x^2 - 8x = x^2 - 3 \Rightarrow$
$7x^2 - 8x + 3 = 0 \Rightarrow a = 7, b = -8, c = 3$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(7)(3)}}{2(7)}$

$= \frac{8 \pm \sqrt{64 - 84}}{14} = \frac{8 \pm \sqrt{-20}}{14}$

$= \frac{8 \pm 2i\sqrt{5}}{14} = \frac{4 \pm i\sqrt{5}}{7}$

$x = \frac{4}{7} + \frac{\sqrt{5}}{7}i$ or $x = \frac{4}{7} - \frac{\sqrt{5}}{7}i$

62. $t(t+1) = 3t^2 + 1 \Rightarrow t^2 + t = 3t^2 + 1 \Rightarrow$
$0 = 2t^2 - t + 1 \Rightarrow a = 2, b = -1, c = 1$

$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(1)}}{2(2)}$

$= \frac{1 \pm \sqrt{1 - 8}}{4} = \frac{1 \pm \sqrt{-7}}{4}$

$= \frac{1 \pm i\sqrt{7}}{4}$

$t = \frac{1}{4} + \frac{\sqrt{7}}{4}i$ or $t = \frac{1}{4} - \frac{\sqrt{7}}{4}i$

63. $9k^2 + 25 = 0 \Rightarrow a = 9, b = 0, c = 25$

$t = \frac{0 \pm \sqrt{0^2 - 4(9)(25)}}{2(9)} = \frac{\pm\sqrt{-900}}{18}$

$= \pm\frac{30i}{18} = \pm\frac{5}{3}i$

64. $3k^2 + 4 = 0 \Rightarrow a = 3, b = 0, c = 4$

$t = \frac{0 \pm \sqrt{0^2 - 4(3)(4)}}{2(3)} = \frac{\pm\sqrt{-48}}{6}$

$= \pm\frac{4i\sqrt{3}}{6} = \pm\frac{2\sqrt{3}}{3}i$

1.3 Applying the Concepts

65. $Z_1 = 4 + 3i$ and $Z_2 = 5 - 2i$.
So, $Z_1 + Z_2 = (4 + 3i) + (5 - 2i) = 9 + i$.

66. $I = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(4+3i)(5-2i)}{(4+3i)+(5-2i)}$

$= \frac{20 - 8i + 15i - 6i^2}{9 + i} = \frac{20 + 7i + 6}{9 + i} = \frac{26 + 7i}{9 + i}$

Now simplify the fraction by multiplying the numerator and denominator by $9 - i$.

$\frac{26 + 7i}{9 + i} = \frac{(26 + 7i)(9 - i)}{(9 + i)(9 - i)}$

$= \frac{234 - 26i + 63i - 7i^2}{81 + 1}$

$= \frac{234 + 37i + 7}{82} = \frac{241 + 37i}{82}$

$= \frac{241}{82} + \frac{37}{82}i$

67. $Z = \frac{V}{I}$, $I = 7 + 5i$, $V = 35 + 70i$. Then,

$Z = \frac{35 + 70i}{7 + 5i}$. Simplify the fraction by multiplying the numerator and denominator by $7 - 5i$.

$\frac{35 + 70i}{7 + 5i} = \frac{(35 + 70i)(7 - 5i)}{(7 + 5i)(7 - 5i)}$

$= \frac{245 - 175i + 490i - 350i^2}{49 + 25}$

$= \frac{245 + 315i + 350}{74} = \frac{595 + 315i}{74}$

$= \frac{595}{74} + \frac{315}{74}i$

68. $Z = \frac{V}{I}$, $I = 7 + 4i$, $V = 45 + 88i$. Then,

$Z = \frac{45 + 88i}{7 + 4i}$. Simplify the fraction by multiplying the numerator and denominator by $7 - 4i$.

$\frac{45 + 88i}{7 + 4i} = \frac{(45 + 88i)(7 - 4i)}{(7 + 4i)(7 - 4i)}$

$= \frac{315 - 180i + 616i - 352i^2}{49 + 16}$

$= \frac{315 + 436i + 352}{65} = \frac{667 + 436i}{65}$

$= \frac{667}{65} + \frac{436}{65}i$

69. $Z = \frac{V}{I}$, $Z = 5 - 7i$, $I = 2 + 5i$. Then,

$$\begin{aligned} V = ZI &= (5-7i)(2+5i) \\ &= 10 + 25i - 14i - 35i^2 \\ &= 10 + 11i + 35 = 45 + 11i \end{aligned}$$

70. $Z = \frac{V}{I}$, $Z = 7 - 8i$, $I = \frac{1}{3} + \frac{1}{6}i$. Then,

$$\begin{aligned} V = ZI &= (7-8i)\left(\frac{1}{3} + \frac{1}{6}i\right) \\ &= \frac{7}{3} + \frac{7}{6}i - \frac{8}{3}i - \frac{8}{6}i^2 \\ &= \frac{7}{3} + \frac{7}{6}i - \frac{16}{6}i + \frac{4}{3} \\ &= \frac{11}{3} - \frac{9}{6}i = \frac{11}{3} - \frac{3}{2}i \end{aligned}$$

71. $Z = \frac{V}{I}$, $V = 12 + 10i$, $Z = 12 + 6i$. Then, $I = \frac{V}{Z} = \frac{12+10i}{12+6i}$. Simplify the fraction by multiplying the numerator and denominator by $12 - 6i$.

$$\begin{aligned} \frac{12+10i}{12+6i} &= \frac{(12+10i)(12-6i)}{(12+6i)(12-6i)} \\ &= \frac{144 - 72i + 120i - 60i^2}{144+36} \\ &= \frac{144 + 48i + 60}{180} \\ &= \frac{204 + 48i}{180} = \frac{17}{15} + \frac{4}{15}i \end{aligned}$$

72. $Z = \frac{V}{I}$, $V = 29 + 18i$, $Z = 25 + 6i$. Then, $I = \frac{V}{Z} = \frac{29+18i}{25+6i}$. Simplify the fraction by multiplying the numerator and denominator by $25 - 6i$.

$$\begin{aligned} \frac{29+18i}{25+6i} &= \frac{(29+18i)(25-6i)}{(25+6i)(25-6i)} \\ &= \frac{725 - 174i + 450i - 108i^2}{625+36} \\ &= \frac{725 + 276i + 108}{661} \\ &= \frac{833 + 276i}{661} = \frac{833}{661} + \frac{276}{661}i \end{aligned}$$

1.3 Beyond the Basics

73. To find i^{17}, first divide 17 by 4. The remainder is 1, so $i^{17} = i^1 = i$.

74. To find i^{125}, first divide 125 by 4. The remainder is 1, so $i^{125} = i^1 = i$.

75. To find i^{-7}, first rewrite it as $\frac{1}{i^7}$. Then divide 7 by 4. The remainder is 3, so $\frac{1}{i^7} = \frac{1}{i^3}$. Simplify the fraction by multiplying the numerator and denominator by i.

$\frac{1}{i^3} \cdot \frac{i}{i} = \frac{i}{i^4} = i$.

76. To find i^{-24}, first rewrite it as $\frac{1}{i^{24}}$. Then divide 24 by 4. The remainder is 0, so $\frac{1}{i^{24}} = \frac{1}{i^0} = 1$.

77. To find i^{10}, first divide 10 by 4. The remainder is 2, so $i^{10} = i^2 = -1$. So $i^{10} + 7 = -1 + 7 = 6$.

78. $i^3 = -i$, so $9 + i^3 = 9 - i$.

79. To find i^5, first divide 5 by 4. The remainder is 1, so $i^5 = i$. So $3i^5 = 3i$. $i^3 = -i$, so $-2i^3 = 2i$. Then, $3i^5 - 2i^3 = 3i + 2i = 5i$.

80. To find i^6, first divide 6 by 4. The remainder is 2, so $i^6 = i^2 = -1$. So $5i^6 = -5$. $i^4 = 1$, so $-3i^4 = -3$. Then, $5i^6 - 3i^4 = -5 - 3 = -8$.

81. $i^3 = -i$, so $2i^3 = -2i$. $i^4 = 1$, so $1 + i^4 = 1 + 1 = 2$. Then $2i^3(1 + i^4) = -2i(2) = -4i$.

82. To find i^5, first divide 5 by 4. The remainder is 1, so $i^5 = i$. So $5i^5 = 5i$. $i^3 = -i$, so $i^3 - i = -i - i = -2i$. Then, $5i^5(i^3 - i) = 5i(-2i) = -10i^2 = 10$.

83.
$$\begin{aligned} \frac{1}{a+bi} &= \frac{1(a-bi)}{(a+bi)(a-bi)} = \frac{a-bi}{a^2+b^2} \\ &= \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i \end{aligned}$$

84. $z = a + bi$, so $\text{Re}(z) = a$.

$$\frac{z + \bar{z}}{2} = \frac{(a+bi)+(a-bi)}{2} = \frac{2a}{2} = a = \text{Re}(z)$$

85. $z = a + bi$, so $\text{Im}(z) = b$.

$$\frac{z - \overline{z}}{2i} = \frac{(a+bi)-(a-bi)}{2i} = \frac{2bi}{2i} = b = \text{Im}(z)$$

86. $$\text{Re}\left(\frac{z}{z+w}\right) = \text{Re}\left(\frac{a+bi}{(a+bi)+(c+di)}\right) = \text{Re}\left(\frac{a+bi}{(a+c)+(b+d)i}\right) = \text{Re}\left(\frac{(a+bi)\left[(a+c)-(b+d)i\right]}{\left[(a+c)+(b+d)i\right]\left[(a+c)-(b+d)i\right]}\right)$$
$$= \frac{a(a+c)-bi(b+d)i}{(a+c)^2+(b+d)^2} = \frac{a(a+c)+b(b+d)}{(a+c)^2+(b+d)^2}$$

Note that when we simplify the fraction, in the numerator we multiply only the first and last terms because we need only the real terms. Multiplying the inside and outside terms give the imaginary terms

$$\text{Re}\left(\frac{w}{z+w}\right) = \text{Re}\left(\frac{c+di}{(a+bi)+(c+di)}\right) = \text{Re}\left(\frac{c+di}{(a+c)+(b+d)i}\right) = \text{Re}\left(\frac{(c+di)\left[(a+c)-(b+d)i\right]}{\left[(a+c)+(b+d)i\right]\left[(a+c)-(b+d)i\right]}\right)$$
$$= \frac{c(a+c)-di(b+d)i}{(a+c)^2+(b+d)^2} = \frac{c(a+c)+d(b+d)}{(a+c)^2+(b+d)^2}$$

So, $\text{Re}\left(\frac{z}{z+w}\right) + \text{Re}\left(\frac{w}{z+w}\right) = \frac{a(a+c)+b(b+d)}{(a+c)^2+(b+d)^2} + \frac{c(a+c)+d(b+d)}{(a+c)^2+(b+d)^2} = \frac{(a+c)^2+(b+d)^2}{(a+c)^2+(b+d)^2} = 1$.

87. $z\overline{z} = (a+bi)(a-bi) = a^2 + b^2$

$a^2 + b^2 = 0$ if and only if $a = 0$ and $b = 0$.

So, $z = 0 + 0i = 0$.

88. $\frac{x}{i} + y = 3 + i \Rightarrow \frac{x}{i} = i \Rightarrow x = i^2 \Rightarrow x = -1$ and $y = 3$.

89. $x - \frac{y}{i} = 4i + 1 \Rightarrow x = 1$

$-\frac{y}{i} = 4i \Rightarrow -y = 4i^2 \Rightarrow -y = -4 \Rightarrow y = 4$

90. $\frac{x+yi}{i} = 5 - 7i \Rightarrow x + yi = i(5-7i)$

$\Rightarrow x + yi = 7 + 5i \Rightarrow x = 7, y = 5.$

91. $\frac{5x+yi}{2-i} = 2 + i \Rightarrow 5x + yi = (2+i)(2-i)$

$\Rightarrow 5x + yi = 5 \Rightarrow x = 1, y = 0.$

92. $$\frac{1-2i}{5-5i} = \frac{1-2i}{5-5i}\cdot\frac{5+5i}{5+5i} = \frac{5+5i-10i-10i^2}{25-25i^2}$$
$$= \frac{5-5i+10}{25+25} = \frac{15-5i}{50} = \frac{3}{10} - \frac{1}{10}i$$
$$\frac{2-3i}{9-7i} = \frac{2-3i}{9-7i}\cdot\frac{9+7i}{9+7i} = \frac{18+14i-27i-21i^2}{81-49i^2}$$
$$= \frac{18-13i+21}{81+49} = \frac{39-13i}{130} = \frac{3}{10} - \frac{1}{10}i$$
$$\frac{1-2i}{5-5i} = \frac{3}{10} - \frac{1}{10}i = \frac{2-3i}{9-7i} \Rightarrow \frac{1-2i}{5-5i} = \frac{2-3i}{9-7i}$$

93. $$\frac{1+i}{1-i} \div \frac{2+i}{1+2i} = \frac{1+i}{1-i}\cdot\frac{1+2i}{2+i} = \frac{1+2i+i+2i^2}{2+i-2i-i^2}$$
$$= \frac{1+3i-2}{2-i+1} = \frac{-1+3i}{3-i}$$
$$= \frac{-1+3i}{3-i}\cdot\frac{3+i}{3+i} = \frac{-3-i+9i+3i^2}{9-i^2}$$
$$= \frac{-3+8i-3}{9+1} = \frac{-6+8i}{10} = -\frac{3}{5} + \frac{4}{5}i$$

94. $$(1+3i)z + (2+4i) = 7 - 3i$$
$$(1+3i)z = 7 - 3i - (2+4i) = 5 - 7i$$
$$z = \frac{5-7i}{1+3i} = \frac{5-7i}{1+3i}\cdot\frac{1-3i}{1-3i}$$
$$= \frac{5-15i-7i+21i^2}{1-9i^2}$$
$$= \frac{5-22i-21}{1+9} = \frac{-16-22i}{10}$$
$$= -\frac{8}{5} - \frac{11}{5}i$$

95. $z = 2 - 3i,\ w = 1 + 2i$

a. $$\overline{(zw)} = \overline{((2-3i)(1+2i))} = \overline{(2+4i-3i-6i^2)}$$
$$= \overline{(8+i)} = 8 - i$$
$$(\overline{z})(\overline{w}) = (2+3i)(1-2i) = 2 - 4i + 3i - 6i^2$$
$$= 8 - i$$

b. $$\frac{z}{w} = \frac{2-3i}{1+2i} = \frac{2-3i}{1+2i}\cdot\frac{1-2i}{1-2i} = \frac{2-4i-3i+6i^2}{1-4i^2}$$
$$= \frac{-4-7i}{5} = -\frac{4}{5} - \frac{7}{5}i$$
$$\overline{\left(\frac{z}{w}\right)} = -\frac{4}{5} + \frac{7}{5}i$$

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(*continued*)

$$\frac{\overline{z}}{\overline{w}} = \frac{2+3i}{1-2i} = \frac{2+3i}{1-2i}\cdot\frac{1+2i}{1+2i}$$
$$= \frac{2+4i+3i+6i^2}{1-4i^2} = \frac{-4+7i}{5} = -\frac{4}{5}+\frac{7}{5}i$$

1.3 Critical Thinking/Discussion/Writing

96. a. True. Every real number a can be written as a complex number $a+0i$.

b. False.

c. False. A complex number with the form $a+0i$ does not have an imaginary component.

d. True

e. True. $(a+bi)(a-bi)=a^2+b^2$. There is no imaginary component.

f. True

97. $\frac{1+i}{1-i} = \frac{1+i}{1-i}\cdot\frac{1+i}{1+i} = \frac{1+2i-1}{2} = \frac{2i}{2} = i$

$i^n = 1 \Rightarrow n = 4$

98. If $i=0$, then $i^2=0^2 \Rightarrow -1=0$, which is a contradiction. If $i<0$, then $i\cdot i > 0\cdot i$ (since i is negative) $\Rightarrow i^2>0 \Rightarrow -1>0$, a contradiction.

If $i>0$, then $i\cdot i>0\cdot i \Rightarrow i^2>0 \Rightarrow -1>0$, a contradiction. Thus, the set of complex numbers does not have the ordering properties of the set of real numbers.

1.3 Maintaining Skills

99.
$$\begin{aligned} x^3-x^2-9x+9 &= \left(x^3-x^2\right)-(9x-9) \\ &= x^2(x-1)-9(x-1) \\ &= \left(x^2-9\right)(x-1) \\ &= (x-3)(x+3)(x-1) \end{aligned}$$

100.
$$\begin{aligned} -x^3-2x^2+25x+50 &= -\left(x^3+2x^2\right)+(25x+50) \\ &= -x^2(x+2)+25(x+2) \\ &= \left(25-x^2\right)(x+2) \\ &= -\left(x^2-25\right)(x+2) \\ &= -(x-5)(x+5)(x+2) \end{aligned}$$

101. $\frac{1}{x-1}, \frac{1}{x}$

LCD $= x(x-1)$

102. $\frac{3}{2-x}, \frac{x}{x-2}$

Since $x-2=-(2-x)$, the LCD is $x-2$.

103. $\frac{15}{x+3}, \frac{2x+1}{x-3}$

LCD $=(x+3)(x-3)$

104. $\frac{3x}{x^2-25}, \frac{9}{5-x}$

Since $x^2-25=(x-5)(x+5)$ and $5-x=-(x-5)$, the LCD is x^2-25.

105. $\frac{x+1}{(x-3)(2-x)}, \frac{12}{(x-2)(x+1)}$

Note that $x-2=-(2-x)$.

The LCD is $(x-3)(x-2)(x+1)$.

106. $\frac{x}{4x^2-1}, \frac{7}{2x-1}, \frac{1}{x}$

Note that $4x^2-1=(2x-1)(2x+1)$.

The LCD is $x(2x-1)(2x+1)$.

107. $27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

108. $8^{4/3} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$

109. $4^{5/2} = \left(\sqrt{4}\right)^5 = 2^5 = 32$

110. $25^{3/2} = \left(\sqrt{25}\right)^3 = 5^3 = 125$

111. $\left(\sqrt{3+x^2}\right)^2 = 3+x^2$

112.
$$\begin{aligned} \left(1+\sqrt{x+1}\right)^2 &= \left(1+\sqrt{x+1}\right)\left(1+\sqrt{x+1}\right) \\ &= 1+2\sqrt{x+1}+x+1 \\ &= x+2\sqrt{x+1}+2 \end{aligned}$$

113. $\left(\left(3x^2+7\right)^{1/3}\right)^3 = \left(3x^2+7\right)^{(1/3)(3)} = 3x^2+7$

114. $\left((5x-2)^{3/5}\right)^{5/3} = (5x-2)^{(3/5)(5/3)} = 5x-2$

1.4 Solving Other Types of Equations

1.4 Practice Problems

1. $x^3 + 2x^2 - x - 2 = 0$

$x^2(x+2) - (x+2) = 0$

$(x^2 - 1)(x+2) = 0$

$(x-1)(x+1)(x+2) = 0$

$x - 1 = 0 \Rightarrow x = 1$ or $x + 1 = 0 \Rightarrow x = -1$ or $x + 2 = 0 \Rightarrow x = -2$

Solution set: $\{-2, -1, 1\}$

2. $x^4 = 4x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2(x-2)(x+2) = 0$

$x^2 = 0 \Rightarrow x = 0$ or $x - 2 = 0 \Rightarrow x = 2$ or $x + 2 = 0 \Rightarrow x = -2$

Solution set: $\{-2, 0, 2\}$

3. $x^3 - 5x^2 = 4x - 20$

$x^3 - 5x^2 - 4x + 20 = 0$

$x^2(x-5) - 4(x-5) = 0$

$(x^2 - 4)(x-5) = 0$

$(x-2)(x+2)(x-5) = 0$

$x - 2 = 0 \Rightarrow x = 2$ or $x + 2 = 0 \Rightarrow x = -2$ or $x - 5 = 0 \Rightarrow x = 5$

Solution set: $\{-2, 2, 5\}$

4. $\frac{1}{x} - \frac{12}{5x+10} = \frac{1}{5}$

$\frac{1}{x} - \frac{12}{5(x+2)} = \frac{1}{5}$

Multiply by the LCD, $5x(x+2)$.

$5(x+2) - 12x = x(x+2)$

$5x + 10 - 12x = x^2 + 2x$

$0 = x^2 + 9x - 10$

$0 = (x+10)(x-1)$

$x + 10 = 0 \Rightarrow x = -10$ or $x - 1 = 0 \Rightarrow x = 1$

Be sure to check the solutions in the original equation.

Solution set: $\{-10, 1\}$

5. $\frac{x}{x-2} - \frac{2}{x+2} = \frac{4x}{x^2-4}$

$\frac{x}{x-2} - \frac{2}{x+2} = \frac{4x}{(x-2)(x+2)}$

Multiply by the common denominator $(x-2)(x+2)$.

$x(x+2) - 2(x-2) = 4x$

$x^2 + 2x - 2x + 4 = 4x$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0 \Rightarrow x = 2$

Since neither $\frac{x}{x-2}$ nor $\frac{4x}{x^2-4}$ is defined for $x = 2$, this is an extraneous solution.

Solution set: $\varnothing$

6. $x = \sqrt{x^3 - 6x}$

$x^2 = \left(\sqrt{x^3 - 6x}\right)^2 = x^3 - 6x$

$0 = x^3 - x^2 - 6x = x(x^2 - x - 6)$

$= x(x-3)(x+2)$

$x = 0$ or $x - 3 = 0 \Rightarrow x = 3$ or $x + 2 = 0 \Rightarrow x = -2$

Now check to see if -2, 0, or 3 are extraneous solutions. If $x = -2$, then $\sqrt{(-2)^3 - 6(-2)} = \sqrt{4} = 2 \neq -2$, so -2 is an extraneous solution. If $x = 0$, then $\sqrt{0^3 - 6(0)} = \sqrt{0} = 0$, so 0 is a solution.

If $x = 3$, then $\sqrt{3^3 - 6(3)} = \sqrt{9} = 3$, so 3 is a solution.

Solution set: $\{0, 3\}$

7. $\sqrt{6x+4} + 2 = x$

$\sqrt{6x+4} = x - 2$

$\left(\sqrt{6x+4}\right)^2 = (x-2)^2$

$6x + 4 = x^2 - 4x + 4$

$0 = x^2 - 10x = x(x-10)$

$x = 0$ or $x - 10 = 0 \Rightarrow x = 10$

Now check to see if 0 or 10 are extraneous solutions. If $x = 0$, then $\sqrt{6(0)+4} + 2 = \sqrt{4} + 2 = 2 + 2 = 4 \neq 0$, so 0 is an extraneous solution. If $x = 10$, then $\sqrt{6(10)+4} + 2 = \sqrt{64} + 2 = 8 + 2 = 10$, so 10 is a solution.

Solution set: $\{10\}$

8. $\sqrt{x-5}+\sqrt{x}=5$

$$\sqrt{x-5}=5-\sqrt{x}$$
$$\left(\sqrt{x-5}\right)^2=\left(5-\sqrt{x}\right)^2$$
$$x-5=25-10\sqrt{x}+x$$
$$-30=-10\sqrt{x}$$
$$3=\sqrt{x}\Rightarrow 9=x$$

Now check to see if 9 is an extraneous solution.
$\sqrt{9-5}+\sqrt{9}=\sqrt{4}+\sqrt{9}=2+3=5,$ so 9 is a solution.
Solution set: {9}

9. a. $3(x-2)^{3/5}+4=7$

$$3(x-2)^{3/5}=3$$
$$(x-2)^{3/5}=1$$
$$\left[(x-2)^{3/5}\right]^{5/3}=1^{5/3}$$
$$x-2=1$$
$$x=3$$

Be sure to check that 3 satisfies the original equation.
Solution set: {3}

b. $(2x+1)^{4/3}-7=9$

$$(2x+1)^{4/3}=16$$
$$\left[(2x+1)^{4/3}\right]^{3/4}=\pm\left(16^{3/4}\right)$$
$$2x+1=\pm 8$$

$2x+1=-8$	$2x+1=8$
$2x=-9$	$2x=7$
$x=-\frac{9}{2}$	$x=\frac{7}{2}$

Check:

$$\left[2\left(-\frac{9}{2}\right)+1\right]^{4/3}-7\stackrel{?}{=}9$$
$$(-9+1)^{4/3}-7\stackrel{?}{=}9$$
$$(-8)^{4/3}-7\stackrel{?}{=}9$$
$$16-7=9$$
$$\left[2\left(\frac{7}{2}\right)+1\right]^{4/3}-7\stackrel{?}{=}9$$
$$(7+1)^{4/3}-7\stackrel{?}{=}9$$
$$(8)^{4/3}-7\stackrel{?}{=}9$$
$$16-7=9$$

Solution set: $\left\{-\frac{9}{2}, \frac{7}{2}\right\}$

10. $x^{2/3}-7x^{1/3}+6=0$

Let $u=x^{1/3}$. Then the original equation becomes
$u^2-7u+6=0\Rightarrow(u-1)(u-6)=0\Rightarrow$
$u=1, 6$
Now solve for x:
$1=x^{1/3}\Rightarrow 1^3=\left(x^{1/3}\right)^3\Rightarrow 1=x$
$6=x^{1/3}\Rightarrow 6^3=\left(x^{1/3}\right)^3\Rightarrow 216=x$
Be sure to check that both solutions satisfy the original equation.
Solution set: {1, 216}

11. $\left(1+\frac{1}{x}\right)^2-6\left(1+\frac{1}{x}\right)+8=0$

Let $u=\left(1+\frac{1}{x}\right)$. Then the original equation becomes
$u^2-6u+8=0\Rightarrow(u-2)(u-4)=0\Rightarrow u=2, 4$
Now solve for x:
$1+\frac{1}{x}=2\Rightarrow x+1=2x\Rightarrow x=1$
$1+\frac{1}{x}=4\Rightarrow x+1=4x\Rightarrow 1=3x\Rightarrow\frac{1}{3}=x$
Be sure to check that both solutions satisfy the original equation.
Solution set: $\left\{\frac{1}{3}, 1\right\}$

12. $t_0=t\sqrt{1-\frac{v^2}{c^2}},\ t_0=20,\ t=25$

$20=25\sqrt{1-\frac{v^2}{c^2}}\Rightarrow\frac{4}{5}=\sqrt{1-\frac{v^2}{c^2}}\Rightarrow$
$\frac{16}{25}=1-\frac{v^2}{c^2}\Rightarrow\frac{v^2}{c^2}=1-\frac{16}{25}=\frac{9}{25}\Rightarrow$
$\frac{v}{c}=\frac{3}{5}\Rightarrow v=0.6c$
The spacecraft must have been traveling at 60% of the speed of light.

1.4 Basic Concepts and Skills

1. If an apparent solution does not satisfy the equation, it is called an extraneous solution.
2. When solving a rational equation, we multiply both sides by the least common denominator (LCD).
3. If $x^{3/4}=8$, then $x=16$.

4. If $x^{4/3} = 16$, then $x = \underline{-8}$ or $x = \underline{8}$.

5. False

6. True

7. $x^3 = 2x^2 \Rightarrow x^3 - 2x^2 = 0 \Rightarrow x^2(x-2) = 0 \Rightarrow$
$x^2 = 0$ or $x - 2 = 0 \Rightarrow x = 0$ or $x = 2$

8. $3x^4 - 27x^2 = 0$
Divide both sides by 3.
$x^4 - 9x^2 = 0$
$x^2(x^2 - 9) = 0$
$x^2(x-3)(x+3) = 0$
$x^2 = 0$ or $x - 3 = 0$ or $x + 3 = 0$
$x = 0$ or $x = 3$ or $x = -3$

9. $\left(\sqrt{x}\right)^3 = \sqrt{x} \Rightarrow \left(\sqrt{x}\right)^2 \sqrt{x} = \sqrt{x} \Rightarrow$
$x\sqrt{x} = \sqrt{x} \Rightarrow x\sqrt{x} - \sqrt{x} = 0 \Rightarrow$
$\sqrt{x}(x-1) = 0 \Rightarrow \sqrt{x} = 0$ or $(x-1) = 0 \Rightarrow$
$x = 0$ or $x = 1$

10. $\left(\sqrt{x}\right)^5 = 16\sqrt{x} \Rightarrow \left(\sqrt{x}\right)^4 \sqrt{x} = 16\sqrt{x} \Rightarrow$
$x^2\sqrt{x} = 16\sqrt{x} \Rightarrow x^2\sqrt{x} - 16\sqrt{x} = 0 \Rightarrow$
$\sqrt{x}(x^2 - 16) = 0 \Rightarrow \sqrt{x}(x-4)(x+4) = 0 \Rightarrow$
$\sqrt{x} = 0$ or $(x-4)$ or $(x+4) = 0 \Rightarrow$
$x = 0$ or $x = 4$ or $x = -4$
We are looking for real roots only, so we reject $x = -4$. Solution set: $\{0, 4\}$

11. $x^3 + x = 0 \Rightarrow x(x^2 + 1) = 0 \Rightarrow$
$x = 0$ or $x^2 + 1 = 0 \Rightarrow x = 0$ or $x^2 = -1 \Rightarrow$
$x = 0$ or $x = i$
We are looking for real roots only, so we reject $x = i$. Solution set: $\{0\}$

12. $x^3 - 1 = 0 \Rightarrow x^3 = 1 \Rightarrow x = 1$. Note that $x^3 - 1 = 0$ is the difference of two cubes and can be factored into $(x-1)(x^2 + x + 1) = 0$.
Solving $x^2 + x + 1 = 0$ gives $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.
However, we are looking for the real roots only, so we reject these roots.
Solution set: $\{1\}$

13.
$$x^4 - x^3 = x^2 - x$$
$$x^4 - x^3 - x^2 + x = 0$$
Factor by grouping.
$$x^3(x-1) - x(x-1) = 0$$
$$(x^3 - x)(x-1) = 0$$
$$x(x^2 - 1)(x-1) = 0$$
$$x(x+1)(x-1)(x-1) = 0$$
$$x = 0 \text{ or } x = -1 \text{ or } x = 1$$

14.
$$x^3 - 36x = 16(x-6)$$
$$x(x^2 - 36) = 16(x-6)$$
$$x(x+6)(x-6) = 16(x-6)$$
$$x(x+6)(x-6) - 16(x-6) = 0$$
$$(x-6)[x(x+6) - 16] = 0$$
$$(x-6)(x^2 + 6x - 16) = 0$$
$$(x-6)(x+8)(x-2) = 0$$
$$x = 6 \text{ or } x = -8 \text{ or } x = 2$$

15.
$$x^4 = 27x$$
$$x^4 - 27x = 0$$
$$x(x^3 - 27) = 0$$
$$x(x^3 - 3^3) = 0$$
$$x(x-3)(x^2 + 3x + 9) = 0$$
$$x = 0 \text{ or } x = 3 \text{ or } x^2 + 3x + 9 = 0$$
Solving $x^2 + 3x + 9 = 0$ gives $x = -\frac{3}{2} \pm \frac{3i\sqrt{3}}{2}$. However, we are looking for the real roots only, so we reject these roots.
Solution set: $\{0, 3\}$

16.
$$3x^4 = 24x$$
$$3x^4 - 24x = 0$$
$$3x(x^3 - 8) = 0$$
$$3x(x^3 - 2^3) = 0$$
$$3x(x-2)(x^2 + 2x + 4) = 0$$
$$x = 0 \text{ or } x = 2 \text{ or } x^2 + 2x + 4 = 0$$
Solving $x^2 + 2x + 4 = 0$ gives $x = -1 \pm i\sqrt{3}$.
However, we are looking for the real roots only, so we reject these roots.
Solution set: $\{0, 2\}$

17.
$$\frac{x+1}{3x-2} = \frac{5x-4}{3x+2}$$
$$(x+1)(3x+2) = (5x-4)(3x-2)$$
$$3x^2 + 5x + 2 = 15x^2 - 22x + 8$$
$$-12x^2 + 27x - 6 = 0$$
$$-3(4x^2 - 9x + 2) = 0$$
$$4x^2 - 9x + 2 = 0 \Rightarrow (4x-1)(x-2) = 0$$
$$4x - 1 = 0 \text{ or } x - 2 = 0 \Rightarrow x = \frac{1}{4} \text{ or } x = 2$$

18. $$\frac{x}{2x+1} = \frac{3x+2}{4x+3}$$
$$x(4x+3) = (2x+1)(3x+2)$$
$$4x^2 + 3x = 6x^2 + 7x + 2$$
$$-2x^2 - 4x - 2 = 0$$
$$-2(x^2 + 2x + 1) = 0$$
$$x^2 + 2x + 1 = 0$$
$$(x+1)^2 = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$$

19. $$\frac{1}{x} + \frac{2}{x+1} = 1$$
$$(x)(x+1)\left(\frac{1}{x} + \frac{2}{x+1}\right) = (x)(x+1)(1)$$
$$(x+1) + 2x = x^2 + x$$
$$3x + 1 = x^2 + x$$
$$0 = x^2 - 2x - 1$$
Solve using the quadratic formula.
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2}$$
$$x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

20. $$\frac{x}{x+1} + \frac{x+1}{x+2} = 1$$
$$(x+1)(x+2)\left(\frac{x}{x+1} + \frac{x+1}{x+2}\right) = (x+1)(x+2)(1)$$
$$x(x+2) + (x+1)(x+1) = x^2 + 3x + 2$$
$$x^2 + 2x + x^2 + 2x + 1 = x^2 + 3x + 2$$
$$2x^2 + 4x + 1 = x^2 + 3x + 2$$
$$x^2 + x - 1 = 0$$
Solve using the quadratic formula.
$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$$
$$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

21. $$\frac{6x-7}{x} - \frac{1}{x^2} = 5$$
$$x^2\left(\frac{6x-7}{x} - \frac{1}{x^2}\right) = 5x^2$$
$$x(6x-7) - 1 = 5x^2$$
$$6x^2 - 7x - 1 = 5x^2$$
$$x^2 - 7x - 1 = 0$$
Solve using the quadratic formula.
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-1)}}{2}$$
$$x = \frac{7 \pm \sqrt{49+4}}{2} = \frac{7 \pm \sqrt{53}}{2} = \frac{7}{2} \pm \frac{\sqrt{53}}{2}$$

22. $$\frac{1}{x} + \frac{2}{x+1} + \frac{3}{x+2} = 0$$
$$x(x+1)(x+2)\left(\frac{1}{x} + \frac{2}{x+1} + \frac{3}{x+2} = 0\right)$$
$$(x+1)(x+2) + 2x(x+2) + 3x(x+1) = 0$$
$$x^2 + 3x + 2 + 2x^2 + 4x + 3x^2 + 3x = 0$$
$$6x^2 + 10x + 2 = 0$$
$$2(3x^2 + 5x + 1) = 0$$
$$3x^2 + 5x + 1 = 0$$
Solve using the quadratic formula.
$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(1)}}{2(3)}$$
$$x = \frac{-5 \pm \sqrt{25-12}}{6} = \frac{-5 \pm \sqrt{13}}{6} = -\frac{5}{6} \pm \frac{\sqrt{13}}{6}$$

23. $$\frac{1}{x} + \frac{1}{x-3} = \frac{7}{3x-5}$$
$$x(x-3)(3x-5)\left(\frac{1}{x} + \frac{1}{x-3} = \frac{7}{3x-5}\right)$$
$$(x-3)(3x-5) + x(3x-5) = 7x(x-3)$$
$$3x^2 - 14x + 15 + 3x^2 - 5x = 7x^2 - 21x$$
$$6x^2 - 19x + 15 = 7x^2 - 21x$$
$$-x^2 + 2x + 15 = 0$$
$$-1(x^2 - 2x - 15) = 0$$
$$x^2 - 2x - 15 = 0$$
$$(x-5)(x+3) = 0 \Rightarrow$$
$$x - 5 = 0 \text{ or } x + 3 = 0 \Rightarrow x = 5 \text{ or } x = -3$$

24. $$\frac{x+3}{x-1} + \frac{x+4}{x+1} = \frac{8x+5}{x^2-1}$$
$$(x^2-1)\left(\frac{x+3}{x-1} + \frac{x+4}{x+1}\right) = (x^2-1)\left(\frac{8x+5}{x^2-1}\right)$$
$$(x+1)(x+3) + (x-1)(x+4) = 8x + 5$$
$$x^2 + 4x + 3 + x^2 + 3x - 4 = 8x + 5$$
$$2x^2 + 7x - 1 = 8x + 5$$
$$2x^2 - x - 6 = 0$$
$$(2x+3)(x-2) = 0$$
$$2x + 3 = 0 \text{ or } x - 2 = 0 \Rightarrow x = -\frac{3}{2} \text{ or } x = 2$$

25. $\frac{5}{x+1}-\frac{4}{2x+2}+\frac{2}{2x-1}=\frac{13}{18}$

$\frac{5}{x+1}-\frac{4}{2(x+1)}+\frac{2}{2x-1}=\frac{13}{18}$

$18(x+1)(2x-1)\left(\frac{5}{x+1}-\frac{4}{2(x+1)}+\frac{2}{2x-1}\right)$

$=18(x+1)(2x-1)\left(\frac{13}{18}\right)$

$90(2x-1)-36(2x-1)+36(x+1)$

$=13(x+1)(2x-1)$

$180x-90-72x+36+36x+36$

$=13(2x^2+x-1)$

$144x-18=26x^2+13x-13$

$0=26x^2-131x+5$

$0=(26x-1)(x-5)$

$26x-1=0$ or $x-5=0$

$x=\frac{1}{26}$ or $x=5$

26. $\frac{6}{2x-2}-\frac{1}{x+1}=\frac{2}{2x+2}+1$

$\frac{3}{x-1}-\frac{1}{x+1}=\frac{1}{x+1}+1$

$(x+1)(x-1)\left(\frac{3}{x-1}-\frac{1}{x+1}\right)=$

$(x+1)(x-1)\left(\frac{1}{x+1}+1\right)$

$3(x+1)-(x-1)=x-1+x^2-1$

$3x+3-x+1=x^2+x-2$

$2x+4=x^2+x-2$

$0=x^2-x-6$

$0=(x-3)(x+2)$

$x-3=0$ or $x+2=0$

$x=3$ or $x=-2$

27. $\frac{x}{x-5}-\frac{5}{x+5}=\frac{10x}{x^2-25}$

$x(x+5)-5(x-5)=10x$

$x^2+5x-5x+25=10x$

$x^2-10x+25=0$

$(x-5)^2=0\Rightarrow x=5$

Since $x = 5$ makes the denominator in the first fraction equal zero, there is no solution. Solution set: $\varnothing$

28. $\frac{x}{x-4}-\frac{4}{x+4}=\frac{8x}{x^2-16}$

$x(x+4)-4(x-4)=8x$

$x^2+4x-4x+16=8x$

$x^2-8x+16=0$

$(x-4)^2=0\Rightarrow x=4$

Since $x = 4$ makes the denominator in the first fraction equal zero, there is no solution. Solution set: $\varnothing$

29. $\frac{x}{x-3}+\frac{3}{x+3}=\frac{6x}{x^2-9}$

$x(x+3)+3(x-3)=6x$

$x^2+3x+3x-9=6x$

$x^2-9=0$

$(x+3)(x-3)=0\Rightarrow x=-3,\ 3$

Since $x = -3$ makes the denominator in the second fraction equal zero and $x = 3$ makes the denominator in the first fraction equal zero, there is no solution. Solution set: $\varnothing$

30. $\frac{x}{x+7}+\frac{7}{x-7}=\frac{14x}{x^2-49}$

$x(x-7)+7(x+7)=14x$

$x^2-7x+7x+49=14x$

$x^2-14x+49=0$

$(x-7)^2=0\Rightarrow x=7$

Since $x = 7$ makes the denominator in the second fraction equal zero, there is no solution. Solution set: $\varnothing$

31. $\frac{1}{x-1}+\frac{x}{x+3}=\frac{4}{x^2+2x-3}$

$\frac{1}{x-1}+\frac{x}{x+3}=\frac{4}{(x-1)(x+3)}$

$(x+3)+x(x-1)=4$

$x+3+x^2-x=4\Rightarrow x^2=1\Rightarrow x=\pm1$

If $x = 1$, then the denominator in the first fraction equals zero, so 1 is an extraneous solution. Solution set: $\{-1\}$

32. $\frac{x}{x-2}+\frac{2}{x+3}=-\frac{10}{x^2+x-6}$

$\frac{x}{x-2}+\frac{2}{x+3}=-\frac{10}{(x+3)(x-2)}$

$x(x+3)+2(x-2)=-10$

$x^2+3x+2x-4=-10$

$x^2+5x+6=0$

$(x+2)(x+3)=0\Rightarrow x=-2,\ -3$

If $x = -3$, then the denominator in the second fraction equals zero, so -3 is an extraneous solution. Solution set: $\{-2\}$

33.
$$\frac{2x}{x+3}-\frac{x}{x-1}=\frac{14}{x^2+2x-3}$$
$$\frac{2x}{x+3}-\frac{x}{x-1}=\frac{14}{(x+3)(x-1)}$$
$$2x(x-1)-x(x+3)=14$$
$$2x^2-2x-x^2-3x=14$$
$$x^2-5x-14=0$$
$$(x-7)(x+2)=0 \Rightarrow x=7,\ -2$$
Solution set: $\{-2, 7\}$

34.
$$\frac{2(x+1)}{x-2}-\frac{x}{x+1}=\frac{9}{x^2-x-2}$$
$$\frac{2(x+1)}{x-2}-\frac{x}{x+1}=\frac{9}{(x-2)(x+1)}$$
$$2(x+1)^2-x(x-2)=9$$
$$2x^2+4x+2-x^2+2x=9$$
$$x^2+6x-7=0$$
$$(x+7)(x-1)=0 \Rightarrow x=-7,\ 1$$
Solution set: $\{-7, 1\}$

35. $\sqrt[3]{3x-1}=2 \Rightarrow \left(\sqrt[3]{3x-1}\right)^3=2^3 \Rightarrow$
$3x-1=8 \Rightarrow 3x=9 \Rightarrow x=3$

36. $\sqrt[3]{2x+3}=3 \Rightarrow \left(\sqrt[3]{2x+3}\right)^3=3^3 \Rightarrow$
$2x+3=27 \Rightarrow 2x=24 \Rightarrow x=12$

37. There is no solution for $\sqrt{x-1}=-2$ because the square root is not negative. The solution set is $\varnothing$.

38. There is no solution for $\sqrt{3x+4}=-1$ because the square root is not negative. The solution set is $\varnothing$.

39. $x+\sqrt{x+6}=0 \Rightarrow x=-\sqrt{x+6} \Rightarrow$
$x^2=\left(-\sqrt{x+6}\right)^2 \Rightarrow x^2=x+6 \Rightarrow$
$x^2-x-6=0 \Rightarrow (x-3)(x+2)=0 \Rightarrow$
$x=3$ or $x=-2$
Now check to see if 3 or –2 are extraneous roots. If $x=3$, then $x+\sqrt{x+6}=3+\sqrt{3+6}=$ $3+3\neq 0$. So 3 is extraneous. If $x=-2$, then $x+\sqrt{x+6}=-2+\sqrt{-2+6}=-2+\sqrt{4}=$ $-2+2=0$. Solution set: $\{-2\}$

40.
$$x-\sqrt{6x+7}=0$$
$$x=\sqrt{6x+7}$$
$$x^2=\left(\sqrt{6x+7}\right)^2$$
$$x^2=6x+7$$
$$x^2-6x-7=0$$
$$(x-7)(x+1)=0 \Rightarrow x=7 \text{ or } x=-1$$
Now check to see if 7 or –1 are extraneous roots. If $x=7$, then $x-\sqrt{6x+7}=$ $7-\sqrt{6(7)+7}=7-\sqrt{49}=7-7=0$. So 7 is a solution. If $x=-1$, then $x-\sqrt{6x+7}=$ $-1-\sqrt{6(-1)+7}=-1-\sqrt{1}=-1-1=-2\neq 0$. So –2 is extraneous. Solution set: $\{7\}$

41. $\sqrt{y+6}=y \Rightarrow \left(\sqrt{y+6}\right)^2=y^2 \Rightarrow y+6=y^2 \Rightarrow$
$0=y^2-y-6 \Rightarrow 0=(y-3)(y+2) \Rightarrow$
$y=3$ or $y=-2$
Now check to see if 3 or –2 are extraneous roots. If $y=3$, then $\sqrt{y+6}=y \Rightarrow$ $\sqrt{3+6}=3 \Rightarrow \sqrt{9}=3 \Rightarrow 3=3$. So 3 is a solution. If $y=-2$, then $\sqrt{y+6}=y \Rightarrow$ $\sqrt{-2+6}=-2 \Rightarrow \sqrt{4}=-2 \Rightarrow 2\neq -2$. So –2 is extraneous. Solution set: $\{3\}$

42.
$$r+11=6\sqrt{r+3}$$
$$(r+11)^2=\left(6\sqrt{r+3}\right)^2$$
$$r^2+22r+121=36(r+3)$$
$$r^2+22r+121=36r+108$$
$$r^2-14r+13=0$$
$$(r-13)(r-1)=0 \Rightarrow r=13 \text{ or } r=1$$
Check to see if 13 or 1 are extraneous roots. If $r=13$, then $r+11=6\sqrt{r+3} \Rightarrow$
$13+11=6\sqrt{13+3} \Rightarrow 24=6\sqrt{16} \Rightarrow$
$24=6(4) \Rightarrow 24=24$. So 13 is a solution.
If $r=1$, then $r+11=6\sqrt{r+3} \Rightarrow$
$1+11=6\sqrt{1+3} \Rightarrow 12=6\sqrt{4} \Rightarrow$
$12=6(2) \Rightarrow 12=12$. So 1 is a solution.
Solution set: $\{1, 13\}$

43. $\sqrt{6y-11} = 2y-7$

$\left(\sqrt{6y-11}\right)^2 = (2y-7)^2$

$6y-11 = 4y^2 - 28y + 49$

$0 = 4y^2 - 34y + 60$

$0 = 2y^2 - 17y + 30$

$0 = (2y-5)(y-6) \Rightarrow y = \frac{5}{2}$ or $y = 6$

Now check to see if 5/2 or 6 are extraneous roots. If $y = \frac{5}{2}$, then $\sqrt{6y-11} = 2y-7 \Rightarrow$

$\sqrt{6\left(\frac{5}{2}\right)-11} = 2\left(\frac{5}{2}\right)-7 \Rightarrow \sqrt{15-11} = 5-7 \Rightarrow$

$\sqrt{4} = -2 \Rightarrow 2 \neq -2$. So 5/2 is extraneous.

If $y = 6$, then $\sqrt{6y-11} = 2y-7 \Rightarrow$

$\sqrt{6(6)-11} = 2(6)-7 \Rightarrow \sqrt{36-11} = 12-7 \Rightarrow$

$\sqrt{25} = 5 \Rightarrow 5 = 5$. Solution set: {6}

44. $\sqrt{3y+1} = y-1$

$\left(\sqrt{3y+1}\right)^2 = (y-1)^2$

$3y+1 = y^2 - 2y + 1$

$0 = y^2 - 5y$

$0 = y(y-5) \Rightarrow y = 0$ or $y = 5$

Now check to see if 0 or 5 are extraneous roots. If $y = 0$, then $\sqrt{3y+1} = y-1 \Rightarrow$

$\sqrt{3(0)+1} = 0-1 \Rightarrow \sqrt{1} = -1 \Rightarrow 1 \neq -1$.

So 0 is extraneous.

If $y = 5$, then $\sqrt{3y+1} = y-1 \Rightarrow$

$\sqrt{3(5)+1} = 5-1 \Rightarrow \sqrt{15+1} = 4 \Rightarrow \sqrt{16} = 4 \Rightarrow$

$4 = 4$.

Solution set: {5}

45. $t - \sqrt{3t+6} = -2$

$t + 2 = \sqrt{3t+6}$

$(t+2)^2 = \left(\sqrt{3t+6}\right)^2$

$t^2 + 4t + 4 = 3t + 6$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0 \Rightarrow t = -2$ or $t = 1$

Now check to see if –2 or 1 are extraneous roots. If $t = -2$, then $t - \sqrt{3t+6} = -2 \Rightarrow$

$-2 - \sqrt{3(-2)+6} = -2 \Rightarrow -2 - \sqrt{-6+6} \Rightarrow$

$-2 - 0 = -2$. So –2 is a solution.

If $t = 1$, then $t - \sqrt{3t+6} = -2 \Rightarrow$

$1 - \sqrt{3(1)+6} = -2 \Rightarrow 1 - \sqrt{9} = -2 \Rightarrow$

$1 - 3 = -2 \Rightarrow -2 = -2$. So 1 is a solution.

Solution set: {–2, 1}

46. $\sqrt{5x^2 - 10x + 9} = 2x - 1$

$\left(\sqrt{5x^2 - 10x + 9}\right)^2 = (2x-1)^2$

$5x^2 - 10x + 9 = 4x^2 - 4x + 1$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0 \Rightarrow x = 2$ or $x = 4$

Now check to see if 2 or 4 are extraneous roots.

If $x = 2$, then $\sqrt{5x^2 - 10x + 9} = 2x - 1 \Rightarrow$

$\sqrt{5(2^2) - 10(2) + 9} = 2(2) - 1 \Rightarrow$

$\sqrt{20 - 20 + 9} = 3 \Rightarrow 3 = 3$. So 2 is a solution.

If $x = 4$, then $\sqrt{5x^2 - 10x + 9} = 2x - 1 \Rightarrow$

$\sqrt{5(4^2) - 10(4) + 9} = 2(4) - 1 \Rightarrow$

$\sqrt{80 - 40 + 9} = 7 \Rightarrow 7 = 7$. So 4 is a solution.

Solution set: {2, 4}

47. $\sqrt{x-3} = \sqrt{2x-5} - 1$

$\left(\sqrt{x-3}\right)^2 = \left(\sqrt{2x-5} - 1\right)^2$

$x - 3 = (2x-5) - 2\sqrt{2x-5} + 1$

$x - 3 = 2x - 4 - 2\sqrt{2x-5}$

$2\sqrt{2x-5} = x - 1 \Rightarrow \left(2\sqrt{2x-5}\right)^2 = (x-1)^2$

$4(2x-5) = x^2 - 2x + 1$

$8x - 20 = x^2 - 2x + 1$

$0 = x^2 - 10x + 21$

$0 = (x-7)(x-3) \Rightarrow x = 7$ or $x = 3$

Now check to see if 3 or 7 are extraneous roots. If $x = 3$, then $\sqrt{x-3} = \sqrt{2x-5} - 1 \Rightarrow$

$\sqrt{3-3} = \sqrt{2(3)-5} - 1 \Rightarrow 0 = 0$. So 3 is a solution.

If $x = 7$, then $\sqrt{x-3} = \sqrt{2x-5} - 1 \Rightarrow$

$\sqrt{7-3} = \sqrt{2(7)-5} - 1 \Rightarrow \sqrt{4} = \sqrt{9} - 1 \Rightarrow$

$2 = 2$. So 7 is a solution.

Solution set: {3, 7}

48. $x + \sqrt{x+1} = 5$

$\sqrt{x+1} = 5 - x$

$\left(\sqrt{x+1}\right)^2 = (5-x)^2$

$x + 1 = 25 - 10x + x^2$

$0 = x^2 - 11x + 24$

$0 = (x-3)(x-8) \Rightarrow x = 3$ or $x = 8$

(*continued on next page*)

(*continued*)

Now check to see if 3 or 8 are extraneous roots. If $x = 3,$ then $x + \sqrt{x+1} = 5 \Rightarrow$ $3 + \sqrt{3+1} = 5 \Rightarrow 3 + \sqrt{4} = 5 \Rightarrow 3 + 2 = 5 \Rightarrow$ $5 = 5.$ So 3 is a solution.

If $x = 8,$ then $x + \sqrt{x+1} = 5 \Rightarrow 8 + \sqrt{8+1} = 5 \Rightarrow$ $8 + 3 = 5 \Rightarrow 11 \neq 5.$ So 8 is extraneous. Solution set: $\{3\}$

49.
$$\sqrt{2y+9} = 2 + \sqrt{y+1}$$
$$\left(\sqrt{2y+9}\right)^2 = \left(2 + \sqrt{y+1}\right)^2$$
$$2y + 9 = 4 + 4\sqrt{y+1} + y + 1$$
$$y + 4 = 4\sqrt{y+1}$$
$$(y+4)^2 = \left(4\sqrt{y+1}\right)^2$$
$$y^2 + 8y + 16 = 16(y+1)$$
$$y^2 + 8y + 16 = 16y + 16$$
$$y^2 - 8y = 0$$
$$y(y-8) = 0 \Rightarrow y = 0 \text{ or } y = 8$$
Now check to see if 0 or 8 are extraneous roots. If $y = 0,$ then $\sqrt{2y+9} = 2 + \sqrt{y+1} \Rightarrow$ $\sqrt{2(0)+9} = 2 + \sqrt{0+1} \Rightarrow \sqrt{9} = 2 + 1 \Rightarrow 3 = 3.$ So 0 is a solution. If $y = 8,$ then $\sqrt{2y+9} = 2 + \sqrt{y+1} \Rightarrow \sqrt{2(8)+9} =$ $2 + \sqrt{8+1} \Rightarrow \sqrt{25} = 2 + \sqrt{9} \Rightarrow 5 = 5.$ So 8 is a solution. Solution set: $\{0, 8\}$

50.
$$\sqrt{m} - 1 = \sqrt{m-5}$$
$$\left(\sqrt{m} - 1\right)^2 = \left(\sqrt{m-5}\right)^2$$
$$m - 2\sqrt{m} + 1 = m - 5$$
$$-2\sqrt{m} = -6$$
$$\sqrt{m} = 3 \Rightarrow m = 9$$
Now check to see if 9 is an extraneous root. $\sqrt{9} - 1 = \sqrt{9-5} \Rightarrow 3 - 1 = \sqrt{4} \Rightarrow 2 = 2.$ Solution set: $\{9\}$

51.
$$\sqrt{7z+1} - \sqrt{5z+4} = 1$$
$$\sqrt{7z+1} = \sqrt{5z+4} + 1$$
$$\left(\sqrt{7z+1}\right)^2 = \left(\sqrt{5z+4} + 1\right)^2$$
$$7z + 1 = 5z + 4 + 2\sqrt{5z+4} + 1$$
$$2z - 4 = 2\sqrt{5z+4}$$
$$(2z-4)^2 = \left(2\sqrt{5z+4}\right)^2$$
$$4z^2 - 16z + 16 = 4(5z+4)$$
$$4z^2 - 16z + 16 = 20z + 16$$
$$4z^2 - 36z = 0 \Rightarrow 4z(z-9) = 0 \Rightarrow$$
$$z = 0 \text{ or } z = 9$$
Now check to see if 0 or 9 are extraneous roots. If $z = 0,$ then $\sqrt{7z+1} - \sqrt{5z+4} = 1 \Rightarrow$ $\sqrt{7(0)+1} - \sqrt{5(0)+4} = 1 \Rightarrow \sqrt{1} - \sqrt{4} = 1 \Rightarrow$ $1 - 2 \neq 1.$ So 0 is extraneous. If $z = 9,$ then $\sqrt{7z+1} - \sqrt{5z+4} = 1 \Rightarrow \sqrt{7(9)+1} - \sqrt{5(9)+4}$ $= 1 \Rightarrow \sqrt{64} - \sqrt{49} \Rightarrow 8 - 7 = 1 \Rightarrow 1 = 1.$ So 9 is a solution. Solution set: $\{9\}$

52.
$$\sqrt{3q+1} - \sqrt{q-1} = 2$$
$$\left(\sqrt{3q+1}\right)^2 = \left(\sqrt{q-1} + 2\right)^2$$
$$3q + 1 = q - 1 + 4\sqrt{q-1} + 4$$
$$2q - 2 = 4\sqrt{q-1}$$
$$(2q-2)^2 = \left(4\sqrt{q-1}\right)^2$$
$$4q^2 - 8q + 4 = 16(q-1)$$
$$4q^2 - 8q + 4 = 16q - 16$$
$$4q^2 - 24q + 20 = 0$$
$$4(q-1)(q-5) = 0 \Rightarrow q = 1 \text{ or } q = 5$$
Now check to see if 1 or 5 are extraneous roots. If $q = 1,$ then $\sqrt{3q+1} - \sqrt{q-1} = 2 \Rightarrow$ $\sqrt{3(1)+1} - \sqrt{1-1} = 2 \Rightarrow \sqrt{4} - \sqrt{0} = 2 \Rightarrow$ $2 - 0 = 2.$ So 1 is a solution. If $q = 5,$ then $\sqrt{3q+1} - \sqrt{q-1} = 2 \Rightarrow \sqrt{3(5)+1} - \sqrt{5-1} = 2 \Rightarrow$ $\sqrt{16} - \sqrt{4} \Rightarrow 4 - 2 = 2 \Rightarrow 2 = 2.$ So 5 is a solution. Solution set: $\{1, 5\}$

53.
$$\sqrt{2x+5} + \sqrt{x+6} = 3$$
$$\sqrt{2x+5} = 3 - \sqrt{x+6}$$
$$\left(\sqrt{2x+5}\right)^2 = \left(3 - \sqrt{x+6}\right)^2$$
$$2x + 5 = 9 - 6\sqrt{x+6} + x + 6$$
$$x - 10 = -6\sqrt{x+6}$$
$$(x-10)^2 = \left(-6\sqrt{x+6}\right)^2$$
$$x^2 - 20x + 100 = 36x + 216$$
$$x^2 - 56x - 116 = 0$$
$$(x-58)(x+2) = 0 \Rightarrow x = 58 \text{ or } x = -2$$
Now check to see if 58 or –2 are extraneous roots. If $x = 58,$ then $\sqrt{2x+5} + \sqrt{x+6} = 3 \Rightarrow$ $\sqrt{2(58)+5} + \sqrt{58+6} = 3 \Rightarrow \sqrt{121} + \sqrt{64} = 3 \Rightarrow$ $11 + 8 \neq 3.$ So 58 is extraneous. If $x = -2,$ then $\sqrt{2x+5} + \sqrt{x+6} = 3 \Rightarrow \sqrt{2(-2)+5} +$ $\sqrt{-2+6} = 3 \Rightarrow \sqrt{1} + \sqrt{4} \Rightarrow 1 + 2 = 3.$ So –2 is a solution. Solution set: $\{-2\}$

54. $\sqrt{5x-9}-\sqrt{x+4}=1$

$$\sqrt{5x-9}=1+\sqrt{x+4}$$
$$\left(\sqrt{5x-9}\right)^2=\left(1+\sqrt{x+4}\right)^2$$
$$5x-9=1+2\sqrt{x+4}+x+4$$
$$4x-14=2\sqrt{x+4}$$
$$(4x-14)^2=\left(2\sqrt{x+4}\right)^2$$
$$16x^2-112x+196=4(x+4)$$
$$16x^2-112x+196=4x+16$$
$$16x^2-116x+180=0$$
$$4(4x^2-29x+45)=0$$
$$(4x-9)(x-5)=0 \Rightarrow x=\frac{9}{4} \text{ or } x=5$$

Check to see if $9/4$ or 5 are extraneous roots.

If $x=9/4$, then $\sqrt{5x-9}-\sqrt{x+4}=1 \Rightarrow$

$\sqrt{5\left(\frac{9}{4}\right)-9}-\sqrt{\frac{9}{4}+4}=1 \Rightarrow \sqrt{\frac{9}{4}}-\sqrt{\frac{25}{4}}=1 \Rightarrow$

$\frac{3}{2}-\frac{5}{2}=1 \Rightarrow -1 \neq 1.$ So $9/4$ is extraneous.

If $x=5$, then $\sqrt{5x-9}-\sqrt{x+4}=1 \Rightarrow$

$\sqrt{5(5)-9}-\sqrt{5+4}=1 \Rightarrow \sqrt{16}-\sqrt{9}=1 \Rightarrow$

$4-3=1.$

So 5 is a solution. Solution set: $\{5\}$

55. $\sqrt{2x-5}-\sqrt{x-3}=1$

$$\sqrt{2x-5}=1+\sqrt{x-3}$$
$$\left(\sqrt{2x-5}\right)^2=\left(1+\sqrt{x-3}\right)^2$$
$$2x-5=1+2\sqrt{x-3}+x-3$$
$$x-3=2\sqrt{x-3}$$
$$(x-3)^2=\left(2\sqrt{x-3}\right)^2$$
$$x^2-6x+9=4x-12$$
$$x^2-10x+21=0$$
$$(x-7)(x-3)=0$$
$$x=7 \text{ or } x=3$$

Now check to see if 7 or 3 are extraneous roots. If $x=7$, then $\sqrt{2x-5}-\sqrt{x-3}=1 \Rightarrow$ $\sqrt{2(7)-5}-\sqrt{7-3}=1 \Rightarrow \sqrt{9}-\sqrt{4}=1 \Rightarrow$ $3-2=1.$ So 7 is a solution. If $x=3$, then $\sqrt{2x-5}-\sqrt{x-3}=1 \Rightarrow \sqrt{2(3)-5}-$ $\sqrt{3-3}=1 \Rightarrow \sqrt{1}-\sqrt{0}=1 \Rightarrow 1=1.$ So 3 is a solution. Solution set: $\{3, 7\}$

56. $\sqrt{3x+5}+\sqrt{6x+3}=3$

$$\sqrt{3x+5}=3-\sqrt{6x+3}$$
$$\left(\sqrt{3x+5}\right)^2=\left(3-\sqrt{6x+3}\right)^2$$
$$3x+5=9-6\sqrt{6x+3}+6x+3$$
$$-3x-7=-6\sqrt{6x+3}$$
$$(-3x-7)^2=\left(-6\sqrt{6x+3}\right)^2$$
$$9x^2+42x+49=36(6x+3)$$
$$9x^2+42x+49=216x+108$$
$$9x^2-174x-59=0$$
$$(3x+1)(3x-59)=0 \Rightarrow x=-\frac{1}{3} \text{ or } \frac{59}{3}$$

Now check to see if –1/3 or 59/3 are extraneous roots.

If $x=-\frac{1}{3}$, then $\sqrt{3x+5}+\sqrt{6x+3}=3 \Rightarrow$

$\sqrt{3\left(-\frac{1}{3}\right)+5}+\sqrt{6\left(-\frac{1}{3}\right)+3}=3 \Rightarrow$

$\sqrt{4}+\sqrt{1}=3 \Rightarrow 2+1=3.$ So $-\frac{1}{3}$ is a solution.

If $x=\frac{59}{3}$, then $\sqrt{3x+5}+\sqrt{6x+3}=3 \Rightarrow$

$\sqrt{3\left(\frac{59}{3}\right)+5}+\sqrt{6\left(\frac{59}{3}\right)+3}=3 \Rightarrow$

$\sqrt{64}+\sqrt{121}=3 \Rightarrow 8+11 \neq 3.$

So $\frac{59}{3}$ is extraneous. Solution set: $\left\{-\frac{1}{3}\right\}$

57. $(x-4)^{3/2}=27$

$$\left[(x-4)^{3/2}\right]^{2/3}=27^{2/3}$$
$$x-4=9 \Rightarrow x=13$$

Verify that 13 satisfies the original equation. Solution set: $\{13\}$

58. $(x+7)^{3/2}=64$

$$\left[(x+7)^{3/2}\right]^{2/3}=64^{2/3}$$
$$x+7=16 \Rightarrow x=9$$

Verify that 9 satisfies the original equation. Solution set: $\{9\}$

59. $(5x-3)^{2/3}-5=4$

$(5x-3)^{2/3}=9$

$\left[(5x-3)^{2/3}\right]^{3/2}=\pm 9^{3/2}$

$5x-3=\pm 27$

$5x-3=-27$	$5x-3=27$
$5x=-24$	$5x=30$
$x=-\frac{24}{5}$	$x=6$

If $x=-24/5$, then

$\left(5\left(-\frac{24}{5}\right)-3\right)^{2/3}-5=(-24-3)^{2/3}-5$

$=(-27)^{2/3}-5$

$=9(-1)^{2/3}-5=4,$

so $x=-\frac{24}{5}$ is a solution. Verify that 6 satisfies the original equation.

Solution set: $\left\{-\frac{24}{5}, 6\right\}$

60. $(2x-6)^{2/3}+9=13$

$(2x-6)^{2/3}=4$

$\left[(2x-6)^{2/3}\right]^{3/2}=\pm 4^{3/2}$

$2x-6=\pm 8$

$2x-6=-8$	$2x-6=8$
$2x=-2$	$2x=14$
$x=-1$	$x=7$

Verify that –1 and 7 satisfy the original equation. Solution set: {–1, 7}

61. Let $u=\sqrt{x}$. Then $u^2=x$. So the equation becomes $u^2-5u+6=0\Rightarrow$ $(u-3)(u-2)=0\Rightarrow u=3$ or $u=2$. Solve for x: $3=\sqrt{x}\Rightarrow x=9$ or $2=\sqrt{x}\Rightarrow 4=x$. Check to make sure that neither solution is extraneous. If $x=9$, then $9-5\sqrt{9}+6=0\Rightarrow$ $9-15+6=0\Rightarrow 0=0$. If $x=4$, then $4-5\sqrt{4}+6=0\Rightarrow 4-10+6=0\Rightarrow 0=0$.
Solution set: {4, 9}

62. Let $u=\sqrt{x}$. Then $u^2=x$. So the equation becomes $u^2-3u+2=0\Rightarrow$ $(u-2)(u-1)=0\Rightarrow u=2$ or $u=1$.
Now solve for x: $2=\sqrt{x}\Rightarrow 4=x$ or $1=\sqrt{x}\Rightarrow 1=x$. Check to make sure that neither solution is extraneous.
If $x=4$, then $4-3\sqrt{4}+2=0\Rightarrow$ $4-6+2=0\Rightarrow 0=0$.
If $x=1$, then
$1-3\sqrt{1}+2=0\Rightarrow 1-3+2=0\Rightarrow 0=0$.
Solution set: {1, 4}

63. Let $u=\sqrt{y}$. Then $u^2=y$. So the equation becomes $2u^2-15u=-7\Rightarrow$ $2u^2-15u+7=0\Rightarrow(2u-1)(u-7)=0\Rightarrow$ $u=\frac{1}{2}$ or $u=7$. Solve for y: $\frac{1}{2}=\sqrt{y}\Rightarrow$ $\frac{1}{4}=y$ or $7=\sqrt{y}\Rightarrow 49=y$. Now check to make sure that neither solution is extraneous.
If $y=\frac{1}{4}$, then $2\left(\frac{1}{4}\right)-15\sqrt{\frac{1}{4}}=-7\Rightarrow$ $\frac{1}{2}-\frac{15}{2}=-7\Rightarrow -7=-7$.
If $y=49$, then
$2(49)-15\sqrt{49}=-7\Rightarrow 98-105=-7\Rightarrow$ $-7=-7$. Solution set: $\left\{\frac{1}{4}, 49\right\}$

64. Let $u=\sqrt{y}$. Then $u^2=y$. So the equation becomes $u^2+44=15u\Rightarrow$ $u^2-15u+44=0\Rightarrow(u-11)(u-4)=0\Rightarrow$ $u=11$ or $u=4$. Now solve for y: $11=\sqrt{y}\Rightarrow$ $121=y$ or $4=\sqrt{y}\Rightarrow 16=y$. Now check to make sure that neither solution is extraneous.
If $y=121$, then $121+44=15\sqrt{121}\Rightarrow$ $165=15(11)\Rightarrow 165=165$. If $y=16$, then $16+44=15\sqrt{16}\Rightarrow 60=15(4)\Rightarrow 60=60$.
Solution set {16, 121}

65. $x^{-2}-x^{-1}-42=0$
Let $u=x^{-1}$. Then the equation becomes $u^2-u-42=0\Rightarrow(u-7)(u+6)=0\Rightarrow$ $u=7,-6$. Now solve for x:
$7=x^{-1}\Rightarrow x=\frac{1}{7}; -6=x^{-1}\Rightarrow x=-\frac{1}{6}$
Be sure to check that neither of the solutions are extraneous.
Solution set: $\left\{-\frac{1}{6}, \frac{1}{7}\right\}$

66. $x^{1/2}+3-4x^{-1/2}=0$
Let $u=x^{1/2}$. Then the equation becomes
$u+3-\frac{4}{u}=0\Rightarrow u^2+3u-4=0\Rightarrow$
$(u+4)(u-1)=0\Rightarrow u=-4, 1$
Now solve for x: $-4=x^{1/2}\Rightarrow x$ is not a real number. $1=x^{1/2}\Rightarrow x=1$
Be sure to check that 1 is not an extraneous solution. Solution set: $\{1\}$

67. $x^{2/3}-6x^{1/3}+8=0$
Let $u=x^{1/3}$. Then the equation becomes
$u^2-6x+8=0\Rightarrow(u-4)(u-2)=0\Rightarrow$
$u=4, 2$. Now solve for x:
$x^{1/3}=4\Rightarrow x=64;\ x^{1/3}=2\Rightarrow x=8$
Be sure to check that neither of the solutions are extraneous. Solution set: $\{8, 64\}$

68. $x^{2/5}+x^{1/5}-2=0$
Let $u=x^{1/5}$. Then the equation becomes
$u^2+u-2=0\Rightarrow(u+2)(u-1)=0\Rightarrow$
$u=-2, 1$. Now solve for x:
$x^{1/5}=-2\Rightarrow x=-32;\ x^{1/5}=1\Rightarrow x=1$
Be sure to check that neither of the solutions are extraneous. Solution set: $\{-32, 1\}$

69. $2x^{1/2}+3x^{1/4}-2=0$
Let $u=x^{1/4}$. Then the equation becomes
$2u^2+3u-2=0\Rightarrow(2u-1)(u+2)=0\Rightarrow$
$u=\frac{1}{2}, -2$. Now solve for x:
$x^{1/4}=\frac{1}{2}\Rightarrow x=\frac{1}{16};\ x^{1/4}=-2\Rightarrow x$ is not a real number. Be sure to check that $\frac{1}{16}$ is not extraneous. Solution set: $\left\{\frac{1}{16}\right\}$

70. $2x^{1/2}-x^{1/4}-1=0$
Let $u=x^{1/4}$. Then the equation becomes
$2u^2-u-1=0\Rightarrow(2u+1)(u-1)=0\Rightarrow$
$u=-\frac{1}{2}, 1$. Now solve for x:
$x^{1/4}=-\frac{1}{2}\Rightarrow x$ is not a real number.
$x^{1/4}=1\Rightarrow x=1$.
Be sure to check that 1 is not extraneous.
Solution set: $\{1\}$

71. Let $u=x^2$. Then $x^4=u^2$. So the equation becomes $u^2-13u+36=0\Rightarrow$
$(u-4)(u-9)=0\Rightarrow u=4$ or $u=9$. Now solve for x: $4=x^2\Rightarrow\pm2=x$ or
$9=x^2\Rightarrow\pm3=x$. Now check to make sure that none of the solutions are extraneous.
If $x=\pm2$, then $(\pm2)^4-13(\pm2)^2+36=0\Rightarrow$
$16-52+36=0\Rightarrow0=0$. If $x=\pm3$, then
$(\pm3)^4-13(\pm3)^2+36=0\Rightarrow81-117+36=0$
$\Rightarrow0=0$. The solutions are ±2 and ±3.

72. Let $u=x^2$. Then $x^4=u^2$. The equation becomes $u^2-7u+12=0\Rightarrow$
$(u-3)(u-4)=0\Rightarrow u=3$ or $u=4$. Solve for x: $3=x^2\Rightarrow\pm\sqrt{3}=x$ or $4=x^2\Rightarrow\pm2=x$.
Check to make sure that none of the solutions are extraneous. If $x=\pm\sqrt{3}$, then
$(\sqrt{\pm3})^4-7(\sqrt{\pm3})^2+12=0\Rightarrow$
$9-21+12=0\Rightarrow0=0$. If $x=\pm2$, then
$(\pm2)^4-7(\pm2)^2+12=0\Rightarrow$
$16-28+12=0\Rightarrow0=0$.
The solutions are ±2 and $\pm\sqrt{3}$.

73. Let $u=t^2$. Then $t^4=u^2$. So the equation becomes $2u^2+u-1=0\Rightarrow(2u-1)(u+1)=0$
$\Rightarrow u=\frac{1}{2}$ or $u=-1$. Now solve for t:
$\frac{1}{2}=t^2\Rightarrow\pm\sqrt{\frac{1}{2}}=\pm\frac{\sqrt{2}}{2}=t$ or $-1=t^2\Rightarrow$
$\pm i=t$. Now check to make sure that none of the solutions are extraneous.
If $t=\pm\frac{\sqrt{2}}{2}$, then $2\left(\pm\frac{\sqrt{2}}{2}\right)^4+\left(\frac{\sqrt{2}}{2}\right)^2-1=0\Rightarrow$
$\frac{1}{2}+\frac{1}{2}-1=0\Rightarrow0=0$. If $x=\pm i$, then
$2(\pm i)^4+(\pm i)^2-1=0\Rightarrow2-1-1=0\Rightarrow$
$0=0$. The solutions are $\pm i$ and $\pm\frac{\sqrt{2}}{2}$.

74. Let $u = y^2$. Then $y^4 = u^2$. So the equation becomes $81u^2 + 1 = 18u \Rightarrow 81u^2 - 18u + 1 = 0 \Rightarrow (9u-1)^2 = 0 \Rightarrow u = 1/9$. Now solve for y: $\frac{1}{9} = y^2 \Rightarrow \pm\frac{1}{3} = y.$ Check to make sure that neither solution is extraneous. If $y = \pm 1/3$, then $81\left(\pm\frac{1}{3}\right)^4 + 1 = 18\left(\pm\frac{1}{3}\right)^2 \Rightarrow$
$1 + 1 = 2 \Rightarrow 2 = 2.$
Solution set: $\left\{\pm\frac{1}{3}\right\}.$

75. Let $u = \sqrt{p^2 - 3}$. Then $p^2 - 3 = u^2$. So the equation becomes $u^2 + 4u - 5 = 0 \Rightarrow$
$(u+5)(u-1) = 0 \Rightarrow u = -5$ or $u = 1$. Now solve for p: $-5 = \sqrt{p^2 - 3}$, which is not possible, or $1 = \sqrt{p^2 - 3} \Rightarrow$
$1 = p^2 - 3 \Rightarrow 1 = p^2 - 3 \Rightarrow 4 = p^2 \Rightarrow \pm 2 = p.$
Now check to make sure that none of the solutions are extraneous. If $p = \pm 2$, then
$(\pm 2)^2 - 3 + 4\sqrt{(\pm 2)^2 - 3} - 5 = 0 \Rightarrow$
$4 - 3 + 4\sqrt{1} - 5 = 0 \Rightarrow 0 = 0.$
Solution set: $\{\pm 2\}$.

76. Let $u = \sqrt{x^2 - 3}$. Then $x^2 - 3 = u^2$. So the equation becomes $u^2 - 4u - 12 = 0 \Rightarrow$
$(u+2)(u-6) = 0 \Rightarrow u = -2$ or $u = 6$. Now solve for x: $6 = \sqrt{x^2 - 3} \Rightarrow 36 = x^2 - 3 \Rightarrow$
$39 = x^2 \Rightarrow \pm\sqrt{39} = x$ or $-2 = \sqrt{x^2 - 3}$, which is not possible. Now check to make sure that none of the solutions are extraneous.
If $x = \pm\sqrt{39}$, then
$(\pm\sqrt{39})^2 - 3 - 4\sqrt{(\pm 39)^2 - 3} - 12 = 0 \Rightarrow$
$39 - 3 - 4\sqrt{36} - 12 \Rightarrow 0 = 0.$
Solution set: $\left\{\pm\sqrt{39}\right\}.$

77. Let $u = 3t + 1$. Then the equation becomes $u^2 - 3u + 2 = 0 \Rightarrow (u-2)(u-1) = 0 \Rightarrow$
$u = 2$ or $u = 1$. Now solve for t: $2 = 3t + 1 \Rightarrow$
$\frac{1}{3} = t$ or $1 = 3t + 1 \Rightarrow 0 = t$. Now check to make sure that neither solution is extraneous.
If $t = 0$,
$(3 \bullet 0 + 1)^2 - 3(3 \bullet 0 + 1) + 2 = 0 \Rightarrow$
$1 - 3 + 2 = 0 \Rightarrow 0 = 0$, so 0 is a solution. If $t = \frac{1}{3}$, then $\left(3\left(\frac{1}{3}\right) + 1\right)^2 - 3\left(3\left(\frac{1}{3}\right) + 1\right) + 2 = 0 \Rightarrow$
$4 - 6 + 2 = 0 \Rightarrow 0 = 0$. So 1/3 is a solution.
Solution set: $\left\{0, \frac{1}{3}\right\}.$

78. Let $u = 7z + 5$. Then the equation becomes $u^2 + 2u - 15 = 0 \Rightarrow (u+5)(u-3) = 0 \Rightarrow$
$u = -5$ or $u = 3$. Now solve for z:
$-5 = 7z + 5 \Rightarrow z = -\frac{10}{7}$ or
$3 = 7z + 5 \Rightarrow z = -\frac{2}{7}.$
Now check to make sure that neither solution is extraneous.
If $z = -10/7$, then
$\left[7\left(-\frac{10}{7}\right) + 5\right]^2 + 2\left[7\left(-\frac{10}{7}\right) + 5\right] - 15 = 0 \Rightarrow$
$25 - 10 - 15 = 0 \Rightarrow 0 = 0$. So $-10/7$ is a solution. If $z = -2/7$, then
$\left[7\left(-\frac{2}{7}\right) + 5\right]^2 + 2\left[7\left(-\frac{2}{7}\right) + 5\right] - 15 = 0 \Rightarrow$
$9 + 6 - 15 = 0 \Rightarrow 0 = 0$. So $-2/7$ is a solution.
Solution set: $\left\{-\frac{2}{7}, -\frac{10}{7}\right\}.$

79. Let $u = \sqrt{y} + 5$. Then the equation becomes $u^2 - 9u + 20 = 0 \Rightarrow (u-5)(u-4) = 0 \Rightarrow$
$u = 5$ or $u = 4$. Now solve for y:
$5 = \sqrt{y} + 5 \Rightarrow y = 0$ or $4 = \sqrt{y} + 5 \Rightarrow$
$-1 = \sqrt{y}$, which is not defined. Now check to see if $y = 0$ is extraneous.
$\left(\sqrt{0} + 5\right)^2 - 9\left(\sqrt{0} + 5\right) + 20 = 0 \Rightarrow$
$25 - 45 + 20 = 0 \Rightarrow 0 = 0.$
Solution set: $\{0\}$

80. Let $u = 2\sqrt{t} + 1$. Then the equation becomes $u^2 - 2u - 3 = 0 \Rightarrow (u-3)(u+1) = 0 \Rightarrow u = 3$ or $u = -1$. Now solve for t: $3 = 2\sqrt{t} + 1 \Rightarrow$ $2 = 2\sqrt{t} \Rightarrow 1 = \sqrt{t} \Rightarrow t = 1$ or $-1 = 2\sqrt{t} + 1 \Rightarrow -2 = 2\sqrt{t} \Rightarrow -1 = \sqrt{t}$, which is not defined. Now check to see if $t = 1$ is extraneous. $(2\sqrt{1}+1)^2 - 2(2\sqrt{1}+1) - 3 = 0 \Rightarrow$ $9 - 6 - 3 = 0 \Rightarrow 0 = 0.$
Solution set: $\{1\}$.

81. Let $u = x^2 - 4$. So the equation becomes $u^2 - 3u - 4 = 0 \Rightarrow (u-4)(u+1) = 0 \Rightarrow$ $u = 4$ or $u = -1$. Now solve for x: $4 = x^2 - 4 \Rightarrow x = \pm 2\sqrt{2}$ or $-1 = x^2 - 4 \Rightarrow x = \pm\sqrt{3}$. Now check to make sure that neither solution is extraneous. If $x = \pm 2\sqrt{2}$, then

$$\left[(\pm 2\sqrt{2})^2 - 4\right]^2 - 3\left[(\pm 2\sqrt{2})^2 - 4\right] - 4 = 0 \Rightarrow$$

$16 - 12 - 4 = 0 \Rightarrow 0 = 0$. So $\pm 2\sqrt{2}$ are solutions.
If $x = \pm\sqrt{3}$, then

$$\left[(\pm\sqrt{3})^2 - 4\right]^2 - 3\left[(\pm\sqrt{3})^2 - 4\right] - 4 = 0 \Rightarrow$$

$1 + 3 - 4 = 0 \Rightarrow 0 = 0$. So $\pm\sqrt{3}$ are solutions.
Solution set: $\left\{\pm\sqrt{3}, \pm 2\sqrt{2}\right\}$.

82. Let $u = x^2 + 2$. So the equation becomes $u^2 - 5u - 6 = 0 \Rightarrow (u-6)(u+1) = 0 \Rightarrow u = 6$ or $u = -1$. Now solve for x: $6 = x^2 + 2 \Rightarrow x = \pm 2$ or $-1 = x^2 + 2 \Rightarrow x = \pm i\sqrt{3}$. Check to make sure that none of the solutions are extraneous. If $x = \pm 2$, then

$$\left[(\pm 2)^2 + 2\right]^2 - 5\left[(\pm 2)^2 + 2\right] - 6 =$$

$36 - 30 - 6 = 0$. So ± 2 are solutions.
If $x = i\sqrt{3}$, then

$$\left[\left(i\sqrt{3}\right)^2 + 2\right]^2 - 5\left[\left(i\sqrt{3}\right)^2 + 2\right] - 6 =$$

$$(-3+2)^2 - 5(-3+2) - 6 = 1 + 5 - 6 = 0.$$

If $x = -i\sqrt{3}$, then

$$\left[\left(-i\sqrt{3}\right)^2 + 2\right]^2 - 5\left[\left(-i\sqrt{3}\right)^2 + 2\right] - 6$$

$$= (-3+2)^2 - 5(-3+2) - 6 = 1 + 5 - 6 = 0.$$

So $\pm i\sqrt{3}$ are solutions.
Solution set: $\left\{\pm 2, \pm i\sqrt{3}\right\}$.

83. Let $u = x^2 - 3x$. So the equation becomes $u^2 - 2u - 8 = 0 \Rightarrow (u+2)(u-4) = 0 \Rightarrow$ $u = -2$ or $u = 4$. Now solve for x:
$-2 = x^2 - 3x \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow$ $(x-2)(x-1) = 0 \Rightarrow x = 2$ or $x = 1$ or
$4 = x^2 - 3x \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow$ $(x-4)(x+1) = 0 \Rightarrow x = 4$ or $x = -1$.
Now check to make sure that none of the solutions are extraneous. If $x = 2$, then

$$(2^2 - 3(2))^2 - 2(2^2 - 3(2)) - 8 = 0 \Rightarrow$$

$4 - (-4) - 8 = 0 \Rightarrow 0 = 0$. So 2 is a solution.
If $x = 1$, then

$$(1^2 - 3(1))^2 - 2(1^2 - 3(1)) - 8 = 0 \Rightarrow$$

$4 + 4 - 8 = 0 \Rightarrow 0 = 0$. So 1 is a solution. If $x = 4$, then

$$(4^2 - 3(4))^2 - 2(4^2 - 3(4)) - 8 = 0 \Rightarrow$$

$16 - 8 - 8 = 0 \Rightarrow 0 = 0$. So 4 is a solution.
If $x = -1$, then

$$\left[(-1)^2 - 3(-1)\right]^2 - 2\left[(-1)^2 - 3(-1)\right] - 8 = 0 \Rightarrow$$

$16 - 8 - 8 = 0 \Rightarrow 0 = 0$. So -1 is a solution.
Solution set: $\{-1, 1, 2, 4\}$.

84. Let $u = x^2 - 4x$. So the equation becomes $u^2 + 7u + 12 = 0 \Rightarrow (u+3)(u+4) = 0 \Rightarrow$ $u = -3$ or $u = -4$. Now solve for x:
$-3 = x^2 - 4x \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow$ $(x-3)(x-1) = 0 \Rightarrow x = 3$ or $x = 1$ or
$-4 = x^2 - 4x \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow$ $(x-2)^2 = 0 \Rightarrow x = 2$. Now check to make sure that none of the solutions are extraneous.
If $x = 1$, then

$$(1^2 - 4(1))^2 + 7(1^2 - 4(1)) + 12 = 0 \Rightarrow$$

$9 + 7(-3) + 12 = 0 \Rightarrow 0 = 0$. So, 1 is a solution. If $x = 2$, then

$$(2^2 - 4(2))^2 + 7(2^2 - 4(2)) + 12 = 0 \Rightarrow$$

$16 - 28 + 12 = 0 \Rightarrow 0 = 0$. So, 2 is a solution.
If $x = 3$, then

$$(3^2 - 4(3))^2 + 7(3^2 - 4(3)) + 12 = 0 \Rightarrow$$

$9 + 7(-3) + 12 = 0 \Rightarrow 0 = 0$. So, 3 is a solution.
Solution set: $\{1, 2, 3\}$.

1.4 Applying the Concepts

85. Let x = the denominator. Then $x - 2$ = the numerator.

$$\frac{x-2}{x}+\frac{x}{x-2}=\frac{25}{12}$$
$$12x(x-2)\left(\frac{x-2}{x}+\frac{x}{x-2}\right)=12x(x-2)\left(\frac{25}{12}\right)$$
$$12(x-2)^2+12x^2=25x(x-2)$$
$$24x^2-48x+48=25x^2-50x$$
$$0=x^2-2x-48$$
$$0=(x-8)(x+6)$$
$$x=8 \text{ or } x=-6$$

The solution must be positive. So the fraction is $\frac{6}{8}$.

86. Let x = the denominator. Then $x - 5$ = the numerator.

$$\frac{x-5}{x}+6\left(\frac{x}{x-5}\right)=\frac{25}{2}$$
$$2x(x-5)\left[\frac{x-5}{x}+6\left(\frac{x}{x-5}\right)\right]=2x(x-5)\left(\frac{25}{2}\right)$$
$$2(x-5)^2+12x^2=25x^2-125x$$
$$2x^2-20x+50+12x^2=25x^2-125x$$
$$14x^2-20x+50=25x^2-125x$$
$$-11x^2+105x+50=0$$
$$-1(11x+5)(x-10)=0$$
$$x=-\frac{5}{11} \text{ or } x=10$$

The numerator and denominator must be integers, so the fraction is $5/10$.

87. Let x = the number of shares of stock. Then $1800/x =$ the price of each share of stock.

$$\left(\frac{1800}{x}-18\right)(x+5)=1800$$
$$\frac{1800(x+5)}{x}-18(x+5)=1800$$
$$\frac{1800x+9000}{x}-18x-90=1800$$
$$1800x+9000-18x^2-90x=1800x$$
$$-18x^2-90x+9000=0$$
$$-18(x^2+5x-500)=0$$
$$(x-20)(x+25)=0$$
$$x=20 \text{ or } x=-25$$

The answer must be positive, so we reject –25.

a. Latasha bought 20 shares of stock.

b. She paid $1800/20 = \$90$ for each share.

88. Let x = the number of people in the original group. Then $\frac{575}{x} =$ the cost per person in the original group. $x + 2$ = the number of people in the new group. $\frac{575}{x+2} =$ the cost per person in the new group.

$$\frac{575}{x}-2=\frac{575}{x+2}$$
$$x(x+2)\left(\frac{575}{x}-2\right)=x(x+2)\left(\frac{575}{x+2}\right)$$
$$575(x+2)-2x(x+2)=575x$$
$$575x+1150-2x^2-4x=575x$$
$$-2(x^2+2x-575)=0$$
$$(x+25)(x-23)=0\Rightarrow$$
$$x=-25 \text{ or } x=23$$

The answer must be positive, so we reject –25. There were 23 people in the original group.

89. Let d = the depth of the well and let t_1 = the time it takes for the stone to hit the water. Using Galileo's formula, we have $d=16t_1^2\Rightarrow\frac{\sqrt{d}}{4}=t_1$. Let t_2 = the time it takes for the sound of the splash to return to the top of the well. Then $t_2=\frac{d}{1100}$, and

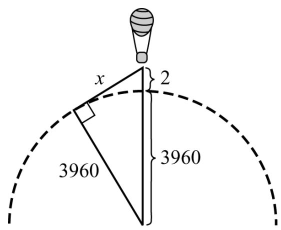
$$t_1+t_2=\frac{\sqrt{d}}{4}+\frac{d}{1100}=4$$
$$1100\left(\frac{\sqrt{d}}{4}+\frac{d}{1100}\right)=1100(4)$$
$$275\sqrt{d}+d=4400$$

Now let $u=\sqrt{d}$ and $u^2=d$. This gives $u^2+275u=4400\Rightarrow u^2+275u-4400=0$. Now solve for u using the quadratic formula with $a = 1$, $b = 275$, and $c = -4400$:

$$u=\frac{-275\pm\sqrt{275^2-4(1)(-4400)}}{2(1)}\approx 15.2.$$

Note that only the positive answer has meaning, so we reject the negative answer. $u^2=d\approx 230$. So the well is approximately 230 feet deep.

90. Let x = the distance to the horizon.



Using the Pythagorean Theorem, we have
$x^2 + 3960^2 = 3962^2 \Rightarrow x^2 = 15844 \Rightarrow$
$x \approx 125.87$ miles.

91. Let x = the speed of the current. Then $10 + x$ = the speed of the motorboat going downstream and $10 - x$ = the speed of the motorboat going upstream. $\frac{12}{10+x} =$ the time the boat took to go downstream and $\frac{12}{10-x} =$ the time the boat took to go upstream. So, we have

$$\frac{12}{10-x} = \frac{12}{10+x} + \frac{1}{2}$$
$$2(10+x)(10-x)\left(\frac{12}{10-x} = \frac{12}{10+x} + \frac{1}{2}\right)$$
$$240 + 24x = 240 - 24x + 100 - x^2$$
$$x^2 + 48x - 100 = 0$$
$$(x+50)(x-2) = 0 \Rightarrow x = -50 \text{ or } x = 2$$

Only the positive answer has meaning, so we reject –50. The rate of the current was 2 mph.

92. Let x = the time of the express train took. Then $x + \frac{5}{2} =$ the time of the freight train took, $\frac{300}{x} =$ the speed of the express train, and $\frac{300}{x + (5/2)} = \frac{600}{2x+5} =$ the speed of the freight train. So, we have

$$\frac{300}{x} = \frac{600}{2x+5} + 20 \Rightarrow$$
$$x(2x+5)\left(\frac{300}{x}\right) = x(2x+5)\left(\frac{600}{2x+5} + 20\right) \Rightarrow$$
$$600x + 1500 = 600x + 40x^2 + 100x \Rightarrow$$
$$0 = 40x^2 + 100x - 1500 \Rightarrow$$
$$0 = 2x^2 + 5x - 75 \Rightarrow 0 = (2x+15)(x-5)$$
$$x = -\frac{15}{2} \text{ or } x = 5$$

Only the positive answer has meaning, so we reject $-15/2$. The express train took 5 hours to travel 300 miles.

93. Let x = the time it took the wife to wash the car alone. Then $x + 20$ = the time it took the husband to wash the car alone. So we have

$$\frac{1}{x} + \frac{1}{x+20} = \frac{1}{24}$$
$$24x(x+20)\left(\frac{1}{x} + \frac{1}{x+20} = \frac{1}{24}\right)$$
$$24x + 480 + 24x = x^2 + 20x$$
$$0 = x^2 - 28x - 480$$
$$0 = (x-40)(x+12)$$
$$x = 40 \text{ or } x = -12$$

Only the positive answer has meaning, so we reject –12. The wife took 40 minutes to wash the car alone.

94. Let x = the time it takes the smaller hose to fill the pool alone. Then $x - 1$ = the time it takes the larger hose to fill the pool alone. It takes 1 hour and 12 minutes = $6/5$ hour for the two hoses to fill the pool together. $\frac{1}{6/5} = \frac{5}{6}$. So we have

$$\frac{1}{x} + \frac{1}{x-1} = \frac{5}{6} \Rightarrow$$
$$6x(x-1)\left(\frac{1}{x} + \frac{1}{x-1}\right) = 6x(x-1)\left(\frac{5}{6}\right) \Rightarrow$$
$$6x - 6 + 6x = 5x^2 - 5x \Rightarrow 0 = 5x^2 - 17x + 6 \Rightarrow$$
$$0 = (5x-2)(x-3) \Rightarrow x = \frac{2}{5} \text{ or } x = 3$$

If the smaller hose can fill the pool in $2/5$ hour alone, then the time it will take the larger hose to fill the pool will be negative; so we reject that answer. It will take the smaller hose 3 hours to fill the pool alone.

95. Let x = the length of the shorter side. Then $100 + x =$ the length of the longer side. Use the Pythagorean theorem to find the length of the diagonal = $\sqrt{x^2 + (x+100)^2}$. So we have

$$\sqrt{x^2 + (x+100)^2} + (x+100) = 3x \Rightarrow$$
$$\sqrt{x^2 + (x+100)^2} = 2x - 100 \Rightarrow$$
$$\left(\sqrt{x^2 + (x+100)^2}\right)^2 = (2x-100)^2 \Rightarrow$$
$$x^2 + (x+100)^2 = 4x^2 - 400x + 10{,}000 \Rightarrow$$
$$2x^2 + 200x + 10{,}000 = 4x^2 - 400x + 10{,}000 \Rightarrow$$
$$0 = 2x^2 - 600x \Rightarrow 0 = 2x(x-300) \Rightarrow$$
$$x = 0 \text{ or } x = 300$$

The shorter side is 300 feet and the longer side is 400 feet. So the area is 120,000 square feet.

96. Let x = the distance from B to C and $130 - x$ = the length from C to the hospital. Use the Pythagorean theorem to find the distance from A to $C = \sqrt{40^2 + x^2}$. The time the boat takes to gets from point A to point C is $\frac{\sqrt{40^2 + x^2}}{40}$ and the time it takes for the ambulance to get from the hospital to point C is $\frac{130 - x}{80}$. This gives

$$\frac{\sqrt{40^2 + x^2}}{40} = \frac{130 - x}{80} \Rightarrow$$
$$2\sqrt{40^2 + x^2} = 130 - x \Rightarrow$$
$$4(1600 + x^2) = 16,900 - 260x + x^2 \Rightarrow$$
$$4x^2 + 6400 = x^2 - 260x + 16,900 \Rightarrow$$
$$3x^2 + 260x - 10,500 = 0$$

Solve for x using the Pythagorean theorem with $a = 3$, $b = 260$, and $c = -10,500$.

$$x = \frac{-260 \pm \sqrt{260^2 - 4(3)(-10,500)}}{2(3)} = 30, -\frac{350}{3}$$

Only the positive root is valid. So, $BC = 30$ km.

97. Use the Pythagorean theorem to find the lengths of AE and BE. $AE = \sqrt{64 + x^2}$ and $BE = \sqrt{(18 - x)^2 + 25}$.

So we have $AE + BE = 23$ or

$$\sqrt{64 + x^2} + \sqrt{(18 - x)^2 + 25} = 23 \Rightarrow$$
$$\sqrt{(18 - x)^2 + 25} = 23 - \sqrt{64 + x^2} \Rightarrow$$
$$\left(\sqrt{(18 - x)^2 + 25}\right)^2 = \left(23 - \sqrt{64 + x^2}\right)^2 \Rightarrow$$
$$(18 - x)^2 + 25 = 529 - 46\sqrt{64 + x^2} + 64 + x^2 \Rightarrow$$
$$324 - 36x + x^2 + 25 = 593 - 46\sqrt{64 + x^2} + x^2 \Rightarrow$$
$$-244 - 36x = -46\sqrt{64 + x^2} \Rightarrow$$
$$(244 + 36x)^2 = \left(46\sqrt{64 + x^2}\right)^2$$
$$1296x^2 + 17,568x + 59,536 = 135,424 + 2116x^2$$
$$-820x^2 + 17,568x - 75,888 = 0$$
$$-4(205x^2 - 4392x + 18,972) = 0$$
$$(x - 6)(205x - 3162) = 0 \Rightarrow$$
$$x = 6 \text{ or } x = \frac{3162}{205} \approx 15.4$$

Check for extraneous solutions. Neither is extraneous, so $CE = 6$ mi or $CE \approx 15.4$ mi.

98. a. Let x = the number of shirts purchased. Then $x - 8$ = the number of shirts sold, $\frac{540}{x} =$ the purchase price of each shirt, and $\frac{540}{x} + 6 =$ the selling price of each shirt.

So we have

$$(x - 8)\left(\frac{540}{x} + 6\right) = 780$$
$$(x - 8)(540 + 6x) = 780x$$
$$6x^2 + 492x - 4320 = 780x$$
$$6x^2 - 288x - 4320 = 0$$
$$6(x - 60)(x + 12) = 0$$
$$x = 60 \text{ or } x = -12$$

Only the positive root is valid. So, 60 shirts were bought initially.

b. The purchase price of each shirt was $\frac{540}{60} = \$9$.

c. The selling price of each shirt was $\frac{540}{60} + 6 = \$15$.

1.4 Beyond the Basics

99. $\left(\frac{x^2 - 3}{x}\right)^2 - 6\left(\frac{x^2 - 3}{x}\right) + 8 = 0$

Let $u = \frac{x^2 - 3}{x}$, so we have

$u^2 - 6u + 8 - 0 \Rightarrow (u - 4)(u - 2) = 0 \Rightarrow$

$u = 4$ or $u = 2$.

Now solve for x.

$$\frac{x^2 - 3}{x} = 4 \Rightarrow x^2 - 3 = 4x \Rightarrow$$
$$x^2 - 4x - 3 = 0 \Rightarrow$$

$x = 2 \pm 2\sqrt{7}$ (using the quadratic formula) or

$$\frac{x^2 - 3}{x} = 2 \Rightarrow x^2 - 3 = 2x \Rightarrow$$
$$x^2 - 2x - 3 = 0 \Rightarrow$$
$$(x - 3)(x + 1) = 0 \Rightarrow x = 3 \text{ or } x = -1$$

Be sure to check to make sure that none of the solutions are extraneous. None are extraneous.

Solution set: $\left\{-1,\ 2 \pm 2\sqrt{7},\ 3\right\}$.

100. $\left(\frac{x+2}{3x+1}\right)^2 - 5\left(\frac{x+2}{3x+1}\right) + 6 = 0$

Let $u = \frac{x+2}{3x+1}$, so we have

$u^2 - 5u + 6 = 0 \Rightarrow (u-2)(u-3) = 0 \Rightarrow$

$u = 2$ or $u = 3$.

Now solve for x.

$\frac{x+2}{3x+1} = 2 \Rightarrow x + 2 = 6x + 2 \Rightarrow x = 0$ or

$\frac{x+2}{3x+1} = 3 \Rightarrow x + 2 = 9x + 3 \Rightarrow x = -\frac{1}{8}$

Be sure to check to make sure that none of the solutions are extraneous. None are extraneous.

Solution set: $\left\{0, -\frac{1}{8}\right\}$.

101. $\sqrt{\sqrt{3x-2} + 2\sqrt{4x+1}} = 2\sqrt{2}$

$\sqrt{3x-2} + 2\sqrt{4x+1} = 8$

$\sqrt{3x-2} = 8 - 2\sqrt{4x+1}$

$3x - 2 = 64 - 32\sqrt{4x+1} + 4(4x+1)$

$3x - 2 = 16x + 68 - 32\sqrt{4x+1}$

$-13x - 70 = -32\sqrt{4x+1}$

$169x^2 + 1820x + 4900 = 4096x + 1024$

$169x^2 - 2276x + 3876 = 0$

Solve for x using the quadratic formula.

$x = \frac{2276 \pm \sqrt{2276^2 - 4(169)(3876)}}{2(169)}$

$x = 2$ or $x \approx 11.47$

Check to make sure that neither solution is extraneous. 11.47 is extraneous.

Solution set: $\{2\}$.

102. $\frac{5}{x-1} - \frac{3x}{x+1} = \frac{9}{x^2-1}$

$(x^2-1)\left(\frac{5}{x-1} - \frac{3x}{x+1}\right) = (x^2-1)\left(\frac{9}{x^2-1}\right)$

$5(x+1) - 3x(x-1) = 9$

$-3x^2 + 8x + 5 = 9$

$3x^2 - 8x + 4 = 0$

$(3x-2)(x-2) = 0$

$x = \frac{2}{3}$ or $x = 2$

Make sure that neither solution is extraneous. Neither is extraneous.

Solution set: $\left\{\frac{2}{3}, 2\right\}$.

103. $\frac{x+2}{x} + \frac{2x+6}{x^2+4x} = -\frac{1}{x+4}$

$x(x+4)\left(\frac{x+2}{x} + \frac{2x+6}{x^2+4x} = -\frac{1}{x+4}\right)$

$(x+4)(x+2) + 2x + 6 = -x$

$x^2 + 9x + 14 = 0$

$(x+7)(x+2) = 0$

$x = -7$ or $x = -2$

Solution set: $\{-7, -2\}$

104. $\left(x^2 + \frac{1}{x^2}\right) - 7\left(x - \frac{1}{x}\right) + 8 = 0$

Let $u = x - \frac{1}{x}$, so $u^2 = x^2 + \frac{1}{x^2} - 2 \Rightarrow$

$u^2 + 2 = x^2 + \frac{1}{x^2}$. Then we have

$u^2 + 2 - 7u + 8 = 0 \Rightarrow u^2 - 7u + 10 = 0 \Rightarrow$

$(u-5)(u-2) = 0 \Rightarrow u = 5$ or $u = 2$.

Now solve for x:

$5 = x - \frac{1}{x} \Rightarrow 5x = x^2 - 1 \Rightarrow x^2 - 5x - 1 = 0$

Use the quadratic formula to solve for x:

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-1)}}{2(1)} = \frac{5}{2} \pm \frac{\sqrt{29}}{2}$

$2 = x - \frac{1}{x} \Rightarrow 2x = x^2 - 1 \Rightarrow x^2 - 2x - 1 = 0$

Using the quadratic formula again, we have:

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = 1 \pm \sqrt{2}$

Check that none of the roots are extraneous.

Solution set: $\left\{\frac{5}{2} \pm \frac{\sqrt{29}}{2}, 1 \pm \sqrt{2}\right\}$.

105. $\left(\frac{t}{t+2}\right)^2 + \frac{2t}{t+2} - 15 = 0$

Let $u = \frac{t}{t+2}$. This gives

$u^2 + 2u - 15 = 0 \Rightarrow (u+5)(u-3) = 0 \Rightarrow$

$u = -5$ or $u = 3$.

Now solve for t.

$-5 = \frac{t}{t+2} \Rightarrow -5t - 10 = t \Rightarrow t = -\frac{5}{3}$ or

$3 = \frac{t}{t+2} \Rightarrow 3t + 6 = t \Rightarrow t = -3.$

Be sure to check to make sure that neither solution is extraneous. Neither is extraneous.

Solution set: $\left\{-3, -\frac{5}{3}\right\}$.

106. $6t^{-2/5} - 17t^{-1/5} + 5 = 0$

Let $u = t^{-1/5}$. So $u^2 = t^{-2/5}$.

$6u^2 - 17u + 5 = 0 \Rightarrow (3u-1)(2u-5) = 0 \Rightarrow$

$u = \frac{1}{3}$ or $u = \frac{5}{2}$.

Now solve for t.

$\frac{1}{3} = t^{-1/5} \Rightarrow \frac{1}{3} = \frac{1}{\sqrt[5]{t}} \Rightarrow \frac{1}{243} = \frac{1}{t} \Rightarrow t = 243$ or

$\frac{5}{2} = t^{-1/5} \Rightarrow \frac{5}{2} = \frac{1}{\sqrt[5]{t}} = \frac{3125}{32} = \frac{1}{t} \Rightarrow t = \frac{32}{3125}$.

Be sure to check to make sure that neither solution is extraneous. Neither is extraneous.

Solution set: $\left\{\frac{32}{3125}, 243\right\}$.

107. $2x^{1/3} + 2x^{-1/3} - 5 = 0$

Let $u = x^{1/3}$. So $\frac{1}{u} = x^{-1/3}$.

$2u + \frac{2}{u} - 5 = 0 \Rightarrow 2u^2 - 5u + 2 = 0 \Rightarrow$

$(2u-1)(u-2) = 0 \Rightarrow u = 1/2$ or $u = 2$.

Now solve for x:

$\frac{1}{2} = x^{1/3} \Rightarrow \frac{1}{8} = x$ or $2 = x^{1/3} \Rightarrow 8 = x$

Make sure that neither solution is extraneous. Neither is extraneous.

Solution set: $x = \left\{\frac{1}{8}, 8\right\}$.

108. $x^{4/3} - 4x^{2/3} + 3 = 0$

Let $u = x^{2/3}$. The equation becomes

$u^2 - 4u + 3 = 0 \Rightarrow (u-1)(u-3) = 0 \Rightarrow$

$u = 1, 3$

$x^{2/3} = 1$	$x^{2/3} = 3$
$\left(x^{2/3}\right)^{3/2} = \pm\left(1^{3/2}\right)$	$\left(x^{2/3}\right)^{3/2} = \pm\left(3^{3/2}\right)$
$x = \pm 1$	$x = \pm\sqrt{27} = \pm 3\sqrt{3}$

Verify that none of the solutions are extraneous. Solution set: $\left\{\pm 1, \pm 3\sqrt{3}\right\}$.

109. $x^{4/3} - 5x^{2/3} + 6 = 0$

Let $u = x^{2/3}$. The equation becomes

$u^2 - 5u + 6 = 0 \Rightarrow (u-2)(u-3) = 0 \Rightarrow$

$u = 2, 3$

$x^{2/3} = 2$	$x^{2/3} = 3$
$\left(x^{2/3}\right)^{3/2} = \pm\left(2^{3/2}\right)$	$\left(x^{2/3}\right)^{3/2} = \pm\left(3^{3/2}\right)$
$x = \pm\sqrt{8}$	$x = \pm\sqrt{27}$
$= \pm 2\sqrt{2}$	$= \pm 3\sqrt{3}$

Verify that none of the solutions are extraneous. Solution set: $\left\{\pm 2\sqrt{2}, \pm 3\sqrt{3}\right\}$.

110. $8\sqrt{\frac{x}{x+3}} - \sqrt{\frac{x+3}{x}} = 2$

Let $u = \sqrt{\frac{x}{x+3}}$. Then $\sqrt{\frac{x+3}{x}} = \frac{1}{u}$. So

$8u - \frac{1}{u} = 2 \Rightarrow 8u^2 - 1 = 2u \Rightarrow$

$8u^2 - 2u - 1 = 0 \Rightarrow (4u+1)(2u-1) = 0 \Rightarrow$

$u = -\frac{1}{4}$ or $u = \frac{1}{2}$

Now solve for x:

$-\frac{1}{4} = \sqrt{\frac{x}{x+3}} \Rightarrow \frac{1}{16} = \frac{x}{x+3} \Rightarrow \frac{1}{5} = x$ or

$\frac{1}{2} = \sqrt{\frac{x}{x+3}} \Rightarrow \frac{1}{4} = \frac{x}{x+3} \Rightarrow x = 1$.

Make sure that neither solution is extraneous. $x = 1/5$ is extraneous.
Solution set: $\{1\}$.

111. a. *Rationalization method*:

$x + \sqrt{x} - 42 = 0 \Rightarrow \sqrt{x} = 42 - x \Rightarrow$

$x = 1764 - 84x + x^2 \Rightarrow$

$x^2 - 85x + 1764 = 0 \Rightarrow$

$(x-36)(x-49) = 0 \Rightarrow x = 36$ or $x = 49$.

Note that $x = 49$ is extraneous.
Solution set: $\{36\}$.

b. *Substitution method*:

Let $u = \sqrt{x}$. Then $u^2 = x$. So,

$u^2 + u - 42 = 0 \Rightarrow (u+7)(u-6) = 0 \Rightarrow$

$u = -7$ or $u = 6$. Now solve for x.

$-7 = \sqrt{x}$, which is not defines, or

$6 = \sqrt{x} \Rightarrow 36 = x$.

Be sure to check to make sure that $x = 36$ is not extraneous. Solution set: $\{36\}$.

112. a. *Substitution method*:

$\left(\frac{x}{x+1}\right)^2 - \frac{2x}{x+1} - 8 = 0.$ Let $u = \frac{x}{x+1} \Rightarrow$

$u^2 - 2u - 8 = 0 \Rightarrow (u-4)(u+2) = 0 \Rightarrow$

$u = 4$ or $u = -2$. Now solve for x:

$4 = \frac{x}{x+1} \Rightarrow 4x + 4 = x \Rightarrow x = -\frac{4}{3}$ or

$-2 = \frac{x}{x+1} \Rightarrow -2x - 2 = x \Rightarrow x = -\frac{2}{3}.$

Solution set: $\left\{-\frac{4}{3}, -\frac{2}{3}\right\}.$

b. *Multiply by LCD method*:

$$\left(\frac{x}{x+1}\right)^2 - \frac{2x}{x+1} - 8 = 0$$
$$\frac{x^2}{(x+1)^2} - \frac{2x}{x+1} - 8 = 0$$
$$(x+1)^2\left(\frac{x^2}{(x+1)^2} - \frac{2x}{x+1} - 8 = 0\right)$$
$$x^2 - 2x(x+1) - 8(x+1)^2 = 0$$
$$x^2 - 2x^2 - 2x - 8x^2 - 16x - 8 = 0$$
$$-9x^2 - 18x - 8 = 0$$
$$(-1)(3x+2)(3x+4) = 0$$
$$x = -\frac{2}{3} \text{ or } x = -\frac{4}{3}$$

Be sure to check to make sure that neither solution is extraneous. Neither is extraneous. Solution set: $\left\{-\frac{4}{3}, -\frac{2}{3}\right\}.$

113. $\frac{x+b}{x-b} = \frac{x-5b}{2x-5b}$

$$(2x-5b)(x+b) = (x-b)(x-5b)$$
$$2x^2 - 3bx - 5b^2 = x^2 - 6bx + 5b^2$$
$$x^2 + 3bx - 10b^2 = 0$$
$$(x+5b)(x-2b) = 0$$
$$x = -5b \text{ or } x = 2b$$

Solution set: $\{-5b, 2b\}$

114. $\frac{1}{x+a} + \frac{1}{x-2a} = \frac{1}{x-3a}$

$$(x+a)(x-2a)(x-3a)\left(\frac{1}{x+a} + \frac{1}{x-2a}\right)$$
$$= (x+a)(x-2a)(x-3a)\frac{1}{x-3a}$$
$$x^2 - 5ax + 6a^2 + x^2 - 2ax - 3a^2$$
$$= x^2 - ax - 2a^2$$
$$2x^2 - 7ax + 3a^2 = x^2 - ax - 2a^2$$
$$x^2 - 6ax + 5a^2 = 0$$
$$(x-5a)(x-a) = 0$$
$$x = 5a \text{ or } x = a$$

Solution set: $\{a, 5a\}$

115. $3x - 2a = \sqrt{a(3x-2a)}$

$$9x^2 - 12ax + 4a^2 = 3ax - 2a^2$$
$$9x^2 - 15ax + 6a^2 = 0$$
$$3(3x-2a)(x-a) = 0$$
$$x = \frac{2a}{3} \text{ or } x = a$$

Solution set: $\left\{\frac{2a}{3}, a\right\}.$

116. $3a - \sqrt{ax} = \sqrt{a(3x+a)}$

$$9a^2 - 6a\sqrt{ax} + ax = 3ax + a^2$$
$$-6a\sqrt{ax} = 2ax - 8a^2$$
$$-3\sqrt{ax} = x - 4a$$
$$9ax = x^2 - 8ax + 16a^2$$
$$0 = x^2 - 17ax + 16a^2$$
$$0 = (x-16a)(x-a)$$
$$x = 16a \text{ or } x = a$$

1.4 Critical Thinking/Discussion/Writing

117. To find the value of x, we can let $x = \sqrt{1+x}$ because the square root contains a copy of itself. Now solve the equation:

$x = \sqrt{1+x} \Rightarrow x^2 = 1 + x \Rightarrow$

$x^2 - x - 1 = 0$. Now use the quadratic formula. $x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}.$

The original expression is positive, so $x = \frac{1-\sqrt{5}}{2}$ cannot be the solution.

Solution set: $\left\{\frac{1+\sqrt{5}}{2}\right\}.$

118. To find the value of x, we can let $x = \sqrt{20+x}$ because the square root contains a copy of itself. Now solve the equation:

$x = \sqrt{20+x} \Rightarrow x^2 = 20 + x \Rightarrow$

$x^2 - x - 20 = 0$. Now use the quadratic formula. $x = \frac{1 \pm \sqrt{1+80}}{2} = \frac{1 \pm 9}{2} \Rightarrow x = 5$ or $x = -4$. The original expression is positive, so $x = -4$ cannot be a solution.

Solution set: $\{5\}$.

119. To find the value of x, we can let $x=\sqrt{n+x}$ because the square root contains a copy of itself. Now solve the equation:
$x=\sqrt{n+x}\Rightarrow x^2=n+x\Rightarrow$
$x^2-x-n=0$. Now use the quadratic formula. $x=\dfrac{1\pm\sqrt{1+4n}}{2}=\dfrac{1}{2}\pm\dfrac{\sqrt{1+4n}}{2}$.
The original expression is positive, so we reject the negative root.
Solution set: $\left\{\dfrac{1}{2}+\dfrac{\sqrt{1+4n}}{2}\right\}$.

120. $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}=4$
Note that $x+2\sqrt{x-1}=(x-1)+1+2\sqrt{x-1}$ $=\left(\sqrt{x-1}+1\right)^2$. Similarly, $x-2\sqrt{x-1}=$ $(x-1)-2\sqrt{x-1}+1=\left(\sqrt{x-1}-1\right)^2$.
Substitute these expressions into the original equation:
$\sqrt{\left(\sqrt{x-1}+1\right)^2}+\sqrt{\left(\sqrt{x-1}-1\right)^2}=4\Rightarrow$
$\sqrt{x-1}+1+\sqrt{x-1}-1=4\Rightarrow$
$2\sqrt{x-1}=4\Rightarrow\sqrt{x-1}=2\Rightarrow x-1=4\Rightarrow$
$x=5$.
Solution set: $\{5\}$

1.4 Maintaining Skills

121.
$$\begin{aligned}12x-4&=28-4x\\16x&=32\\x&=2\end{aligned}$$
Solution set: $\{2\}$

122.
$$\begin{aligned}2(x-3)+17&=4x+1\\2x-6+17&=4x+1\\2x+11&=4x+1\\-2x&=-10\\x&=5\end{aligned}$$
Solution set: $\{5\}$

123.
$$\begin{aligned}2x+3(x-4)&=7x+10\\2x+3x-12&=7x+10\\5x-12&=7x+10\\-2x&=22\\x&=-11\end{aligned}$$
Solution set: $\{-11\}$

124.
$$\begin{aligned}2x-3-(3x-1)&=6\\2x-3-3x+1&=6\\-x-2&=6\\-x&=8\Rightarrow x=-8\end{aligned}$$
Solution set: $\{-8\}$

125.
$$\begin{aligned}\frac{-2x}{3}&=x+\frac{x}{2}\\6\left(\frac{-2x}{3}\right)&=6x+6\left(\frac{x}{2}\right)\\-4x&=6x+3x\\-4x&=9x\\0&=13x\Rightarrow 0=x\end{aligned}$$
Solution set: $\{0\}$

126.
$$\begin{aligned}\frac{x-1}{3}+\frac{2x}{5}&=x+2\\15\left(\frac{x-1}{3}\right)+15\left(\frac{2x}{5}\right)&=15x+2(15)\\5(x-1)+3(2x)&=15x+30\\5x-5+6x&=15x+30\\11x-5&=15x+30\\-4x&=35\\x&=-\frac{35}{4}\end{aligned}$$
Solution set: $\left\{-\dfrac{35}{4}\right\}$

127.
$$\begin{aligned}6x^2-102&=0\\6x^2&=102\\x^2&=17\\x&=\pm\sqrt{17}\end{aligned}$$
Solution set: $\left\{\pm\sqrt{17}\right\}$

128.
$$\begin{aligned}(x+5)^2&=27\\x+5&=\pm\sqrt{27}\\x&=-5\pm\sqrt{27}=-5\pm3\sqrt{3}\end{aligned}$$
Solution set: $\left\{-5\pm3\sqrt{3}\right\}$

129.
$$\begin{aligned}25x^2+1&=10x\\25x^2-10x+1&=0\\(5x-1)^2&=0\\5x-1&=0\\5x&=1\\x&=\frac{1}{5}\end{aligned}$$
Solution set: $\left\{\dfrac{1}{5}\right\}$

130.
$$\begin{aligned}x^2-10x+25&=0\\(x-5)^2&=0\\x-5&=0\\x&=5\end{aligned}$$
Solution set: $\{5\}$

131. $3x^2+3x-8=10$
$3x^2+3x-18=0$
$3\left(x^2+x-6\right)=0$
$x^2+x-6=0$
$(x+3)(x-2)=0 \Rightarrow x=-3,\ 2$
Solution set: $\{-3, 2\}$

132. $5x^2-6x=4x^2+6x-3$
$x^2-12x+3=0$
$$x=\frac{-(-12)\pm\sqrt{(-12)^2-4(1)(3)}}{2(1)}$$
$$=\frac{12\pm\sqrt{132}}{2}=\frac{12\pm2\sqrt{33}}{2}=6\pm\sqrt{33}$$
Solution set: $\left\{6\pm\sqrt{33}\right\}$

133. $\frac{5}{2x}+3=\frac{5+3(2x)}{2x}=\frac{5+6x}{2x}$

134. $6x+\frac{5}{x-3}=\frac{6x(x-3)+5}{x-3}=\frac{6x^2-18x+5}{x-3}$

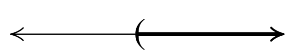
135. $$\frac{3}{x+1}-\frac{x-2}{x+3}=\frac{3(x+3)-(x+1)(x-2)}{(x+1)(x+3)}$$
$$=\frac{3x+9-\left(x^2-x-2\right)}{(x+1)(x+3)}$$
$$=\frac{-x^2+4x+11}{(x+1)(x+3)}$$

136. $$\frac{x}{x^2+6x+9}-\frac{x-3}{x^2+5x+6}$$
$$=\frac{x}{(x+3)^2}-\frac{x-3}{(x+3)(x+2)}$$
$$=\frac{x(x+2)-(x-3)(x+3)}{(x+3)^2(x+2)}$$
$$=\frac{x^2+2x-\left(x^2-9\right)}{(x+3)(x+2)}=\frac{2x+9}{(x+3)(x+2)}$$

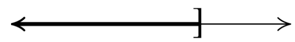
1.5 Inequalities

1.5 Practice Problems

1. a. $4x+9>2(x+6)+1 \Rightarrow 4x+9>2x+13 \Rightarrow$
$2x>4 \Rightarrow x>2$
Solution set: $(2, \infty)$



b. $7-2x\ge-3 \Rightarrow -2x\ge-10 \Rightarrow x\le5$
Solution set: $(-\infty, 5]$



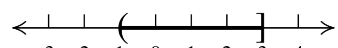
2. Let t = time elapsed since the plane went on autopilot. Then $340t$ = distance the plane has flown in t hours, and $185 + 340t$ = the plane's distance from Miami after t hours.
$185+340t\ge1035 \Rightarrow 340t\ge850 \Rightarrow t\ge2.5$
The tower will suspect trouble if the plane has not arrived in 2.5 hours.

3. a. $2(4-x)+6x<4(x+1)+7$
$8-2x+6x<4x+4+7$
$8+4x<4x+11$
$0<3$
The last inequality is always true, so the solution set is $(-\infty, \infty)$.

b. $3(x-2)+5\ge7(x-1)-4(x-2)$
$3x-6+5\ge7x-7-4x+8$
$3x-1\ge3x+1 \Rightarrow -1\ge1$
The last inequality is always false, so the solution set is $\varnothing$.

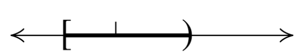
4. $3x-5\ge7$ or $5-2x\ge1$
$3x\ge12$ or $-2x\ge-4$
$x\ge4$ or $x\le2$
Solution set: $(-\infty, 2]\cup[4, \infty)$

5. $2(3-x)-3<5$ and $2(x-5)+7\le3$
$6-2x-3<5$ and $2x-10+7\le3$
$3-2x<5$ and $2x-3\le3$
$-2x<2$ and $2x\le6$
$x>-1$ and $x\le3$
Solution set: $(-1, 3]$



6. $-6\le4x-2<4 \Rightarrow -4\le4x<6 \Rightarrow -1\le x<\frac{3}{2}$

Solution set: $\left[-1, \frac{3}{2}\right)$



7. Start with the interval for x.

$$-3 \le x \le 2$$
$$3(-3) \le 3x \le 3(2)$$
$$-9 \le 3x \le 6$$
$$-9+5 \le 3x+5 \le 6+5$$
$$-4 \le 3x+5 \le 11$$

Therefore, $a = -4$ and $b = 11$.

8.

$$15 < C < 25$$
$$\left(\frac{9}{5}\right)15 < \frac{9}{5}C < \left(\frac{9}{5}\right)25$$
$$27 < \frac{9}{5}C < 45$$
$$27+32 < \frac{9}{5}C+32 < 45+32$$
$$59 < \frac{9}{5}C+32 < 77$$

The temperature range from 15°C to 25°C corresponds to a range from 59°F to 77°F.

9.

$$x^2+2 < 3x+6$$
$$x^2-3x-4 < 0 \Rightarrow (x+1)(x-4) < 0$$

The factors equal 0 at $x = -1$ and $x = 4$.

Interval	Test point	Value of x^2-3x-4	Result
$(-\infty, -1)$	-2	6	+
$(-1, 4)$	0	-4	–
$(4, \infty)$	5	6	+

The solution set is $(-1, 4)$.

10.

$$\frac{2x+5}{x-1} \le 1$$
$$\frac{2x+5}{x-1} - 1 \le 0$$
$$\frac{2x+5-x+1}{x-1} \le 0 \Rightarrow \frac{x+6}{x-1} \le 0$$
$$x+6=0 \Rightarrow x=-6$$
$$x-1=0 \Rightarrow x=1$$

The test points are -6 and 1.

Interval	Test point	Value of $\frac{x+6}{x-1}$	Result
$(-\infty, -6)$	-9	$\frac{3}{10}$	+
$(-6, 1)$	0	-6	–
$(1, \infty)$	2	8	+

Note that the expression is undefined for $x = 1$. The solution set is $[-6, 1)$.

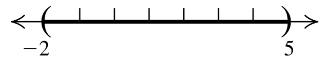
1.5 Basic Concepts and Skills

1. $<$ **2.** $\le$ **3.** $\ge$ **4.** $>$

5. $<$ **6.** $<$ **7.** $\ge$ **8.** $<$

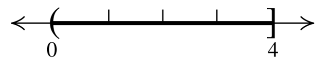
9. $<$ **10.** $\le$

11. $(-2, 5)$

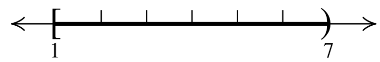


12. $[-5, 0]$

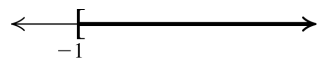
13. $(0, 4]$



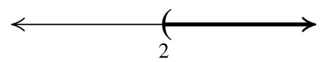
14. $[1, 7)$



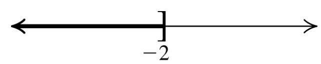
15. $[-1, \infty)$



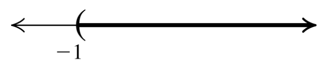
16. $(2, \infty)$



17. $(-\infty, -2]$

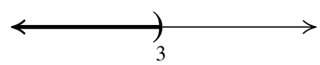


18. $(-1, \infty)$



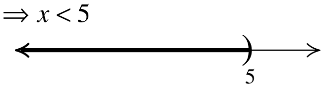
19. $x+3 < 6 \Rightarrow x < 3$

$(-\infty, 3)$



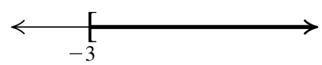
20. $x-2 < 3 \Rightarrow x < 5$

$(-\infty, 5)$



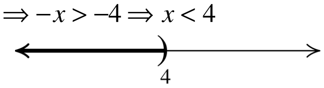
21. $1-x \le 4 \Rightarrow -x \le 3 \Rightarrow x \ge -3$

$[-3, \infty)$



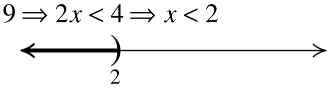
22. $7-x > 3 \Rightarrow -x > -4 \Rightarrow x < 4$

$(-\infty, 4)$



23. $2x+5 < 9 \Rightarrow 2x < 4 \Rightarrow x < 2$

$(-\infty, 2)$

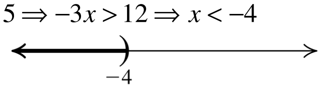


24. $3x+2 \ge 7 \Rightarrow 3x \ge 5 \Rightarrow x \ge \frac{5}{3}$

$\left[\frac{5}{3}, \infty\right)$

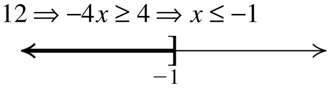
25. $3-3x > 15 \Rightarrow -3x > 12 \Rightarrow x < -4$

$(-\infty, -4)$

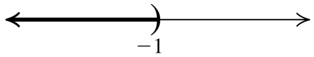


26. $8-4x \ge 12 \Rightarrow -4x \ge 4 \Rightarrow x \le -1$

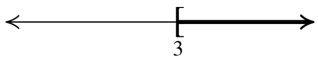
$(-\infty, -1]$



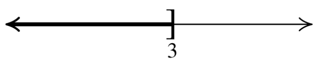
27. $3(x+2)<2x+5 \Rightarrow 3x+6<2x+5 \Rightarrow x<-1$

$(-\infty,-1)$ 

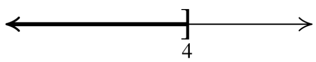
28. $4(x-1)\ge 3x-1 \Rightarrow 4x-4\ge 3x-1 \Rightarrow x\ge 3$

$[3,\infty)$ 

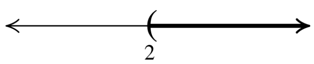
29. $3(x-3)\le 3-x \Rightarrow 3x-9\le 3-x \Rightarrow$
$4x\le 12 \Rightarrow x\le 3$

$(-\infty,3]$ 

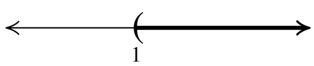
30. $-x-2\ge x-10 \Rightarrow 8\ge 2x \Rightarrow 4\ge x$

$(-\infty,4]$ 

31. $6x+4>3x+10 \Rightarrow 3x>6 \Rightarrow x>2$

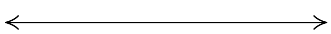
$(2,\infty)$ 

32. $4(x-4)>3(x-5) \Rightarrow 4x-16>3x-15 \Rightarrow$
$x>1$

$(1,\infty)$ 

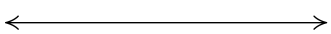
33. $8(x-1)-x\le 7x-12$
$8x-8-x\le 7x-12$
$7x-8\le 7x-12 \Rightarrow -8\le -12$ False

The solution set is $\varnothing$.

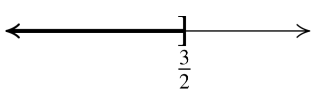


34. $3(x+2)+2x\ge 5x+18$
$3x+6+2x\ge 5x+18$
$5x+6\ge 5x+18 \Rightarrow 6\ge 18$ False

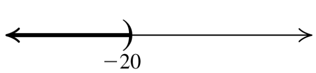
The solution set is $\varnothing$.



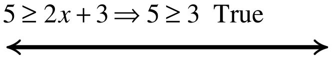
35. $5(x+2)\le 3(x+1)+10 \Rightarrow$
$5x+10\le 3x+3+10 \Rightarrow 5x+10\le 3x+13 \Rightarrow$
$2x\le 3 \Rightarrow x\le \frac{3}{2}$

$\left(-\infty,\frac{3}{2}\right]$ 

36. $x-4>2(x+8) \Rightarrow x-4>2x+16 \Rightarrow$
$-20>x$

$(-\infty,-20)$ 

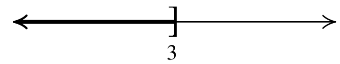
37. $2(x+1)+3\ge 2(x+2)-1$
$2x+2+3\ge 2x+4-1$
$2x+5\ge 2x+3 \Rightarrow 5\ge 3$ True

$(-\infty,\ \infty)$ 

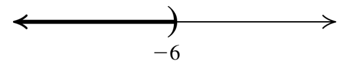
38. $5(2-x)+4x\le 12-x$
$10-5x+4x\le 12-x$
$10-x\le 12-x \Rightarrow 10\le 12$ True

$(-\infty,\ \infty)$ 

39. $2(x+1)-2\le 3(2-x)+9$
$2x+2-2\le 6-3x+9$
$2x\le 15-3x$
$5x\le 15 \Rightarrow x\le 3$

$(\infty,\ 3]$ 

40. $4(1-x)+2x>5(2-x)+4x$
$4-4x+2x>10-5x+4x$
$4-2x>10-x \Rightarrow -6>x \Rightarrow x<-6$

$(-\infty,\ -6)$ 

41. $9x-6\ge \frac{3}{2}x+9$
$18x-12\ge 3x+18$
$15x\ge 30 \Rightarrow x\ge 2$

Solution set: $[2,\ \infty)$

42. $\frac{7x-3}{2}<3x-4 \Rightarrow 7x-3<6x-8 \Rightarrow$
$x<-5$

Solution set: $(-\infty,-5)$

43. $\frac{x-3}{3}\le 2+\frac{x}{2} \Rightarrow 2x-6\le 12+3x \Rightarrow$
$-18\le x \Rightarrow x\ge -18$

Solution set: $[-18,\ \infty)$

44. $\frac{2x-3}{4}\ge 3-\frac{x}{2} \Rightarrow 2x-3\ge 12-2x \Rightarrow$
$4x\ge 15 \Rightarrow x\ge \frac{15}{4}$

Solution set: $\left[\frac{15}{4},\ \infty\right)$

45. $\frac{3x+1}{2}<x-1+\frac{x}{2}$
$3x+1<2x-2+x$
$3x+1<3x-2 \Rightarrow 1<-2$ False

Solution set: $\varnothing$

46. $\frac{2x-1}{3}\ge \frac{x+1}{4}+\frac{x}{12}$
$8x-4\ge 3x+3+x$
$8x-4\ge 4x+3$
$4x\ge 7 \Rightarrow x\ge \frac{7}{4}$

Solution set: $\left[\frac{7}{4},\ \infty\right)$

47. $\frac{x-3}{2} \ge \frac{x}{3}+1$
$3x-9 \ge 2x+6 \Rightarrow x \ge 15$
Solution set: $[15, \infty)$

48. $\frac{2x+1}{3} < \frac{x-1}{2}+\frac{1}{6}$
$4x+2 < 3x-3+1$
$4x+2 < 3x-2 \Rightarrow x < -4$
Solution set: $(-\infty, -4)$

49. $\frac{3x+1}{3}-\frac{x}{2} \le \frac{x+2}{2}$
$6x+2-3x \le 3x+6$
$3x+2 \le 3x+6 \Rightarrow 2 \le 6$ True
Solution set: $(-\infty, \infty)$

50. $\frac{x-1}{3}+\frac{x+1}{4} \le \frac{x}{2}+\frac{x}{12}$
$4x-4+3x+3 \le 6x+x$
$7x-1 \le 7x \Rightarrow -1 \le 0$ True
Solution set: $(-\infty, \infty)$

51. $2x+5<1$ or $2+x>4$
$2x<-4$ or $x>2$
$x<-2$ or
Solution set: $(-\infty, -2)\cup(2, \infty)$

52. $3x-2>7$ or $2(1-x)>1$
$3x>9$ or $2-2x>1$
$x>3$ or $-2x>-1$
$x<-\frac{1}{2}$
Solution set: $\left(-\infty, -\frac{1}{2}\right)\cup(3, \infty)$

53. $\frac{2x-3}{4} \le 2$ or $\frac{4-3x}{2} \ge 2$
$2x-3 \le 8$ or $4-3x \ge 4$
$2x \le 11$ or $-3x \ge 0$
$x \le \frac{11}{2}$ or $x \le 0$
Solution set: $\left(-\infty, \frac{11}{2}\right]$

54. $\frac{5-3x}{3} \ge \frac{1}{6}$ or $\frac{x-1}{3} \le 1$
$10-6x \ge 1$ or $x-1 \le 3$
$-6x \ge -9$ or $x \le 4$
$x \le \frac{3}{2}$
Solution set: $(-\infty, 4]$

55. $\frac{2x+1}{3} \ge x+1$ or $\frac{x}{2}-1 > \frac{x}{3}$
$2x+1 \ge 3x+3$ or $3x-6 > 2x$
$-2 \ge x$ or $-6 > -x$
$x \le -2$ or $6 < x \Rightarrow x > 6$
Solution set: $(-\infty, -2]\cup(6, \infty)$

56. $\frac{x+2}{2} < \frac{x}{3}+1$ or $\frac{x-1}{3} > \frac{x+1}{5}$
$3x+6 < 2x+6$ or $5x-5 > 3x+3$
$x<0$ or $2x>8 \Rightarrow x>4$
Solution set: $(-\infty, 0)\cup(4, \infty)$

57. $\frac{x-1}{2} > \frac{x}{3}-1$ or $\frac{2x+5}{3} \le \frac{x+1}{6}$
$3x-3 > 2x-6$ or $4x+10 \le x+1$
$x>-3$ or $3x \le -9 \Rightarrow x \le -3$
Solution set: $(-\infty, \infty)$

58. $\frac{2x+1}{3} \le \frac{x}{4}+1$ or $\frac{3-x}{2} > \frac{x}{3}-1$
$8x+4 \le 3x+12$ or $9-3x > 2x-6$
$5x \le 8$ or $-5x > -15$
$x \le \frac{8}{5}$ or $x < 3$
Solution set: $(-\infty, 3)$

59. $3-2x \le 7$ and $2x-3 \le 7$
$-2x \le 4$ and $2x \le 10$
$x \ge -2$ and $x \le 5$
Solution set: $[-2, 5]$

60. $6-x \le 3x+10$ and $7x-14 \le 3x+14$
$-4x \le 4$ and $4x \le 28$
$x \ge -1$ and $x \le 7$
Solution set: $[-1, 7]$

61. $2(x+1)+3 \ge 1$ and $2(2-x) > -6$
$2x+2+3 \ge 1$ and $4-2x > -6$
$2x+5 \ge 1$ and $-2x > -10$
$2x \ge -4$ and $x < 5$
$x \ge -2$
Solution set: $[-2, 5)$

62. $3(x+1)-2 \ge 4$ and $3(1-x)+13 > 4$
$3x+3-2 \ge 4$ and $3-3x+13 > 4$
$3x+1 \ge 4$ and $-3x+16 > 4$
$3x \ge 3$ and $-3x > -12$
$x \ge 1$ and $x < 4$
Solution set: $[1, 4)$

63.
$$\begin{aligned} 2(x+1)-3>7 &\quad \text{and} \quad 3(2x+1)+1<10 \\ 2x+2-3>7 &\quad \text{and} \quad 6x+3+1<10 \\ 2x-1>7 &\quad \text{and} \quad 6x+4<10 \\ 2x>8 &\quad \text{and} \quad 6x<6 \\ x>4 &\quad \text{and} \quad x<1 \end{aligned}$$

Since x cannot be less than 1 and greater than 4 at the same time, the solution set is $\varnothing$.

64.
$$\begin{aligned} 5(x+2)+7<2 &\quad \text{and} \quad 2(5-3x)+1<17 \\ 5x+10+7<2 &\quad \text{and} \quad 10-6x+1<17 \\ 5x+17<2 &\quad \text{and} \quad 11-6x<17 \\ 5x<-15 &\quad \text{and} \quad -6x<6 \\ x<-3 &\quad \text{and} \quad x>-1 \end{aligned}$$

Since x cannot be less than −3 and greater than −1 at the same time, the solution set is $\varnothing$.

65.
$$\begin{aligned} 5+3(x-1)<3+3(x+1) &\quad \text{and} \quad 3x-7\le 8 \\ 5+3x-3<3+3x+3 &\quad \text{and} \quad 3x\le 15 \\ 2+3x<6+3x &\quad \text{and} \quad x\le 5 \\ 2<6 \text{ True} & \end{aligned}$$

Solution set: $(-\infty, 5]$

66.
$$\begin{aligned} 2x-3>11 &\quad \text{and} \quad 5(2x+1)<3(4x+1)-2(x-1) \\ 2x>14 &\quad \text{and} \quad 10x+5<12x+3-2x+2 \\ x>7 &\quad \text{and} \quad 10x+5<10x+5 \\ & \quad 0<0 \text{ False} \end{aligned}$$

Solution set: $\varnothing$.

67. $3<x+5<4 \Rightarrow -2<x<-1$ or $(-2,-1)$

68. $9\le x+7\le 12 \Rightarrow 2\le x\le 5$ or $[2,5]$

69. $-4\le x-2<2 \Rightarrow -2\le x<4$ or $[-2,4)$

70. $-3<x+5<4 \Rightarrow -8<x<-1$ or $(-8,-1)$

71. $-9\le 2x+3\le 5 \Rightarrow -12\le 2x\le 2 \Rightarrow$
$-6\le x\le 1$ or $[-6,1]$

72. $-2\le 3x+1\le 7 \Rightarrow -3\le 3x\le 6 \Rightarrow$
$-1\le x\le 2$ or $[-1,2]$

73. $0\le 1-\dfrac{x}{3}<2 \Rightarrow 0\le 3-x<6 \Rightarrow$
$-3\le -x<3 \Rightarrow 3\ge x>-3$ or $(-3,3]$

74. $0<5-\dfrac{x}{2}\le 3 \Rightarrow 0<10-x\le 6 \Rightarrow$
$-10<-x\le -4 \Rightarrow 10>x\ge 4$ or $[4,10)$

75. $-1<\dfrac{2x-3}{5}\le 0 \Rightarrow -5<2x-3\le 0 \Rightarrow$
$-2<2x\le 3 \Rightarrow -1<x\le \dfrac{3}{2}$ or $\left(-1,\dfrac{3}{2}\right]$

76. $-4\le \dfrac{5x-2}{3}\le 0 \Rightarrow -12\le 5x-2\le 0 \Rightarrow$
$-10\le 5x\le 2 \Rightarrow -2\le x\le \dfrac{2}{5}$ or $\left[-2,\dfrac{2}{5}\right]$

77. $5x\le 3x+1<4x-2 \Rightarrow 2x\le 1<x-2 \Rightarrow$
$x\le \dfrac{1}{2}$ and $3<x \Rightarrow x>3$

x cannot be less than or equal to $\dfrac{1}{2}$ at the same time that 3 is less than x, so the solution set is $\varnothing$.

78. $3x+2<2x+3<4x-1 \Rightarrow$
$x+2<3<2x-1 \Rightarrow x+2<3$ and
$3<2x-1 \Rightarrow x<1$ and $4<2x \Rightarrow$
$x<1$ and $2<x$
x cannot be less than 1 at the same time that 2 is less than x, so the solution set is $\varnothing$.

79. $-2<x<1 \Rightarrow -2+7<x+7<1+7 \Rightarrow$
$5<x+7<8 \Rightarrow a=5, b=8$

80. $1<x<5 \Rightarrow 2<2x<10 \Rightarrow$
$2+3<2x+3<10+3 \Rightarrow 5<2x+3<13 \Rightarrow$
$a=5, b=13$

81. $-1<x<1 \Rightarrow 1>-x>-1 \Rightarrow$
$2+1>2-x>2-1 \Rightarrow 3>2-x>1 \Rightarrow$
$1<2-x<3 \Rightarrow a=1, b=3$

82. $3<x<7 \Rightarrow -9>-3x>-21 \Rightarrow$
$1-9>1-3x>1-21 \Rightarrow$
$-8>1-3x>-20 \Rightarrow -20<1-3x<-8 \Rightarrow$
$a=-20, b=-8$

83. $0<x<4 \Rightarrow 0<5x<20 \Rightarrow -1<5x-1<19 \Rightarrow$
$a=-1, b=19$

84. $-4<x<0 \Rightarrow -12<3x<0 \Rightarrow$
$-8<3x+4<4 \Rightarrow a=-8, b=4$

85. $x^2+4x-12\le 0 \Rightarrow (x+6)(x-2)\le 0$.
Now solve the associated equation:
$(x+6)(x-2)=0 \Rightarrow x=-6$ or $x=2$.
So, the intervals are
$(-\infty,-6], [-6,2],$ and $[2,\infty)$.

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Interval	Test point	Value of $x^2+4x-12$	Result
$(-\infty,-6]$	–10	48	+
$[-6,2]$	0	–12	–
$[2,\infty)$	3	9	+

The solution set is $[-6,2]$.

86. $x^2-8x+7>0 \Rightarrow (x-7)(x-1)>0$.
Now solve the associated equation:
$(x-7)(x-1)=0 \Rightarrow x=7$ or $x=1$.
So, the intervals are
$(-\infty, 1), (1, 7)$, and $(7,\infty)$.

Interval	Test point	Value of x^2-8x+7	Result
$(-\infty, 1)$	0	7	+
$(1, 7)$	2	–5	–
$(7,\infty)$	10	27	+

The solution set is $(-\infty, 1)\cup(7,\infty)$.

87. $6x^2+7x-3\ge 0 \Rightarrow (3x-1)(2x+3)\ge 0$.
Now solve the associated equation:
$(3x-1)(2x+3)=0 \Rightarrow x=1/3$ or $x=-3/2$.
The intervals are $(-\infty,-3/2],[-3/2,1/3]$, and $[1/3,\infty)$.

Interval	Test point	Value of $6x^2+7x-3$	Result
$(-\infty,-3/2]$	–2	7	+
$[-3/2,1/3]$	0	–3	–
$[1/3,\infty)$	1	10	+

The solution set is $(-\infty,-3/2]\cup[1/3,\infty)$.

88. $4x^2-2x-2<0 \Rightarrow 2(2x^2-x-1)<0 \Rightarrow$
$2(2x+1)(x-1)<0$. Now solve the associated equation: $2(2x+1)(x-1)=0 \Rightarrow$
$x=-1/2$ or $x=1$. So, the intervals are
$(-\infty,-1/2),(-1/2,1)$, and $(1,\infty)$.

Interval	Test point	Value of $4x^2-2x-2$	Result
$(-\infty,-1/2)$	–1	4	+
$(-1/2,1)$	0	–2	–
$(1,\infty)$	2	10	+

The solution set is $(-1/2,1)$.

89. $(x+3)(x+1)(x-1)\ge 0$.
Now solve the associated equation:
$(x+3)(x+1)(x-1)=0 \Rightarrow$
$x=-3, x=-1$ or $x=1$.
The intervals are
$(-\infty,-3],[-3,-1],[-1,1]$, and $[1,\infty)$.

Interval	Test point	Value of $(x+3)(x+1)(x-1)$	Result
$(-\infty,-3]$	–10	–693	–
$[-3,-1]$	–2	3	+
$[-1,1]$	0	–3	–
$[1,\infty)$	10	1287	+

The solution set is $[-3, -1]\cup[1, \infty)$.

90. $(x+4)(x-1)(x+2)\le 0$.
Now solve the associated equation:
$(x+4)(x-1)(x+2)=0 \Rightarrow x=-4, x=1$ or $x=-2$.
The intervals are
$(-\infty,-4],[-4,-2],[-2,1]$, and $[1,\infty)$.

Interval	Test point	Value of $(x+4)(x-1)(x+2)$	Result
$(-\infty,-4]$	–10	–528	–
$[-4,-2]$	–3	4	+
$[-2,1]$	0	–8	–
$[1,\infty)$	10	1008	+

The solution set is $(-\infty,-4]\cup[-2,1]$.

91. $x^3-4x^2-12x>0$
Solve the associated equation
$x^3-4x^2-12x=0$.
$x^3-4x^2-12x=0 \Rightarrow x(x^2-4x-12)=0 \Rightarrow$
$x(x+2)(x-6)=0 \Rightarrow x=0, -2, 6$

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The intervals are $(-\infty, -2)$, $(-2, 0)$, $(0, 6)$, and $(6, \infty)$.

Interval	Test point	Value of x^3-4x^2-12x	Result
$(-\infty, -2)$	–10	–1280	–
$(-2, 0)$	–1	7	+
$(0, 6)$	1	–15	–
$(6, \infty)$	10	480	+

The solution set is $(-2, 0)\cup(6, \infty)$.

92. $x^3+8x^2+15x>0$
Solve the associated equation
$x^3+8x^2+15x=0.$
$x^3+8x^2+15x=0 \Rightarrow x\left(x^2+8x+15\right)=0 \Rightarrow$
$x(x+3)(x+5)=0 \Rightarrow x=0, -3, -5$
The intervals are $(-\infty, -5)$, $(-5, -3)$, $(-3, 0)$, and $(0, \infty)$.

Interval	Test point	Value of x^3+8x^2+15x	Result
$(-\infty, -5)$	–10	–350	–
$(-5, -3)$	–4	4	+
$(-3, 0)$	–1	–8	–
$(0, \infty)$	1	24	+

The solution set is $(-5, -3)\cup(0, \infty)$.

93. $x^2+2x<-1 \Rightarrow x^2+2x+1<0.$
Now solve the associated equation:
$x^2+2x+1=0 \Rightarrow (x+1)^2=0 \Rightarrow x=-1$.
The intervals are $(-\infty,-1)$ and $(-1,\infty)$.

Interval	Test point	Value of x^2+2x+1	Result
$(\infty,-1)$	–2	1	+
$(-1,\infty)$	0	1	+

Neither interval has a negative solution, so the solution set is $\varnothing$.

94. $4x^2+12x<-9 \Rightarrow 4x^2+12x+9<0.$ Now solve the associated equation:
$4x^2+12x+9=0 \Rightarrow (2x+3)^2=0 \Rightarrow$
$x=-\frac{3}{2}$. The intervals are
$(-\infty,-3/2)$ and $(-3/2,\infty)$.

Interval	Test point	Value of $4x^2+12x+9$	Result
$(-\infty,-3/2)$	–2	1	+
$(-3/2,\infty)$	0	9	+

Neither interval has a negative solution, so the solution set is $\varnothing$.

95. $x^3-x^2\ge 0.$
Now solve the associated equation:
$x^3-x^2=0 \Rightarrow x^2(x-1)=0 \Rightarrow$
$x=0$ or $x=1.$
The intervals are $(-\infty,0],[0,1]$ and $[1,\infty)$.

Interval	Test point	Value of x^3-x^2	Result
$(-\infty,0]$	–1	–2	–
$[0,1]$	$\frac{1}{2}$	$-\frac{1}{8}$	–
$[1,\infty)$	2	4	+

Note that 0 is a solution. The solution set is $\{0\}\cup[1,\infty)$.

96. $x^3-9x^2\ge 0.$
Now solve the associated equation:
$x^3-9x^2=0 \Rightarrow x^2(x-9)=0 \Rightarrow$
$x=0$ or $x=9.$
The intervals are $(-\infty,0]$, $[0,9]$ and $[9,\infty)$.
Note that 0 is a solution.

Interval	Test point	Value of x^3-9x^2	Result
$(-\infty,0]$	–1	–10	–
$[0,9]$	1	–8	–
$[9,\infty)$	10	100	+

The solution set is $[9,\infty)\cup\{0\}$.

97. $x^2 \ge 1 \Rightarrow x^2 - 1 \ge 0.$
Now solve the associated equation:
$x^2 - 1 = 0 \Rightarrow (x-1)(x+1) = 0 \Rightarrow$
$x = -1$ or $x = 1.$
The intervals are $(-\infty, -1], [-1, 1],$ and $[1, \infty).$

Interval	Test point	Value of $x^2 - 1$	Result
$(-\infty, -1]$	–2	3	+
$[-1, 1]$	0	–1	–
$[1, \infty)$	2	3	+

The solution set is $(-\infty, -1] \cup [1, \infty).$

98. $x^4 \le 16 \Rightarrow x^4 - 16 \le 0.$
Now solve the associated equation:
$x^4 - 16 = 0 \Rightarrow (x^2 - 4)(x^2 + 4) = 0 \Rightarrow$
$(x-2)(x+2)(x^2+4) = 0 \Rightarrow x = 2, x = -2$ or
$x^2 + 4 = 0.$
Solving $x^2 + 4 = 0$ using the quadratic formula gives $x = \pm 2i.$ We cannot use complex intervals, so the only intervals we examine are $(-\infty, -2],$ $[-2, 2],$ and $[2, \infty).$

Interval	Test point	Value of $x^4 - 16$	Result
$(-\infty, -2]$	–10	9984	+
$[-2, 2]$	0	–16	–
$[2, \infty)$	10	9984	+

The solution set is $[-2,\ 2].$

99. $x^3 < -8 \Rightarrow x^3 + 8 < 0.$
Now solve the associated equation:
$x^3 + 8 = 0 \Rightarrow (x+2)(x^2 - 2x + 4) = 0 \Rightarrow$
$x = -2$ or $x^2 - 2x + 4 = 0.$
Solve $x^2 - 2x + 4 = 0$ using the quadratic formula:
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = 1 \pm i\sqrt{3}.$$
We cannot use complex intervals, so the only intervals we examine are $(-\infty, -2)$ and $(-2, \infty).$

Interval	Test point	Value of $x^3 + 8$	Result
$(-\infty, -2)$	–10	–992	–
$(-2, \infty)$	10	1008	+

The solution set is $(-\infty, -2).$

100. $x^4 > 9 \Rightarrow x^4 - 9 > 0.$
Now solve the associated equation:
$x^4 - 9 = 0 \Rightarrow (x^2 - 3)(x^2 + 3) = 0 \Rightarrow$
$\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x^2 + 3) = 0 \Rightarrow x = -\sqrt{3},$
$x = \sqrt{3},$ or $x^2 + 3 = 0.$
Solving $x^2 + 3 = 0$ using the quadratic formula gives $x = \pm i\sqrt{3}.$ We cannot use complex intervals, so the only intervals we examine are $\left(-\infty, -\sqrt{3}\right),$ $\left(-\sqrt{3}, \sqrt{3}\right),$ or $\left(\sqrt{3}, \infty\right).$

Interval	Test point	Value of $x^4 - 9$	Result
$\left(-\infty, -\sqrt{3}\right)$	–10	9991	+
$\left(-\sqrt{3}, \sqrt{3}\right)$	0	–9	–
$\left(\sqrt{3}, \infty\right)$	10	9991	+

The solution set is $\left(-\infty,\ -\sqrt{3}\right) \cup \left(\sqrt{3},\ \infty\right).$

101. $\dfrac{x+2}{x-5} < 0.$
Now solve $x + 2 = 0 \Rightarrow x = -2$ and $x - 5 = 0 \Rightarrow x = 5$.
So the intervals are $(-\infty, -2), (-2, 5),$ and $(5, \infty).$

Interval	Test point	Value of $\dfrac{x+2}{x-5}$	Result
$(-\infty, -2)$	–10	8/15	+
$(-2, 5)$	0	−2/5	–
$(5, \infty)$	10	12/5	+

Note that the fraction is undefined if $x = 5.$
The solution set is $(-2,\ 5).$

102. $\frac{x-3}{x+1} > 0$. Now solve $x-3=0 \Rightarrow x=3$ and $x+1=0 \Rightarrow x=-1$. So the intervals are $(-\infty,-1), (-1,3),$ and $(3,\infty)$.

Interval	Test point	Value of $\frac{x-3}{x+1}$	Result
$(-\infty,-1)$	–10	13/9	+
$(-1,3)$	0	–3	–
$(3,\infty)$	10	7/11	+

Note that the fraction is undefined if $x = -1$. The solution set is $(-\infty,-1)\cup(3,\infty)$.

103. $\frac{x+4}{x} < 0$.
Solve $x+4=0 \Rightarrow x=-4$ and $x=0$. The fraction is undefined if $x=0$. So the intervals are $(-\infty,-4),\ (-4,0),$ and $(0,\infty)$.

Interval	Test point	Value of $\frac{x+4}{x}$	Result
$(-\infty,-4)$	–10	3/5	+
$(-4,0)$	–2	–1	–
$(0,\infty)$	2	3	+

The solution set is $(-4,\ 0)$.

104. $\frac{x}{x-2} > 0$.
Now solve $x=0$ and $x-2=0 \Rightarrow x=2$. So the intervals are $(-\infty,0), (0,2),$ and $(2,\infty)$.

Interval	Test point	Value of $\frac{x}{x-2}$	Result
$(-\infty,0)$	–1	1/3	+
$(0,2)$	1	–1	–
$(2,\infty)$	3	3	+

Note that the fraction is undefined if $x=0$. The solution set is $(-\infty,0)\cup(2,\infty)$.

105. $\frac{x+1}{x+2} \le 3 \Rightarrow \frac{x+1}{x+2} - 3 \le 0 \Rightarrow \frac{x+1-3(x+2)}{x+2} \le 0 \Rightarrow \frac{-2x-5}{x+2} \le 0.$
Now solve $-2x-5=0 \Rightarrow x=-5/2$ and $x+2=0 \Rightarrow x=-2$. So the intervals are $(-\infty,-5/2],\ [-5/2,-2),$ and $(-2,\infty)$. The original fraction is not defined if $x = -2$, so –2 is not included in the intervals.

Interval	Test point	Value of $\frac{-2x-5}{x+2}$	Result
$(-\infty,-5/2]$	–3	–1	–
$[-5/2,-2)$	–9/4	2	+
$(-2,\infty)$	3	–11/5	–

The solution set is $(-\infty,\ -5/2]\cup(-2,\ \infty)$.

106. $\frac{x-1}{x-2} \ge 3 \Rightarrow \frac{x-1}{x-2} - 3 \ge 0 \Rightarrow \frac{x-1-3(x-2)}{x-2} \ge 0 \Rightarrow \frac{-2x+5}{x-2} \ge 0.$
Now solve $-2x+5=0 \Rightarrow x=5/2$ and $x-2=0 \Rightarrow x=2$.
So the intervals are $(-\infty,2),\ (2,5/2],$ and $[5/2,\infty)$. The original fraction is not defined if $x = 2$, so 2 is not included in the intervals.

Interval	Test point	Value of $\frac{-2x+5}{x-2}$	Result
$(-\infty,2)$	0	–2/5	–
$(2,5/2]$	9/4	2	+
$[5/2,\infty)$	3	–1	–

The solution set is $(2,5/2]$.

107. $\frac{(x-2)(x+2)}{x} > 0.$
Now we have $x=0$ and $(x-2)(x+2)=0 \Rightarrow x=2$ and $x=-2$.
So the intervals are $(-\infty,-2), (-2,0), (0,2),$ and $(2,\infty)$. Note that the original fraction is not defined if $x = 0$, so 0 is not included in the intervals.

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Interval	Test point	Value of $\frac{(x-2)(x+2)}{x}$	Result
$(-\infty,-2)$	–3	$-5/3$	–
$(-2,0)$	–1	3	+
$(0,2)$	1	–3	–
$(2,\infty)$	3	$5/3$	+

The solution set is $(-2,\ 0)\cup(2,\ \infty)$.

108. $\frac{(x-1)(x+3)}{x-2}<0.$

Now we have $x-2=0\Rightarrow\ x=2$ and $(x-1)(x+3)=0\Rightarrow x=1$ and $x=-3$.

So the intervals are $(-\infty,-3),(-3,1),\ (1,2)$, and $(2,\infty)$. Note that the original fraction is not defined if $x = 0$, so 0 is not included in the intervals.

Interval	Test point	Value of $\frac{(x-1)(x+3)}{x-2}$	Result
$(-\infty,-3)$	–4	$-5/6$	–
$(-3,1)$	0	$3/2$	+
$(1,2)$	$3/2$	$-9/2$	–
$(2,\infty)$	3	12	+

The solution set is $(-\infty,-3)\cup(1,2)$.

109. $\frac{(x-2)(x+1)}{(x-3)(x+5)}\ge 0.$

Set the numerator and denominator equal to zero and solve for x.

$(x-2)(x+1)=0\Rightarrow x=2,\ -1$

$(x-3)(x+5)=0\Rightarrow x=3,\ -5$

The intervals are $(-\infty,\ -5),\ (-5,\ -1],$ $[-1,\ 2],\ [2,\ 3)$, and $(3,\ \infty)$.

Interval	Test point	Value of $\frac{(x-2)(x+1)}{(x-3)(x+5)}$	Result
$(-\infty,\ -5)$	–6	$40/9$	+
$(-5,\ -1]$	–2	$-4/15$	–
$[-1,\ 2]$	0	$2/15$	+
$[2,\ 3)$	$5/2$	$-5/15$	–
$(3,\ \infty)$	5	$9/10$	+

The solution set is $(-\infty,\ -5)\cup[-1,\ 2]\cup(3,\ \infty)$.

110. $\frac{(x-1)(x-3)}{(x+2)(x+4)}\ge 0.$

Set the numerator and denominator equal to zero and solve for x.

$(x-1)(x-3)=0\Rightarrow x=1,\ 3$

$(x+2)(x+4)=0\Rightarrow x=-2,\ -4$

The intervals are $(-\infty,\ -4),\ (-4,\ -2),$ $(-2,\ 1],\ [1,\ 3]$, and $[3,\ \infty)$.

Interval	Test point	Value of $\frac{(x-1)(x-3)}{(x+2)(x+4)}$	Result
$(-\infty,\ -4)$	–6	$63/8$	+
$(-4,\ -2)$	–3	–24	–
$(-2,\ 1]$	0	$3/8$	+
$[1,\ 3]$	2	$-1/24$	–
$[3,\ \infty)$	5	$8/63$	+

The solution set is $(-\infty,\ -4)\cup(-2,\ 1]\cup[3,\ \infty)$.

111. $\frac{x^2-1}{x^2-4}\le 0.$

Set the numerator and denominator equal to zero and solve for x.

$x^2-1=0\Rightarrow x=\pm1;\ x^2-4=0\Rightarrow x=\pm2$

The intervals are $(-\infty,\ -2),\ (-2,\ -1],$ $[-1,\ 1],\ [1,\ 2)$, and $[2,\ \infty)$.

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Interval	Test point	Value of $\frac{x^2-1}{x^2-4}$	Result
$(-\infty, -2)$	-3	$8/5$	+
$(-2, -1]$	$-3/2$	$-5/7$	–
$[-1, 1]$	0	$1/4$	+
$[1, 2)$	$3/2$	$-5/7$	–
$[2, \infty)$	3	$8/5$	+

The solution set is $(-2,-1]\cup[1, 2)$.

112. $\dfrac{x^2-9}{x^2-64} \le 0.$
Set the numerator and denominator equal to zero and solve for *x*.
$x^2-9=0 \Rightarrow x=\pm 3;\ x^2-64=0 \Rightarrow x=\pm 8$
The intervals are $(-\infty, -8)$, $(-8, -3]$, $[-3, 3]$, $[3, 8)$, and $(8, \infty)$.

Interval	Test point	Value of $\frac{x^2-9}{x^2-64}$	Result
$(-\infty, -8)$	-10	$91/36$	+
$(-8, -3]$	-5	$-16/39$	–
$[-3, 3]$	0	$9/64$	+
$[3, 8)$	5	$-16/39$	–
$[8, \infty)$	10	$91/36$	+

The solution set is $(-8, -3]\cup[3, 8)$.

113. $\dfrac{x+4}{3x-2} \ge 1 \Rightarrow \dfrac{x+4}{3x-2} - 1 \ge 0 \Rightarrow$
$\dfrac{x+4-3x+2}{3x-2} \ge 0 \Rightarrow \dfrac{-2x+6}{3x-2} \ge 0.$
Now we have $-2x+6=0 \Rightarrow x=3$ and $3x-2=0 \Rightarrow x=2/3$. So the intervals are $(-\infty, 2/3)$, $(2/3, 3]$, and $[3, \infty)$. Note that the original fraction is not defined if $x=2/3$, so $2/3$ is not included in the intervals.

Interval	Test point	Value of $\frac{x+4}{3x-2}-1$	Result
$(-\infty, 2/3)$	0	-3	–
$(2/3, 3]$	1	4	+
$[3, \infty)$	4	$-1/5$	–

The solution set is $(2/3, 3]$.

114. $\dfrac{2x-3}{x+3} \le 1 \Rightarrow \dfrac{2x-3}{x+3} - 1 \le 0 \Rightarrow$
$\dfrac{2x-3-x-3}{x+3} \le 0 \Rightarrow \dfrac{x-6}{x+3} \le 0.$
Now we have $x-6=0 \Rightarrow x=6$ and $x+3=0 \Rightarrow x=-3$. So the intervals are $(-\infty, -3)$, $(-3, 6]$, and $[6, \infty)$. Note that the original fraction is not defined if $x=-3$, so -3 is not included in the intervals.

Interval	Test point	Value of $\frac{2x-3}{x+3}-1$	Result
$(-\infty, -3)$	-6	4	+
$(-3, 6]$	0	-2	–
$[6, \infty)$	9	$1/4$	+

The solution set is $(-3, 6]$.

115. $3 \le \dfrac{2x+6}{2x+1} \Rightarrow \dfrac{2x+6}{2x+1} - 3 \ge 0 \Rightarrow$
$\dfrac{2x+6-3(2x+1)}{2x+1} \ge 0 \Rightarrow \dfrac{-4x+3}{2x+1} \ge 0.$
This gives $-4x+3=0 \Rightarrow x=3/4$ and $2x+1=0 \Rightarrow x=-1/2$. The intervals are $(-\infty, -1/2)$, $(-1/2, 3/4]$, and $[3/4, \infty)$. The original fraction is not defined if $x=-1/2$, so $-1/2$ is not included in the intervals.

Interval	Test point	Value of $\frac{2x+6}{2x+1}-3$	Result
$(-\infty, -1/2)$	-1	-7	–
$(-1/2, 3/4]$	0	3	+
$[3/4, \infty)$	1	$-1/3$	–

The solution set is $(-1/2, 3/4]$.

116. $\frac{x-2}{2x+1} < -1 \Rightarrow \frac{x-2}{2x+1} + 1 < 0 \Rightarrow$
$\frac{x-2+2x+1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0.$
Now we have $3x-1=0 \Rightarrow x = 1/3$ and $2x+1=0 \Rightarrow x = -1/2$. So the intervals are $(-\infty, -1/2)$, $(-1/2, 1/3)$, and $(1/3, \infty)$. Note that the original fraction is not defined if $x = -1/2$, so $-1/2$ is not included in the intervals.

Interval	Test point	Value of $\frac{x-2}{2x+1}+1$	Result
$(-\infty, -1/2)$	–1	4	+
$(-1/2, 1/3)$	0	–1	–
$(1/3, \infty)$	1	2/3	+

The solution set is $(-1/2, 1/3)$.

117. $\frac{x+2}{x-3} \geq \frac{x-1}{x+3} \Rightarrow \frac{x+2}{x-3} - \frac{x-1}{x+3} \geq 0 \Rightarrow$
$\frac{(x+2)(x+3)-(x-1)(x-3)}{(x-3)(x+3)} \geq 0 \Rightarrow$
$\frac{(x^2+5x+6)-(x^2-4x+3)}{(x-3)(x+3)} \geq 0 \Rightarrow$
$\frac{9x+3}{(x-3)(x+3)} \geq 0$
Set the numerator and the denominator equal to zero and solve for *x*. $9x+3=0 \Rightarrow x = -1/3$ and $(x-3)(x+3)=0 \Rightarrow x = 3$ or $x = -3$. The intervals are $(-\infty, -3)$, $(-3, -1/3]$, $[-1/3, 3)$, and $(3, \infty)$. Note that the original fractions are not defined if $x = -3$ or $x = 3$, so –3 and 3 are not included in the intervals.

Interval	Test point	Value of $\frac{x+2}{x-3} - \frac{x-1}{x+3}$	Result
$(-\infty, -3)$	–6	$-17/9$	–
$(-3, -1/3]$	–1	3/4	+
$[-1/3, 3)$	1	$-3/2$	–
$(3, \infty)$	6	19/9	+

The solution set is $(-3, -1/3] \cup (3, \infty)$.

118. $\frac{x+1}{x-2} \geq \frac{x}{x-1} \Rightarrow \frac{x+1}{x-2} - \frac{x}{x-1} \geq 0 \Rightarrow$
$\frac{(x+1)(x-1)-x(x-2)}{(x-1)(x-2)} \geq 0 \Rightarrow$
$\frac{(x^2-1)-(x^2-2x)}{(x-1)(x-2)} \geq 0 \Rightarrow \frac{2x-1}{(x-1)(x-2)} \geq 0$
Set the numerator and the denominator equal to zero and solve for *x*. $2x-1=0 \Rightarrow x = 1/2$ and $(x-1)(x-2)=0 \Rightarrow x = 1$ or $x = 2$. The intervals are $(-\infty, 1/2]$, $[1/2, 1)$, $(1, 2)$, and $(2, \infty)$. Note that the original fractions are not defined if $x = 1$ or $x = 2$, so 1 and 2 are not included in the intervals.

Interval	Test point	Value of $\frac{x+1}{x-2} - \frac{x}{x-1}$	Result
$(-\infty, 1/2]$	0	$-1/2$	–
$[1/2, 1)$	3/4	8/5	+
$(1, 2)$	3/2	–8	–
$(2, \infty)$	3	5/2	+

The solution set is $[1/2, 1) \cup (2, \infty)$.

119. $\frac{x-1}{x+1} \leq \frac{x+2}{x-3} \Rightarrow \frac{x-1}{x+1} - \frac{x+2}{x-3} \leq 0 \Rightarrow$
$\frac{(x-1)(x-3)-(x+2)(x+1)}{(x+1)(x-3)} \leq 0 \Rightarrow$
$\frac{(x^2-4x+3)-(x^2+3x+2)}{(x+1)(x-3)} \leq 0 \Rightarrow$
$\frac{-7x+1}{(x+1)(x-3)} \leq 0$
Set the numerator and the denominator equal to zero and solve for *x*. $-7x+1=0 \Rightarrow x = 1/7$ and $(x+1)(x-3)=0 \Rightarrow x = -1$ or $x = 3$. The intervals are $(-\infty, -1)$, $(-1, 1/7]$, $[1/7, 3)$, and $(3, \infty)$. Note that the original fractions are not defined if $x = -1$ or $x = 3$, so –1 and 3 are not included in the intervals.

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Interval	Test point	Value of $\frac{x-1}{x+1}-\frac{x+2}{x-3}$	Result
$(-\infty, -1)$	−2	3	+
$(-1, 1/7]$	0	$-1/3$	−
$[1/7, 3)$	1	$3/2$	+
$(3, \infty)$	4	$-27/5$	−

The solution set is $(-1, 1/7]\cup(3, \infty)$.

120. $\frac{x+3}{x+1} \le \frac{x-1}{x-2} \Rightarrow \frac{x+3}{x+1} - \frac{x-1}{x-2} \le 0$

$\frac{(x+3)(x-2)-(x-1)(x+1)}{(x+1)(x-2)} \le 0 \Rightarrow$

$\frac{(x^2+x-6)-(x^2-1)}{(x+1)(x-2)} \le 0 \Rightarrow \frac{x-5}{(x+1)(x-2)} \le 0$

Set the numerator and the denominator equal to zero and solve for x. $x-5=0 \Rightarrow x=5$ and $(x+1)(x-2)=0 \Rightarrow x=-1$ or $x=2$. The intervals are $(-\infty, -1)$, $(-1, 2)$, $(2, 5]$, and $[5, \infty)$. Note that the original fractions are not defined if $x = -1$ or $x = 2$, so -1 and 2 are not included in the intervals.

Interval	Test point	Value of $\frac{x+3}{x+1}-\frac{x-1}{x-2}$	Result
$(-\infty, -1)$	−2	$-7/4$	−
$(-1, 2)$	0	$5/2$	+
$(2, 5]$	3	$-1/2$	−
$[5, \infty)$	7	$1/20$	+

The solution set is $(-\infty, -1)\cup(2, 5]$.

1.5 Applying the Concepts

121. Let x = the selling price of the refrigerator. Then
$1750 + 0.15(1750) \le x \le 1750 + 0.20(1750) \Rightarrow$
$2012.50 \le x \le 2100$
The refrigerator's selling price ranges from \$2012.50 to \$2100.

122. Let r = the interest rate per year. Then
$200 \le 5000r \le 275 \Rightarrow 0.04 \le r \le 0.55$
The interest rate ranges from 4% to 5.5%.

123. Let x = the amount of gasoline in the car at the start of the trip. Then
$300 \le 40x \le 480 \Rightarrow 7.5 \le x \le 12$
The car had between 7.5 gallons and 12 gallons of gas at the start of the trip.

124. Let x = Sean's score on the fourth exam. Then
$\frac{85+72+77+x}{4} \ge 80 \Rightarrow 234 + x \ge 320 \Rightarrow$
$x \ge 86$
Sean needs to score between 86 and 100 on the last exam to earn a B.

125. Let x = the amount of cream.
Then $270 - x$ = the amount of milk. So,
$0.3x + 0.03(270 - x) \ge 0.045(270) \Rightarrow$
$0.3x + 8.1 - 0.03x \ge 12.15 \Rightarrow$
$0.27x \ge 4.05 \Rightarrow x \ge 15$
At least 15 quarts of cream must be added.

126. Let x = the number of pedometers to be sold. Then $2x$ = the amount of profit per pedometer.
$2x \ge 4000 + 3000 \Rightarrow 2x \ge 7000 \Rightarrow x \ge 3500$
At least 3500 pedometers must be sold.

127. Let x = the number of 4-door sedans. Then $3x$ = the number of SUV's and $2x$ = the number of convertibles.
$x + 3x + 2x \ge 48 \Rightarrow 6x \ge 48 \Rightarrow x \ge 8$
There are at least 8 four-door sedans.

128. Let C = degrees Celsius. Then
$\frac{5}{9}(68-32) \le C \le \frac{5}{9}(86-32) \Rightarrow 20 \le C \le 30$
The range of temperatures is 20°C to 30°C.

129. $132t - t^2 \ge 3200 \Rightarrow -t^2 + 132t - 3200 \ge 0 \Rightarrow$
$t^2 - 132t + 3200 \le 0 \Rightarrow (t-100)(t-32) \le 0$
Solving the associated equation gives $t = 100$ or $t = 32$. The intervals to be checked are $(-\infty, 32], [32, 100],$ and $[100, \infty)$. Checking a number in each interval shows that the temperature must fall in the range [32°F, 100°F].

130. $-16t^2 + 32t + 1584 \geq 1200 \Rightarrow$
$-16t^2 + 32t + 384 \geq 0 \Rightarrow t^2 - 2t - 24 \leq 0$
Solving the associated equation gives $t = 6$ or $t = -4$. The time can't be less than 0, so the time when the object is at least 1200 feet is from 0 seconds to 6 seconds.

131. Probability must be less than 1, so we have
$0.5 < \frac{64 - 0.2x}{208 - x} \leq 1 \Rightarrow$
$0.5 < \frac{64 - 0.2x}{208 - x}$ and $\frac{64 - 0.2x}{208 - x} \leq 1$
Solving each inequality independently gives
$0.5 < \frac{64 - 0.2x}{208 - x} \Rightarrow 0 < \frac{64 - 0.2x}{208 - x} - 0.5 \Rightarrow$
$0 < \frac{64 - 0.2x - 0.5(208 - x)}{208 - x} \Rightarrow$
$0 < \frac{-40 + 0.3x}{208 - x}$
Now we have $-40 + 0.3x = 0 \Rightarrow x = 400/3$ and $208 - x = 0 \Rightarrow 208 = x$. The original fraction is not defined if $x = 208$, so 208 is not in the solution set. So there need to be more than 133 cards. (Note that the value of x is rounded up to account for the partial value.)
$\frac{64 - 0.2x}{208 - x} \leq 1 \Rightarrow \frac{64 - 0.2x}{208 - x} - 1 \leq 0 \Rightarrow$
$\frac{64 - 0.2x - 208 + x}{208 - x} \leq 0 \Rightarrow \frac{0.8x - 144}{208 - x} \leq 0 \Rightarrow$
$0.8x - 144 \leq 0 \Rightarrow 0.8x \leq 144 \Rightarrow x \leq 180$
So, the likelihood that the next card dealt would be a jack, queen, king, or ace is greater than 50% if more than 133 cards and less than 180 cards are dealt.

132. Let h = the height of the triangle.
Then $h + 3$ = the base of the triangle.
$\frac{h(h+3)}{2} \geq 5 \Rightarrow h^2 + 3h \geq 10 \Rightarrow h^2 + 3h - 10 \geq 0$
Solving the associated equation gives $h = 2$ or $h = -5$. We reject the negative answer. So the height must be at least 2 cm.

1.5 Beyond the Basics

133. $2x^2 + kx + 2 = 0$ has two real solutions if the discriminant is greater than zero. So $k^2 - 4(2)(2) > 0 \Rightarrow k^2 - 16 > 0$. Solving the associated equation gives $k = -4$ or $k = 4$. The intervals to be tested are $(-\infty, -4), (-4, 4)$, and $(4, \infty)$. $k^2 - 16 > 0$ for $(-\infty, -4) \cup (4, \infty)$.

134. $2x^2 + kx + 2 = 0$ has no real solutions if the discriminant is less than zero. So $k^2 - 4(2)(2) < 0 \Rightarrow k^2 - 16 < 0$. Solving the associated equation gives $k = -4$ or $k = 4$. The intervals to be tested are $(-\infty, -4), (-4, 4)$, and $(4, \infty)$. $k^2 - 16 < 0$ for $(-4, 4)$.

135. $x^2 + kx + k = 0$ has two real solutions if the discriminant is greater than zero. So $k^2 - 4k > 0 \Rightarrow k(k-4) > 0$. Solving the associated equation gives $k = 0$ or $k = 4$. The intervals to be tested are $(-\infty, 0)$, $(0, 4)$, and $(4, \infty)$. $k^2 - 4k > 0$ for $(-\infty, 0) \cup (4, \infty)$.

136. $x^2 + kx + k = 0$ has no real solutions if the discriminant is less than zero. So $k^2 - 4k < 0 \Rightarrow k(k-4) < 0$. Solving the associated equation gives $k = 0$ or $k = 4$. The intervals to be tested are $(-\infty, 0)$, $(0, 4)$, and $(4, \infty)$. $k^2 - 4k < 0$ for $(0, 4)$.

137. $\frac{x}{2x+1} \geq \frac{1}{4}$ and $\frac{6x}{4x-1} < \frac{1}{2}$
We will solve each inequality independently and then determine where the solution sets intersect.
$\frac{x}{2x+1} \geq \frac{1}{4} \Rightarrow \frac{x}{2x+1} - \frac{1}{4} \geq 0 \Rightarrow$
$\frac{4x - (2x+1)}{4(2x+1)} \geq 0 \Rightarrow \frac{2x-1}{8x+4} \geq 0$
Now, we have $2x - 1 = 0 \Rightarrow x = 1/2$ and $8x + 4 = 0 \Rightarrow x = -1/2$. Note that the original fraction $\frac{x}{2x+1}$ is not defined if $x = -1/2$, so the intervals to be tested are $(-\infty, -1/2)$, $(-1/2, 1/2]$, and $[1/2, \infty)$.

Interval	Test point	Value of $\frac{x}{2x+1} - \frac{1}{4}$	Result
$(-\infty, -1/2)$	-1	$3/4$	+
$(-1/2, 1/2]$	0	$-1/4$	–
$[1/2, \infty)$	1	$1/12$	+

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The solution set is $(-\infty, -1/2) \cup [1/2, \infty)$.

$$\frac{6x}{4x-1} < \frac{1}{2} \Rightarrow \frac{6x}{4x-1} - \frac{1}{2} < 0 \Rightarrow$$
$$\frac{12x-(4x-1)}{2(4x-1)} < 0 \Rightarrow \frac{8x+1}{8x-2} < 0$$

The original fractions $\frac{6x}{4x-1}$ and $\frac{8x+1}{8x-2}$ are not defined if $x = 1/4$. Also, $8x+1=0 \Rightarrow x = -1/8$. The intervals to be tested are $(-\infty, -1/8)$, $(-1/8, 1/4)$, and $(1/4, \infty)$.

Interval	Test point	Value of $\frac{6x}{4x-1} - \frac{1}{2}$	Result
$(-\infty, -1/8)$	−1	7/10	+
$(-1/8, 1/4)$	0	−1/2	−
$(1/4, \infty)$	1	3/2	+

The solution set is $(-1/8, 1/4)$.

The two solution sets do not intersect, so the solution set of $\frac{x}{2x+1} \ge \frac{1}{4}$ and $\frac{6x}{4x-1} < \frac{1}{2}$ is $\varnothing$.

138. $\frac{2x-1}{x-7} > 1$ and $\frac{x-10}{x-8} > 2$

We will solve each inequality independently and then determine where the solution sets intersect.

$$\frac{2x-1}{x-7} > 1 \Rightarrow \frac{2x-1}{x-7} - 1 > 0 \Rightarrow$$
$$\frac{2x-1-(x-7)}{x-7} > 0 \Rightarrow \frac{x+6}{x-7} > 0$$

Now, we have $x+6=0 \Rightarrow x=-6$ and $x-7=0 \Rightarrow x=7$. The intervals to be tested are $(-\infty, -6)$, $(-6, 7)$, and $(7, \infty)$.

Interval	Test point	Value of $\frac{2x-1}{x-7} - 1$	Result
$(-\infty, -6)$	−10	4/17	+
$(-6, 7)$	0	−6/7	−
$(7, \infty)$	10	16/3	+

The solution set is $(-\infty, -6) \cup (7, \infty)$.

$$\frac{x-10}{x-8} > 2 \Rightarrow \frac{x-10}{x-8} - 2 > 0 \Rightarrow$$
$$\frac{(x-10)-2(x-8)}{x-8} > 0 \Rightarrow \frac{-x+6}{x-8} > 0$$

Now, we have $-x+6=0 \Rightarrow x=6$ and $x-8=0 \Rightarrow x=8$. The intervals to be tested are $(-\infty, 6)$, $(6, 8)$, and $(8, \infty)$.

Interval	Test point	Value of $\frac{x-10}{x-8} - 2$	Result
$(-\infty, 6)$	0	−3/4	−
$(6, 8)$	7	1	+
$(8, \infty)$	10	−2	−

The solution set is $(6, 8)$.

The two solution sets intersect at $(7, 8)$, so the solution set of $\frac{2x-1}{x-7} > 1$ and $\frac{x-10}{x-8} > 2$ is $(7, 8)$.

139. Let x = one number. Then $c - x$ = the other number. So we have

$$x(c-x) = 36 \Rightarrow cx - x^2 = 36 \Rightarrow$$
$$x^2 - cx + 36 = 0$$

$x^2 - cx + 36 = 0$ has one or two real solutions if the discriminant is greater than or equal to zero. So

$$(-c)^2 - 4(1)(36) \ge 0 \Rightarrow c^2 - 144 \ge 0 \Rightarrow$$
$$c^2 > 144 \Rightarrow c \ge 12 \text{ or } c \le -12$$

Solution set: $(-\infty, -12] \cup [12, \infty)$

140. Let x = one number. Then $12 - x$ = the other number. So we have

$$x^2 + (12 - x)^2 = c$$
$$x^2 + 144 - 24x + x^2 = c$$
$$2x^2 - 24x + 144 = c$$
$$2x^2 - 24x + 144 - c = 0$$

$2x^2 - 24x + 144 - c$ has one or two real solutions if the discriminant is greater than or equal to zero. So

$$(-24)^2 - 4(2)(144 - c) \ge 0 \Rightarrow$$
$$576 - 1152 + 8c \ge 0 \Rightarrow -576 + 8c \ge 0$$
$$8c > 576 \Rightarrow c \ge 72$$

Solution set: $[72, \infty)$

141. Let x = the total number of radios imported. Then $x - 1000$ = the number of radios subject to the penalty tax. So we have

$$10x + 0.05x(x - 1000) \le 640,000$$
$$10x + 0.05x^2 - 50x \le 640,000$$
$$0.05x^2 - 40x - 640,000 \le 0$$
$$x^2 - 800x - 12,800,000 \le 0$$
$$(x - 4000)(x + 3200) \le 0$$

Solving the associated equation gives x = 4000 or x = –3200. Checking the intervals $[0, 4000]$ and $[4000, \infty)$, we find that the solution is in the range $[0, 4000]$. So they can import no more than 4000 radios.

142. Let x = the number of bonus questions. Then the number of questions answered correctly is $x + 10$, and the amount won for each correct answer is $100 + 50x$. So we have

$$(x + 10)(50x + 100) \ge 3500$$
$$50x^2 + 600x + 1000 \ge 3500$$
$$50x^2 + 600x - 2500 \ge 0$$
$$x^2 + 12x - 50 \ge 0$$

Using the quadratic formula to solve for x, we have

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(-50)}}{2(1)} \approx 3.3 \text{ or } -15.3.$$

We reject the negative root, so there must have been more than 3.3 bonus questions answered correctly. Since there cannot be a fraction of a question, we round up to 4. Therefore the contestant must have answered at least 14 questions correctly to have earned more than \$3500.

1.5 Critical Thinking/Discussion/Writing

In Exercises 143 and 144, answers may vary. Sample responses are given.

143. a. $(x + 4)(x - 5) < 0$

b. $(x + 2)(x - 6) \le 0$

c. $x^2 \ge 0$ **d.** $x^2 < 0$

e. $(x - 3)^2 \le 0$ **f.** $(x - 2)^2 > 0$

144. a. $\frac{x - 4}{x + 2} \le 0$ **b.** $\frac{x - 3}{x - 5} \le 0$

c. It is not possible to have a quadratic inequality with solution set (2, 5]. If we try $(x - 2)(x - 5) < 0$, then the solution set will be (2, 5). If we try $(x - 2)(x - 5) \le 0$, then the solution set will be [2, 5].

1.5 Maintaining Skills

145. $|-3| = 3$ **146.** $|3 - 7| = |-4| = 4$

147. $|6 - 4| = |2| = 2$ **148.** $|-\sqrt{2}| = \sqrt{2}$

149. $|0| = 0$ **150.** $|-15.8| = 15.8$

151. $d = |5 - (-2)| = |7| = 7$

152. $d = |-8 - (-15)| = |-7| = 7$

153. $d = |5.7 - 2.3| = |3.4| = 3.4$

154. $d = |0 - (-5)| = |5| = 5$

155. $|x - (-2)| = 5$ or $|x + 2| = 5$

156. $|x| = 3$ **157.** $|x - 4| \le 2$

158. $|x| > 5$ **159.** $|x - 5| \le 3$

160. $|x - 2| < |x - 6|$

1.6 Equations and Inequalities Involving Absolute Value

1.6 Practice Problems

1. a. $|x-2| = 0 \Rightarrow x-2 = 0 \Rightarrow x = 2$

Solution set: $\{2\}$

b. $|6x-3| - 8 = 1$

$|6x-3| = 9$

$6x-3 = 9$ or $6x-3 = -9$

$6x = 12$ $\quad$ $6x = -6$

$x = 2$ $\quad$ $x = -1$

Solution set: $\{-1, 2\}$

2. $|x+2| = |x-3|$

$x+2 = x-3$ or $x+2 = -(x-3)$

$0 = -5$ False $\quad$ $x+2 = -x+3$

$2x = 1 \Rightarrow x = \dfrac{1}{2}$

Solution set: $\left\{\dfrac{1}{2}\right\}$

3. $|3x-4| = 2(x-1)$

$3x-4 = 2(x-1)$ or $3x-4 = -2(x-1)$

$3x-4 = 2x-2$ $\quad$ $3x-4 = -2x+2$

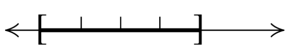
$x = 2$ $\quad$ $5x = 6 \Rightarrow x = \dfrac{6}{5}$

Solution set: $\left\{\dfrac{6}{5}, 2\right\}$

4. $|3x+3| \le 6 \Rightarrow -6 \le 3x+3 \le 6 \Rightarrow$

$-9 \le 3x \le 3 \Rightarrow -3 \le x \le 1$

Solution set: $[-3, 1]$



5. Let x = the actual speed of the search plane, in miles per hour. Then,

$|x-115| \le 25 \Rightarrow -25 \le x-115 \le 25 \Rightarrow$

$90 \le x \le 140$

Thus, the actual speed of the plane is between 90 and 140 miles per hour. Since the plane uses 10 gallons of fuel per hour and has 30 gallons of fuel, it can fly for 3 hours. The actual number of miles the search plane can fly is $3x$:

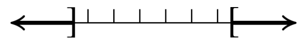
$3(90) \le 3x \le 3(140) \Rightarrow 270 \le x \le 420$

The plane can fly between 270 miles and 420 miles.

6. $|2x+3| \ge 6 \Rightarrow 2x+3 \le -6$ or $2x+3 \ge 6$

$2x+3 \le -6$	$2x+3 \ge 6$
$2x \le -9$	$2x \ge 3$
$x \le -\dfrac{9}{2}$	$x \ge \dfrac{3}{2}$

Solution set: $\left(-\infty, -\dfrac{9}{2}\right] \cup \left[\dfrac{3}{2}, \infty\right)$



7. a. $|5-9x| > -3$

Since absolute value is always nonnegative, $|5-9x| > -3$ is true for all real numbers.

Solution set: $(-\infty, \infty)$

b. $|7x-4| \le -1$

Since absolute value is always nonnegative, $|7x-4| \le -1$ is false for all real numbers.

Solution set: $\varnothing$

8. $\left|\dfrac{x-2}{x+4}\right| < 4 \Rightarrow -4 < \dfrac{x-2}{x+4} < 4$

$0 < \dfrac{x-2}{x+4} + 4$	$\dfrac{x-2}{x+4} - 4 < 0$
$0 < \dfrac{x-2+4(x+4)}{x+4}$	$\dfrac{x-2-4(x+4)}{x+4} < 0$
$0 < \dfrac{x-2+4x+16}{x+4}$	$\dfrac{x-2-4x-16}{x+4} < 0$
$0 < \dfrac{5x+14}{x+4}$	$\dfrac{-3x-18}{x+4} < 0$

$5x+14 = 0 \Rightarrow x = -\dfrac{14}{5}$; $\; x+4 = 0 \Rightarrow x = -4$

$-3x-18 = 0 \Rightarrow x = -6$

Interval	Test point	Value of $\dfrac{5x+14}{x+4}$	Result
$(-\infty, -6)$	−10	6	+
$\left(-4, -\frac{14}{5}\right)$	−3	−1	−
$\left(-\frac{14}{5}, \infty\right)$	−1	3	+

Note that the expression is undefined for $x = -4$. The solution set is for this part of the original inequality is $(-\infty, -6) \cup \left(-\frac{14}{5}, \infty\right)$.

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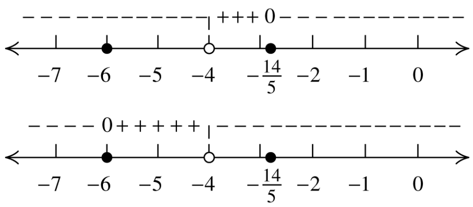
Interval	Test point	Value of $\frac{-3x-18}{x+4}$	Result
$(-\infty, -6)$	-7	-1	$-$
$(-6, -4)$	-5	3	$+$
$(-4, \infty)$	-1	-5	$-$

Note that the expression is undefined for $x = 4$. The solution set is for this part of the original inequality is $(-\infty, -6) \cup (-4, \infty)$.



The figure shows that both inequalities are true on $(-\infty, -6) \cup \left(-\frac{14}{5}, \infty\right)$, so the solution set is $(-\infty, -6) \cup \left(-\frac{14}{5}, \infty\right)$.

1.6 Basic Concepts and Skills

1. The solution set of the equation $|x| = a$ is $\underline{\{-a, a\}}$.
2. The solution set of the inequality $|x| < a$ is $\underline{(-a, a)}$.
3. The solution set of the inequality $|x| \ge a$ is $\underline{(-\infty, -a] \cup [a, \infty)}$.
4. The equation $|u| = |v|$ if $u = \underline{v}$ or $u = \underline{-v}$.
5. True
6. True
7. $|3x| = 9 \Rightarrow 3x = 9$ or $3x = -9 \Rightarrow$ $x = 3$ or $x = -3$
8. $|4x| = 24 \Rightarrow 4x = 24$ or $4x = -24 \Rightarrow$ $x = 6$ or $x = -6$
9. $|-2x| = 6 \Rightarrow -2x = 6$ or $-2x = -6 \Rightarrow$ $x = -3$ or $x = 3$
10. $|-x| = 3 \Rightarrow -x = 3$ or $-x = -3 \Rightarrow$ $x = -3$ or $x = 3$
11. $|x+3| = 2 \Rightarrow x + 3 = 2$ or $x + 3 = -2 \Rightarrow$ $x = -1$ or $x = -5$
12. $|x-4| = 1 \Rightarrow x - 4 = 1$ or $x - 4 = -1 \Rightarrow$ $x = 5$ or $x = 3$
13. $|6-2x| = 8 \Rightarrow 6 - 2x = 8$ or $6 - 2x = -8 \Rightarrow$ $x = -1$ or $x = 7$
14. $|6-3x| = 9 \Rightarrow 6 - 3x = 9$ or $6 - 3x = -9 \Rightarrow$ $x = -1$ or $x = 5$
15. $|6x-2| = 9 \Rightarrow 6x - 2 = 9$ or $6x - 2 = -9 \Rightarrow$ $x = \frac{11}{6}$ or $x = -\frac{7}{6}$
16. $|6x-3| = 9 \Rightarrow 6x - 3 = 9$ or $6x - 3 = -9 \Rightarrow$ $x = 2$ or $x = -1$
17. $|2x+3| - 1 = 0 \Rightarrow |2x+3| = 1 \Rightarrow 2x + 3 = 1$ or $2x + 3 = -1 \Rightarrow x = -1$ or $x = -2$
18. $|2x-3| - 1 = 0 \Rightarrow |2x-3| = 1 \Rightarrow 2x - 3 = 1$ or $2x - 3 = -1 \Rightarrow x = 2$ or $x = 1$
19. $\frac{1}{2}|x| = 3 \Rightarrow |x| = 6 \Rightarrow x = -6$ or $x = 6$
20. $\frac{3}{5}|x| = 6 \Rightarrow |x| = 10 \Rightarrow x = -10$ or $x = 10$
21. $\left|\frac{1}{4}x + 2\right| = 3 \Rightarrow \frac{1}{4}x + 2 = -3$or $\frac{1}{4}x + 2 = 3 \Rightarrow$ $x = -20$ or $x = 4$
22. $\left|\frac{3}{2}x - 1\right| = 3 \Rightarrow \frac{3}{2}x - 1 = -3$ or $\frac{3}{2}x - 1 = 3 \Rightarrow$ $x = -\frac{4}{3}$ or $x = \frac{8}{3}$
23. $6|1-2x| - 8 = 10 \Rightarrow 6|1-2x| = 18 \Rightarrow$ $|1-2x| = 3 \Rightarrow 1 - 2x = 3$ or $1 - 2x = -3 \Rightarrow$ $x = -1$ or $x = 2$
24. $5|1-4x| + 10 = 15 \Rightarrow 5|1-4x| = 5 \Rightarrow$ $|1-4x| = 1 \Rightarrow 1 - 4x = 1$ or $1 - 4x = -1 \Rightarrow$ $x = 0$ or $x = \frac{1}{2}$
25. $2|3x-4| + 9 = 7 \Rightarrow 2|3x-4| = -2 \Rightarrow$ $|3x-4| = -1$

    The solution set is $\varnothing$ because an absolute value cannot be negative.

26. $9|2x-3|+2=-7 \Rightarrow 9|2x-3|=-9 \Rightarrow$
$|2x-3|=-1$
The solution set is $\varnothing$ because an absolute value cannot be negative.

27. $|2x+1|=-1$
The solution set is $\varnothing$ because an absolute value cannot be negative.

28. $|3x+7|=-2$
The solution set is $\varnothing$ because an absolute value cannot be negative.

29. $|x^2-4|=0 \Rightarrow x^2-4=0 \Rightarrow x=\pm 2$
Solution set: $\{-2, 2\}$

30. $|9-x^2|=0 \Rightarrow 9-x^2=0 \Rightarrow x=\pm 3$
Solution set: $\{-3, 3\}$

31. $|1-2x|=3 \Rightarrow 1-2x=3$ or $1-2x=-3 \Rightarrow$
$x=-1$ or $x=2$
Solution set: $\{-1, 2\}$

32. $|4-3x|=5 \Rightarrow 4-3x=5$ or $4-3x=-5 \Rightarrow$
$x=-\frac{1}{3}$ or $x=3$
Solution set: $\left\{-\frac{1}{3}, 3\right\}$

33. $\left|\frac{1}{3}-x\right|=\frac{2}{3} \Rightarrow \frac{1}{3}-x=\frac{2}{3}$ or $\frac{1}{3}-x=-\frac{2}{3} \Rightarrow$
$x=-\frac{1}{3}$ or $x=1$
Solution set: $\left\{-\frac{1}{3}, 1\right\}$

34. $\left|\frac{2}{5}-x\right|=\frac{1}{5} \Rightarrow \frac{2}{5}-x=\frac{1}{5}$ or $\frac{2}{5}-x=-\frac{1}{5} \Rightarrow$
$x=\frac{1}{5}$ or $x=\frac{3}{5}$
Solution set: $\left\{\frac{1}{5}, \frac{3}{5}\right\}$

In exercises 35–44, be sure to check answers to eliminate extraneous solutions.

35. $|x+3|=|x+5| \Rightarrow x+3=x+5$ (impossible)
or $x+3=-(x+5) \Rightarrow x+3=-x-5 \Rightarrow x=-4$
The solution set is $\{-4\}$.

36. $|x+4|=|x-8| \Rightarrow x+4=x-8$ (impossible)
or $x+4=-(x-8) \Rightarrow x+4=-x+8 \Rightarrow$
$x=2$
The solution set is $\{2\}$.

37. $|3x-2|=|6x+7| \Rightarrow 3x-2=6x+7 \Rightarrow$
$x=-3$ or $3x-2=-(6x+7) \Rightarrow$
$3x-2=-6x-7 \Rightarrow x=-\frac{5}{9}$
The solution set is $\left\{-3,-\frac{5}{9}\right\}$.

38. $|2x-4|=|4x+6| \Rightarrow 2x-4=4x+6 \Rightarrow$
$x=-5$ or $2x-4=-(4x+6) \Rightarrow$
$2x-4=-4x-6 \Rightarrow x=-\frac{1}{3}$
The solution set is $\left\{-5,-\frac{1}{3}\right\}$.

39. $|2x-1|=x+1 \Rightarrow$
$2x-1=x+1$ or $2x-1=-(x+1)$
$x=2 \qquad 2x-1=-x-1$
$3x=0 \Rightarrow x=0$
Solution set: $\{0, 2\}$

40. $|3x-4|=2(x-1) \Rightarrow$
$3x-4=2(x-1)$ or $3x-4=-2(x-1)$
$3x-4=2x-2 \qquad 3x-4=-2x+2$
$x=2 \qquad 5x=6 \Rightarrow x=\frac{6}{5}$
Solution set: $\left\{\frac{6}{5}, 2\right\}$

41. $|4-3x|=x-1 \Rightarrow$
$4-3x=x-1$ or $4-3x=-(x-1)$
$5=4x \qquad 4-3x=-x+1$
$\frac{5}{4}=x \qquad 3=2x \Rightarrow \frac{3}{2}=x$
Solution set: $\left\{\frac{5}{4}, \frac{3}{2}\right\}$

42. $|2-3x|=2x-1 \Rightarrow$
$2-3x=2x-1$ or $2-3x=-(2x-1)$
$3=5x \qquad 2-3x=-2x+1$
$\frac{3}{5}=x \qquad 1=x$
Solution set: $\left\{\frac{3}{5}, 1\right\}$

43. $|3x+2| = 2(x-1) \Rightarrow$
$3x+2 = 2(x-1)$ or $3x+2 = -2(x-1)$
$3x+2 = 2x-2 \qquad 3x+2 = -2x+2$
$x = -4 \qquad 5x = 0 \Rightarrow x = 0$
If $x = -4$, then $|3x + 2| = |3(-4) + 2| = |-10| = 10$, while $2(x - 1) = 2(-4 - 1) = 2(-5) = -10$. Therefore, -4 is not a solution.
If $x = 0$, then $|3x + 2| = |3(0) + 2| = |2| = 2$, while $2(x - 1) = 2(0 - 1) = 2(-1) = -2$. Therefore, 0 is not a solution.
Solution set: $\varnothing$

44. $|4x+7| = x+1 \Rightarrow$
$4x+7 = x+1$ or $4x+7 = -(x+1)$
$3x = -6 \qquad 4x+7 = -x-1$
$x = -2 \qquad 5x = -8 \Rightarrow x = -\frac{8}{5}$
If $x = -2$, then $|4x + 7| = |4(-2) + 7| = |-1| = 1$, while $x + 1 = -2 + 1 = -1$. Therefore, -2 is not a solution.
If $x = -\frac{8}{5}$, then $|4x+7| = \left|4\left(-\frac{8}{5}\right)+7\right| = \left|\frac{3}{5}\right| = \frac{3}{5}$, while $x+1 = -\frac{8}{5}+1 = -\frac{3}{5}$.
Therefore, $-\frac{8}{5}$ is not a solution.
Solution set: $\varnothing$

45. $|3x| < 12 \Rightarrow -12 < 3x < 12 \Rightarrow -4 < x < 4$
The solution set is $(-4, 4)$.

46. $|2x| \le 6 \Rightarrow -6 \le 2x < 6 \Rightarrow -3 \le x \le 3$
The solution set is $[-3, 3]$.

47. $|4x| > 16 \Rightarrow 4x < -16$ or $4x > 16 \Rightarrow$
$x < -4$ or $x > 4$.
The solution set is $(-\infty, -4) \cup (4, \infty)$.

48. $|3x| > 15 \Rightarrow 3x < -15$ or $3x > 15 \Rightarrow$
$x < -5$ or $x > 5$.
The solution set is $(-\infty, -5) \cup (5, \infty)$.

49. $|x+1| < 3 \Rightarrow -3 < x+1 < 3 \Rightarrow -4 < x < 2$.
The solution set is $(-4, 2)$.

50. $|x-4| < 1 \Rightarrow -1 < x-4 < 1 \Rightarrow 3 < x < 5$.
The solution set is $(3, 5)$.

51. $|x| + 2 \ge -1 \Rightarrow |x| \ge -3$
Since absolute value is always nonnegative, the inequality is true for all real numbers.
Solution set: $(-\infty, \infty)$

52. $|x| + 2 > -7 \Rightarrow |x| > -9$
Since absolute value is always nonnegative, the inequality is true for all real numbers.
Solution set: $(-\infty, \infty)$

53. $|2x-3| < 4 \Rightarrow -4 < 2x-3 < 4 \Rightarrow$
$-1 < 2x < 7 \Rightarrow -\frac{1}{2} < x < \frac{7}{2}$.
The solution set is $\left(-\frac{1}{2}, \frac{7}{2}\right)$.

54. $|4x-6| \le 6 \Rightarrow -6 \le 4x-6 \le 6 \Rightarrow$
$0 \le 4x \le 12 \Rightarrow 0 \le x \le 3$.
The solution set is $[0, 3]$.

55. $|5-2x| > 3 \Rightarrow 5-2x < -3$ or $5-2x > 3 \Rightarrow$
$x > 4$ or $x < 1$.
The solution set is $(-\infty, 1) \cup (4, \infty)$.

56. $|3x-3| \ge 15 \Rightarrow 3x-3 \le -15$ or $3x-3 \ge 15 \Rightarrow$
$x \le -4$ or $x \ge 6$.
The solution set is $(-\infty, -4] \cup [6, \infty)$.

57. $|3x+4| \le 19 \Rightarrow -19 \le 3x+4 \le 19 \Rightarrow$
$-23 \le 3x \le 15 \Rightarrow -\frac{23}{3} \le x \le 5$.
The solution set is $\left[-\frac{23}{3}, 5\right]$.

58. $|9-7x| < 23 \Rightarrow -23 < 9-7x < 23 \Rightarrow$
$-32 < -7x < 14 \Rightarrow \frac{32}{7} > x > -2$.
The inequalities change direction due to division by –7. The solution set is $\left(-2, \frac{32}{7}\right)$.

59. $|2x-15| < 0$. The solution set is $\varnothing$ because an absolute value cannot be negative.

60. $|x+5| \le -3$. The solution set is $\varnothing$ because an absolute value cannot be negative.

61. $\left|\frac{x-2}{x+3}\right| < 1 \Rightarrow -1 < \frac{x-2}{x+3} < 1$

$0 < \frac{x-2}{x+3} + 1$	$\frac{x-2}{x+3} - 1 < 0$
$0 < \frac{x-2+(x+3)}{x+3}$	$\frac{x-2-(x+3)}{x+3} < 0$
$0 < \frac{2x+1}{x+3}$	$\frac{-5}{x+3} < 0$

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$2x+1=0 \Rightarrow x=-\dfrac{1}{2};\ x+3=0 \Rightarrow x=-3$

Interval	Test point	Value of $\frac{2x+1}{x+3}$	Result
$(-\infty, -3)$	−4	7	+
$\left(-3, -\frac{1}{2}\right)$	−2	−3	−
$\left(-\frac{1}{2}, \infty\right)$	2	1	+

Note that the expression is undefined for $x=-3$. The solution set is for this part of the original inequality is $(-\infty, -3)\cup\left(-\frac{1}{2}, \infty\right)$.

Interval	Test point	Value of $\frac{-5}{x+3}$	Result
$(-\infty, -3)$	−4	5	+
$\left(-3, -\frac{1}{2}\right)$	−2	−5	−
$\left(-\frac{1}{2}, \infty\right)$	2	−1	−

Note that the expression is undefined for $x=-3$. The solution set is for this part of the original inequality is $\left(-3, -\frac{1}{2}\right)\cup\left(-\frac{1}{2}, \infty\right)$.

Both inequalities are true on $\left(-\frac{1}{2}, \infty\right)$, so the solution set is $\left(-\frac{1}{2}, \infty\right)$.

62. $\left|\dfrac{x+3}{x-1}\right|<2 \Rightarrow -2<\dfrac{x+3}{x-1}<2$

$$0<\frac{x+3}{x-1}+2 \qquad\qquad \frac{x+3}{x-1}-2<0$$
$$0<\frac{x+3+2(x-1)}{x-1} \qquad\qquad \frac{x+3-2(x-1)}{x-1}<0$$
$$0<\frac{3x+1}{x-1} \qquad\qquad \frac{-x+5}{x-1}<0$$

$3x+1=0 \Rightarrow x=-\dfrac{1}{3};\ x-1=0 \Rightarrow x=1$

$-x+5=0 \Rightarrow x=5$

Interval	Test point	Value of $\frac{3x+1}{x-1}$	Result
$\left(-\infty, -\frac{1}{3}\right)$	−1	1	+
$\left(-\frac{1}{3}, 1\right)$	0	−1	−
$(1, 5)$	2	7	+
$(5, \infty)$	8	$\frac{25}{7}$	+

Note that the expression is undefined for $x=1$. The solution set is for this part of the original inequality is $\left(-\infty, -\frac{1}{3}\right)\cup(1, 5)\cup(5, \infty)$.

Interval	Test point	Value of $\frac{-x+5}{x-1}$	Result
$\left(-\infty, -\frac{1}{3}\right)$	−1	$-\frac{3}{2}$	−
$\left(-\frac{1}{3}, 1\right)$	0	−5	−
$(1, 5)$	2	3	+
$(5, \infty)$	3	−5	−

Note that the expression is undefined for $x=1$. The solution set is for this part of the original inequality is $\left(-\infty, -\frac{1}{3}\right)\cup\left(-\frac{1}{3}, 1\right)\cup(5, \infty)$.

Both inequalities are true on $\left(-\infty, -\frac{1}{3}\right)\cup(5, \infty)$, so the solution set is $\left(-\infty, -\frac{1}{3}\right)\cup(5, \infty)$.

63. $\left|\dfrac{2x-3}{x+1}\right|\le 3 \Rightarrow -3\le\dfrac{2x-3}{x+1}\le 3$

$$0\le\frac{2x-3}{x+1}+3 \qquad\qquad \frac{2x-3}{x+1}-3\le 0$$
$$0\le\frac{2x-3+3(x+1)}{x+1} \qquad\qquad \frac{2x-3-3(x+1)}{x+1}\le 0$$
$$0\le\frac{5x}{x+1} \qquad\qquad \frac{-x-6}{x+1}\le 0$$

$5x=0 \Rightarrow x=0;\ x+1=0 \Rightarrow x=-1$

$-x-6=0 \Rightarrow x=-6$

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(*continued*)

Interval	Test point	Value of $\frac{5x}{x+1}$	Result
$(-\infty, -6]$	-11	$\frac{11}{2}$	+
$[-6, -1)$	-2	10	+
$(-1, 0]$	$-\frac{1}{2}$	-5	−
$[0, \infty)$	4	4	+

Note that the expression is undefined for $x = -1$. The solution set is for this part of the original inequality is
$(-\infty, -6] \cup [-6, -1) \cup [0, \infty)$.

Interval	Test point	Value of $\frac{-x-6}{x+1}$	Result
$(-\infty, -6]$	-11	$-\frac{1}{2}$	−
$[-6, -1)$	-2	4	+
$(-1, 0]$	$-\frac{1}{2}$	-11	−
$[0, \infty)$	4	-2	−

Note that the expression is undefined for $x = -1$. The solution set is for this part of the original inequality is
$(-\infty, -6] \cup (-1, 0] \cup [0, \infty)$.

Both inequalities are true on $(-\infty, -6] \cup [0, \infty)$, so the solution set is $(-\infty, -6] \cup [0, \infty)$.

64. $\left|\frac{2x-1}{3x+2}\right| \le 1 \Rightarrow -1 \le \frac{2x-1}{3x+2} \le 1$

$$0 < \frac{2x-1}{3x+2} + 1 \qquad \frac{2x-1}{3x+2} - 1 < 0$$
$$0 < \frac{2x-1+(3x+2)}{3x+2} \qquad \frac{2x-1-(3x+2)}{3x+2} < 0$$
$$0 < \frac{5x+1}{3x+2} \qquad \frac{-x-3}{3x+2} < 0$$

$5x+1=0 \Rightarrow x = -\frac{1}{5}$; $3x+2=0 \Rightarrow x = -\frac{2}{3}$
$-x-3=0 \Rightarrow x=-3$

Interval	Test point	Value of $\frac{5x+1}{3x+2}$	Result
$(-\infty, -3]$	-10	$\frac{7}{4}$	+
$[-3, -\frac{2}{3})$	-2	$\frac{9}{4}$	+
$(-\frac{2}{3}, -\frac{1}{5}]$	$-\frac{3}{5}$	-10	−
$[-\frac{1}{5}, \infty)$	0	$\frac{1}{2}$	+

Note that the expression is undefined for $x = -\frac{2}{3}$. The solution set is for this part of the original inequality is
$(-\infty, -3] \cup [-3, -\frac{2}{3}) \cup [-\frac{1}{5}, \infty)$.

Interval	Test point	Value of $\frac{-x-3}{3x+2}$	Result
$(-\infty, -3]$	-10	$-\frac{1}{4}$	−
$[-3, -\frac{2}{3})$	-2	$\frac{1}{4}$	+
$(-\frac{2}{3}, -\frac{1}{5}]$	$-\frac{3}{5}$	-12	−
$[-\frac{1}{5}, \infty)$	0	$-\frac{3}{2}$	−

Note that the expression is undefined for $x = -\frac{2}{3}$. The solution set is for this part of the original inequality is
$(-\infty, -3] \cup (-\frac{2}{3}, -\frac{1}{5}] \cup [-\frac{1}{5}, \infty)$.

Both inequalities are true on $(-\infty, -3] \cup [-\frac{1}{5}, \infty)$, so the solution set is $(-\infty, -3] \cup [-\frac{1}{5}, \infty)$.

65. $\left|\frac{x-1}{x+2}\right| \ge 2 \Rightarrow \frac{x-1}{x+2} \le -2$ or $\frac{x-1}{x+2} \ge 2$

$$\frac{x-1}{x+2} \le -2 \qquad \frac{x-1}{x+2} \ge 2$$
$$\frac{x-1}{x+2} + 2 \le 0 \qquad \frac{x-1}{x+2} - 2 \ge 0$$
$$\frac{x-1+2(x+2)}{x+2} \le 0 \qquad \frac{x-1-2(x+2)}{x+2} \ge 0$$
$$\frac{3x+3}{x+2} \le 0 \qquad \frac{-x-5}{x+2} \ge 0$$

$3x+3=0 \Rightarrow x=-1$; $x+2=0 \Rightarrow x=-2$
$-x-5=0 \Rightarrow x=-5$

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(*continued*)

Interval	Test point	Value of $\frac{3x+3}{x+2}$	Result
$(-\infty, -5]$	-6	$\frac{15}{4}$	+
$[-5, -2)$	-3	6	+
$(-2, -1]$	$-\frac{3}{2}$	-3	−
$[-1, \infty)$	1	2	+

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $(-2, -1]$.

Interval	Test point	Value of $\frac{-x-5}{x+2}$	Result
$(-\infty, -5]$	-6	$-\frac{1}{4}$	−
$[-5, -2)$	-3	2	+
$(-2, -1]$	$-\frac{3}{2}$	-7	−
$[-1, \infty)$	1	-2	−

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $[-5, -2)$. Since the original inequality is an "or" inequality, the solution set of the original inequality is the union of the two solution sets.
The solution set is $(-2, -1] \cup [-5, -2)$.

66. $\left|\frac{x+3}{x-2}\right| \ge 3 \Rightarrow \frac{x+3}{x-2} \le -3$ or $\frac{x+3}{x-2} \ge 3$

$$\frac{x+3}{x-2} \le -3 \qquad\qquad \frac{x+3}{x-2} \ge 3$$
$$\frac{x+3}{x-2} + 3 \le 0 \qquad\qquad \frac{x+3}{x-2} - 3 \ge 0$$
$$\frac{x+3+3(x-2)}{x-2} \le 0 \qquad\qquad \frac{x+3-3(x-2)}{x-2} \ge 0$$
$$\frac{4x-3}{x-2} \le 0 \qquad\qquad \frac{-2x+9}{x-2} \ge 0$$

$4x - 3 = 0 \Rightarrow x = \frac{3}{4}$; $x - 2 = 0 \Rightarrow x = 2$

$-2x + 9 = 0 \Rightarrow x = \frac{9}{2}$

Interval	Test point	Value of $\frac{4x-3}{x-2}$	Result
$(-\infty, \frac{3}{4}]$	0	$\frac{3}{2}$	+
$[\frac{3}{4}, 2)$	1	-1	−
$(2, \frac{9}{2}]$	3	9	+
$[\frac{9}{2}, \infty)$	7	5	+

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $[\frac{3}{4}, 2)$.

Interval	Test point	Value of $\frac{-2x+9}{x-2}$	Result
$(-\infty, \frac{3}{4}]$	0	$-\frac{9}{2}$	−
$[\frac{3}{4}, 2)$	1	-7	−
$(2, \frac{9}{2}]$	3	3	+
$[\frac{9}{2}, \infty)$	7	-1	−

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $(2, \frac{9}{2}]$. Since the original inequality is an "or" inequality, the solution set of the original inequality is the union of the two solution sets.
The solution set is $[\frac{3}{4}, 2) \cup (2, \frac{9}{2}]$.

67. $\left|\frac{2x+1}{x-1}\right| > 4 \Rightarrow \frac{2x+1}{x-1} < -4$ or $\frac{2x+1}{x-1} > 4$

$$\frac{2x+1}{x-1} < -4 \qquad\qquad \frac{2x+1}{x-1} > 4$$
$$\frac{2x+1}{x-1} + 4 < 0 \qquad\qquad \frac{2x+1}{x-1} - 4 > 0$$
$$\frac{2x+1+4(x-1)}{x-1} < 0 \qquad\qquad \frac{2x+1-4(x-1)}{x-1} > 0$$
$$\frac{6x-3}{x-1} < 0 \qquad\qquad \frac{-2x+5}{x-1} > 0$$

$6x - 3 = 0 \Rightarrow x = \frac{1}{2}$; $x - 1 = 0 \Rightarrow x = 1$

$-2x + 5 = 0 \Rightarrow x = \frac{5}{2}$

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Interval	Test point	Value of $\frac{6x-3}{x-1}$	Result
$\left(-\infty, \frac{1}{2}\right)$	0	3	+
$\left(\frac{1}{2}, 1\right)$	$\frac{3}{4}$	−6	−
$\left(1, \frac{5}{2}\right)$	2	9	+
$\left(\frac{5}{2}, \infty\right)$	4	7	+

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $\left(\frac{1}{2}, 1\right)$.

Interval	Test point	Value of $\frac{-2x+5}{x-1}$	Result
$\left(-\infty, \frac{1}{2}\right)$	0	−5	−
$\left(\frac{1}{2}, 1\right)$	$\frac{3}{4}$	−14	−
$\left(1, \frac{5}{2}\right)$	2	1	+
$\left(\frac{5}{2}, \infty\right)$	4	−1	−

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $\left(1, \frac{5}{2}\right)$. Since the original inequality is an "or" inequality, the solution set of the original inequality is the union of the two solution sets. The solution set is $\left(\frac{1}{2}, 1\right) \cup \left(1, \frac{5}{2}\right)$.

68. $\left|\frac{2x-1}{3x+2}\right| > 5 \Rightarrow \frac{2x-1}{3x+2} < -5$ or $\frac{2x-1}{3x+2} > 5$

$\frac{2x-1}{3x+2} < -5$	$\frac{2x-1}{3x+2} > 5$
$\frac{2x-1}{3x+2} + 5 < 0$	$\frac{2x-1}{3x+2} - 5 > 0$
$\frac{2x-1+5(3x+2)}{3x+2} < 0$	$\frac{2x-1-5(3x+2)}{3x+2} > 0$
$\frac{17x+9}{3x+2} < 0$	$\frac{-13x-11}{3x+2} > 0$

$17x+9 = 0 \Rightarrow x = -\frac{9}{17}$; $3x+2 = 0 \Rightarrow x = -\frac{2}{3}$

$-13x-11 = 0 \Rightarrow x = -\frac{11}{13}$

Interval	Test point	Value of $\frac{17x+9}{3x+2}$	Result
$\left(-\infty, -\frac{11}{13}\right)$	−1	8	+
$\left(-\frac{11}{13}, -\frac{2}{3}\right)$	$-\frac{10}{13}$	$\frac{53}{4}$	+
$\left(-\frac{2}{3}, -\frac{9}{17}\right)$	$-\frac{10}{17}$	$-\frac{17}{4}$	−
$\left(-\frac{9}{17}, \infty\right)$	0	$\frac{9}{2}$	+

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $\left(-\frac{2}{3}, -\frac{9}{17}\right)$.

Interval	Test point	Value of $\frac{-13x-11}{3x+2}$	Result
$\left(-\infty, -\frac{11}{13}\right)$	−1	−2	−
$\left(-\frac{11}{13}, -\frac{2}{3}\right)$	$-\frac{10}{13}$	$\frac{13}{4}$	+
$\left(-\frac{2}{3}, -\frac{9}{17}\right)$	$-\frac{10}{17}$	$-\frac{57}{4}$	−
$\left(-\frac{9}{17}, \infty\right)$	0	$-\frac{11}{2}$	−

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $\left(-\frac{11}{13}, -\frac{2}{3}\right)$. Since the original inequality is an "or" inequality, the solution set of the original inequality is the union of the two solution sets. The solution set is $\left(-\frac{11}{13}, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, -\frac{9}{17}\right)$.

69. $|x-1| \le 2|2x-5| \Rightarrow \left|\frac{x-1}{2x-5}\right| \le 2 \Rightarrow$

$-2 \le \frac{x-1}{2x-5} \le 2$

$0 \le \frac{x-1}{2x-5} + 2$	$\frac{x-1}{2x-5} - 2 \le 0$
$0 \le \frac{x-1+2(2x-5)}{2x-5}$	$\frac{x-1-2(2x-5)}{2x-5} \le 0$
$0 \le \frac{x-1+4x-10}{2x-5}$	$\frac{x-1-4x+10}{2x-5} \le 0$
$0 \le \frac{5x-11}{2x-5}$	$\frac{-3x+9}{2x-5} \le 0$

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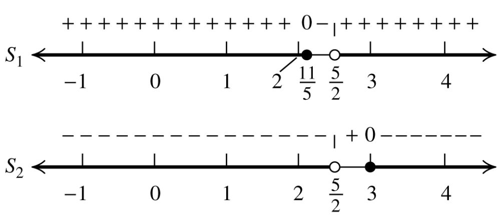
$5x-11=0 \Rightarrow x=\dfrac{11}{5};\ 2x-5=0 \Rightarrow x=\dfrac{5}{2}$
$-3x+9=0 \Rightarrow x=3$

Interval	Test point	Value of $\dfrac{5x-11}{2x-5}$	Result
$\left(-\infty, \frac{11}{5}\right]$	0	$\frac{11}{5}$	+
$\left[\frac{11}{5}, \frac{5}{2}\right)$	$\frac{23}{10}$	$-\frac{5}{4}$	−
$\left(\frac{5}{2}, \infty\right)$	3	4	+

Note that the expression is undefined for $x=\frac{5}{2}$. The solution set is for this part of the original inequality is $S_1=\left(-\infty, \frac{11}{5}\right]\cup\left(\frac{5}{2}, \infty\right)$.

Interval	Test point	Value of $\dfrac{-3x+9}{2x-5}$	Result
$\left(-\infty, \frac{5}{2}\right)$	0	$-\frac{9}{5}$	−
$\left(\frac{5}{2}, 3\right]$	$\frac{11}{4}$	$\frac{3}{2}$	+
$[3, \infty)$	5	$-\frac{6}{5}$	−

Note that the expression is undefined for $x=\frac{5}{2}$. The solution set is for this part of the original inequality is $S_2=\left(-\infty, \frac{5}{2}\right)\cup[3, \infty)$.



The figure shows that both inequalities are true on $\left(-\infty, \frac{11}{5}\right]\cup[3, \infty)$, so the solution set is $\left(-\infty, \frac{11}{5}\right]\cup[3, \infty)$.

70. $2|x-5| \le |2x-3| \Rightarrow 2\left|\dfrac{x-5}{2x-3}\right| \le 1 \Rightarrow$

$\left|\dfrac{x-5}{2x-3}\right| \le \dfrac{1}{2} \Rightarrow -\dfrac{1}{2} \le \dfrac{x-5}{2x-3} \le \dfrac{1}{2}$

$$-\frac{1}{2} \le \frac{x-5}{2x-3}$$
$$0 \le \frac{x-5}{2x-3}+\frac{1}{2}$$
$$0 \le \frac{2(x-5)+(2x-3)}{2(2x-3)}$$
$$0 \le \frac{2x-10+2x-3}{4x-6}$$
$$0 \le \frac{4x-13}{4x-6}$$

$$\frac{x-5}{2x-3} \le \frac{1}{2}$$
$$\frac{x-5}{2x-3}-\frac{1}{2} \le 0$$
$$\frac{2(x-5)-(2x-3)}{2(2x-3)} \le 0$$
$$\frac{2x-10-2x+3}{4x-6} \le 0$$
$$\frac{-7}{4x-6} \le 0$$

Neither expression is defined if $x=\frac{3}{2}$.

$4x-13=0 \Rightarrow x=\frac{13}{4}$.

Interval	Test point	Value of $\dfrac{4x-13}{4x-6}$	Result
$\left(-\infty, \frac{3}{2}\right)$	0	$\frac{13}{6}$	+
$\left(\frac{3}{2}, \frac{13}{4}\right]$	2	$-\frac{5}{2}$	−
$\left[\frac{13}{4}, \infty\right)$	4	$\frac{3}{10}$	+

The solution set is for this part of the original inequality is $S_1=\left(-\infty, \frac{3}{2}\right)\cup\left[\frac{13}{4}, \infty\right)$.

Interval	Test point	Value of $\dfrac{-7}{4x-6}$	Result
$\left(-\infty, \frac{3}{2}\right)$	0	$\frac{7}{6}$	+
$\left(\frac{3}{2}, \infty\right)$	2	$-\frac{7}{2}$	−

The solution set is for this part of the original inequality is $S_2=\left(\frac{3}{2}, \infty\right)$.

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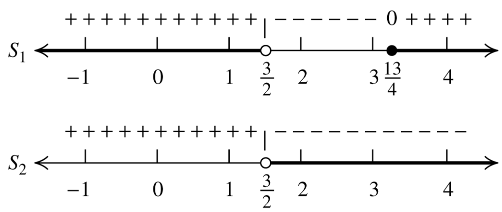

The figure shows that both inequalities are true on $\left[\frac{13}{4}, \infty\right)$, so the solution set is $\left[\frac{13}{4}, \infty\right)$.

1.6 Applying the Concepts

71. $|T-75| = 20 \Rightarrow T-75 = -20$ or $T-75 = 20 \Rightarrow T = 55$ or $T = 95$
The temperatures in Tampa during December are between 55°F and 95°F.

72. $|x-1.14| \le 0.05$

73. $|x-700| \le 50$

74. $|x-480| \le 90$

75. $|x-120| \le 6.75$

76. Let x = the actual number of gallons of gas.
$|x-4| \le \frac{1}{4} \Rightarrow -\frac{1}{4} \le x-4 \le \frac{1}{4} \Rightarrow$
$-1 \le 4x-16 \le 1 \Rightarrow \frac{15}{4} \le x \le \frac{17}{4}$
The motorcycle has between 3.75 and 4.25 gallons of gas, so it can travel between $37(3.75) = 138.75$ miles and $37(6.25) = 157.25$ miles.

77. Let x = the number of people at a party. Then $|120-x| \le 15 \Rightarrow -15 \le 120-x \le 15 \Rightarrow$
$-135 \le -x \le -105 \Rightarrow 105 \le x \le 135$. So, between 105 and 135 people will be at the party. The total spent on food will be between 48(105) = \$5040 and 48(135) = \$6480.

78. Let x = the number of bonuses actually given. Then $|60-x| \le 7 \Rightarrow -7 \le 60-x \le 7 \Rightarrow$
$-67 \le -x \le -53 \Rightarrow 67 \ge x \ge 53$. So, between 53 and 67 people will receive bonuses. The total spent on bonuses will be between 1200(53) = \$63,600 and 1200(67) = \$80,400.

79. Let x = the actual weight of Sarah's catch. Then $|32-x| \le \frac{1}{2} \Rightarrow -\frac{1}{2} \le 32-x \le \frac{1}{2} \Rightarrow$
$-1 \le 64-2x \le 1 \Rightarrow -65 \le -2x \le -63 \Rightarrow$.
$32.5 \ge x \ge 31.5$.
So, her catch is between 31.5 pounds and 32.5 pounds. She will be paid between 0.60(31.5) = \$18.90 and 0.60(32.5) = \$19.50.

80. Let x = the actual number of tickets sold. Then $|460-x| \le 25 \Rightarrow -25 \le 460-x \le 25 \Rightarrow$
$-485 \le -x \le -435 \Rightarrow 485 \ge x \ge 435$. So there were between 435 and 485 tickets sold. The park might take in between 29.50(435) = \$12,832.50 and 29.50(485) = \$14,307.50.

1.6 Beyond the Basics

81. a. $|x^2-9| = x-3 \Rightarrow x^2-9 = -(x-3)$ or $x^2-9 = x-3$
$x^2-9 = -(x-3) \Rightarrow x^2-9 = -x+3 \Rightarrow$
$x^2+x-12 = 0 \Rightarrow (x+4)(x-3) = 0 \Rightarrow$
$x = -4, 3$
$x^2-9 = x-3 \Rightarrow x^2-x-6 = 0 \Rightarrow$
$(x+2)(x-3) = 0 \Rightarrow x = -2, 3$
Checking $x = -4$, $x = -2$, and $x = 3$ in the original equation shows that $x = -4$ and $x = -2$ are extraneous solutions.
The solution set is $\{3\}$.

b. $|x^2-8| = -2x \Rightarrow x^2-8 = -(-2x)$ or $x^2-8 = -2x$
$x^2-8 = -(-2x) \Rightarrow x^2-8 = 2x \Rightarrow$
$x^2-2x+8 = 0 \Rightarrow (x+2)(x-4) = 0 \Rightarrow$
$x = -2, 4$
$x^2-8 = -2x \Rightarrow x^2+2x-8 = 0 \Rightarrow$
$(x+4)(x-2) = 0 \Rightarrow x = -4, 2$
Checking $x = -4$, $x = -2$, $x = 2$, and $x = 4$ in the original equation shows that $x = 2$, and $x = 4$ are extraneous solution.
The solution set is $\{-4, -2\}$.

c. $|x^2-5x| = 6 \Rightarrow x^2-5x = -6$ or $x^2-5x = 6$
$x^2-5x = -6 \Rightarrow x^2-5x+6 = 0 \Rightarrow$
$(x-2)(x-3) = 0 \Rightarrow x = 2, 3$
$x^2-5x = 6 \Rightarrow x^2-5x-6 = 0 \Rightarrow$
$(x+1)(x-6) = 0 \Rightarrow x = -1, 6$
Checking $x = -1$, $x = 2$, $x = 3$, and $x = 6$ in the original equation shows that all values are solutions.
The solution set is $\{-1, 2, 3, 6\}$.

d. $\left|x^2+3x-2\right|=2 \Rightarrow x^2+3x-2=-2$ or $x^2+3x-2=2$. If $x^2+3x-2=-2$, then $x^2+3x=0 \Rightarrow x(x+3)=0 \Rightarrow x=0$ or $x=-3$.

If $x^2+3x-2=2$, then $x^2+3x-4=0 \Rightarrow (x+4)(x-1)=0 \Rightarrow x=1$ or $x=-4$.

Checking $x = –4$, $x = –3$, $x = 0$, and $x = 1$ in the original equation shows that all values are solutions.

The solution set is {–4, –3, 0, 1}.

82. a. $\left|x^2-7\right|=|6x| \Rightarrow x^2-7=6x$ or $x^2-7=-6x$

$x^2-7=6x \Rightarrow x^2-6x-7=0 \Rightarrow (x+1)(x-7)=0 \Rightarrow x=-1, 7$

$x^2-7=-6x \Rightarrow x^2+6x-7=0 \Rightarrow (x+7)(x-1)=0 \Rightarrow x=-7, 1$

Checking $x = −7$, $x = −1$, $x = 1$, and $x = −7$ in the original equation shows that all values are solutions.

The solution set is $\{\pm 1, \pm 7\}$.

b. $\left|x^2-2x\right|=|5x-10| \Rightarrow x^2-2x=5x-10$ or $x^2-2x=-(5x-10)$

$x^2-2x=5x-10 \Rightarrow x^2-7x+10=0 \Rightarrow (x-2)(x-5)=0 \Rightarrow x=2, 5$

$x^2-2x=-(5x-10) \Rightarrow$

$x^2-2x=-5x+10 \Rightarrow x^2+3x-10=0 \Rightarrow (x+5)(x-2)=0 \Rightarrow x=-5, 2$

Checking $x = −5$, $x = 2$, and $x = 5$ in the original equation shows that all values are solutions. The solution set is $\{2, \pm 5\}$.

c. $\left|2x^2-3x+5\right|=\left|x^2-4x+7\right| \Rightarrow$

$2x^2-3x+5=-\left(x^2-4x+7\right)$ or

$2x^2-3x+5=x^2-4x+7$

$2x^2-3x+5=-\left(x^2-4x+7\right) \Rightarrow$

$2x^2-3x+5=-x^2+4x-7 \Rightarrow$

$3x^2-7x+12=0 \Rightarrow$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(3)(12)}}{2(3)} = \frac{7\pm\sqrt{49-144}}{6}=\frac{7\pm i\sqrt{95}}{6}$$

These solutions are extraneous.

$2x^2-3x+5=x^2-4x+7 \Rightarrow$

$x^2+x-2=0 \Rightarrow (x+2)(x-1)=0 \Rightarrow x=-2, 1$

Solution set: {−2, 1}

d. $\left|x^2+x+3\right|=\left|x^2+5x+1\right| \Rightarrow$

$x^2+x+3=-\left(x^2+5x+1\right)$ or

$x^2+x+3=x^2+5x+1$

$x^2+x+3=-\left(x^2+5x+1\right) \Rightarrow$

$x^2+x+3=-x^2-5x-1 \Rightarrow$

$2x^2+6x+4=0 \Rightarrow x^2+3x+2=0 \Rightarrow (x+2)(x+1)=0 \Rightarrow x=-2, -1$

$x^2+x+3=x^2+5x+1 \Rightarrow$

$-4x+2=0 \Rightarrow x=\frac{1}{2}$

Checking $x=-2$, $x=-1$, and $x=\frac{1}{2}$ in the original equation shows that all values are solutions. The solution set is $\left\{-2, -1, \frac{1}{2}\right\}$.

83. $|2x-3|+|x-2|=4$

The points $x=\frac{3}{2}$ and $x = 2$ are the points where the absolute value expressions equal zero. Since these expressions must be negative or positive for other x-values, then these points divide the number line into intervals each of which should be considered separately. Thus, we will consider the intervals $\left(-\infty, \frac{3}{2}\right]$, $\left(\frac{3}{2}, 2\right]$, and $(2, \infty)$.

Interval	Test point	Sign of $2x-3$	Sign of $x-2$
$\left(-\infty, \frac{3}{2}\right]$	0	–	–
$\left(\frac{3}{2}, 2\right]$	$\frac{7}{4}$	+	–
$(2, \infty)$	3	+	+

On the first interval, both absolute-value expressions will have negative values, so change the signs on both of them when taking the bars off.

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(*continued*)

$$-(2x-3)-(x-2)=4$$
$$-2x+3-x+2=4$$
$$-3x+5=4$$
$$-3x=-1 \Rightarrow x=\frac{1}{3}$$

Since $x=\frac{1}{3}$ lies in the interval $\left(-\infty, \frac{3}{2}\right]$, this is a valid solution.

On the second interval, $|2x-3|$ is positive, so just take the bars off. But $|x-2|$ is negative, change the sign when taking the bars off.

$$(2x-3)-(x-2)=4$$
$$2x-3-x+2=4$$
$$x-1=4 \Rightarrow x=5$$

Since $x = 5$ does not lie in the interval $\left(\frac{3}{2}, 2\right]$, it is not a valid solution of the original equation.

On the third interval, the arguments of both absolute values expressions are positive, so just remove the absolute value bars.

$$(2x-3)+(x-2)=4$$
$$2x-3+x-2=4$$
$$3x-5=4$$
$$3x=9 \Rightarrow x=3$$

Since $x = 3$ lies in the interval $(2,\infty)$, the solution is valid. Thus, the solution set for the original equation is $\left\{\frac{1}{3}, 3\right\}$.

84. $2|2x-3|-3|x-2|=5$

The points $x=\frac{3}{2}$ and $x = 2$ are the points where the absolute value expressions equal zero. Since these expressions must be negative or positive for other x-values, then these points divide the number line into intervals each of which should be considered separately. Thus, we will consider the intervals $\left(-\infty, \frac{3}{2}\right]$, $\left(\frac{3}{2}, 2\right]$, and $(2,\infty)$.

Interval	Test point	Sign of $2x-3$	Sign of $x-2$
$\left(-\infty, \frac{3}{2}\right]$	0	–	–
$\left(\frac{3}{2}, 2\right]$	$\frac{7}{4}$	+	–
$(2,\infty)$	3	+	+

On the first interval, both absolute-value expressions will have negative values, so change the signs on both of them when taking the bars off.

$$-2(2x-3)+3(x-2)=5$$
$$-4x+6+3x-6=5$$
$$-x=5 \Rightarrow x=-5$$

Since $x = -5$ lies in the interval $\left(-\infty, \frac{3}{2}\right]$, this is a valid solution.

On the second interval, $|2x-3|$ is positive, so just take the bars off. But $|x-2|$ is negative, change the sign when taking the bars off.

$$2(2x-3)+3(x-2)=5$$
$$4x-6+3x-6=5$$
$$7x-12=5$$
$$7x=17 \Rightarrow x=\frac{17}{7}$$

Since $x=\frac{17}{7}$ does not lie in the interval $\left(\frac{3}{2}, 2\right]$, it is not a valid solution of the original equation.

On the third interval, the arguments of both absolute values expressions are positive, so just remove the absolute value bars.

$$2(2x-3)-3(x-2)=5$$
$$4x-6-3x+6=5$$
$$x=5$$

Since $x = 5$ lies in the interval $(2,\infty)$, the solution is valid. Thus, the solution set for the original equation is $\{\pm 5\}$.

85. Let $u=|x|$. Then $|x|^2-4|x|-7=5 \Rightarrow$
$u^2-4u-7=5 \Rightarrow u^2-4u-12=0 \Rightarrow$
$(u-6)(u+2)=0 \Rightarrow u=-2, 6.$
Now solve for x: $-2=|x|$ is not possible.
$6=|x| \Rightarrow x=6$ or $x=-6$
Solution set: $\{-6, 6\}$.

86. Let $u=|x|$. Then $2|x|^2-|x|+8=11 \Rightarrow$
$2u^2-u+8=11 \Rightarrow 2u^2-u-3=0 \Rightarrow$
$(2u-3)(u+1)=0 \Rightarrow u=\frac{3}{2}, -1.$
Now solve for x: $\frac{3}{2}=|x| \Rightarrow x=\frac{3}{2}$ or $x=-\frac{3}{2}$.
$-1=|x|$ is not possible.
Solution set: $\left\{-\frac{3}{2}, \frac{3}{2}\right\}$

87. $0 < a < b \Rightarrow b - a > 0$ and $0 < c < d \Rightarrow d - c > 0.$ The product of two positive numbers is positive, so
$(b - a)c > 0 \Leftrightarrow bc > ac$ (1) and
$(d - c)a > 0 \Leftrightarrow ad > ac$ (2) . We know that
$bc > ac$ and $bd > ad$, so
$bd - bc > ad - ac \Rightarrow bd + ac > ad + bc.$
Substituting (1) and (2) into the inequality, we have $bd + ac > ad + bc > ac + ac \Rightarrow bd > ac.$

88. Use the results of Exercise 87 in order to show that $ae < bf < cg$ given that $0 < a < b < c$ and $0 < e < f < g$.
$ae < bf$ and $bf < cg$, so $ae < bf < cg$.

89. $x^2 < a \Rightarrow x^2 - a < 0 \Rightarrow \left(x - \sqrt{a}\right)\left(x + \sqrt{a}\right) < 0$
$\left(x - \sqrt{a}\right)\left(x + \sqrt{a}\right) < 0 \Rightarrow$
$\left(x - \sqrt{a}\right) > 0$ and $\left(x + \sqrt{a}\right) < 0 \Rightarrow$
$x > \sqrt{a}$ and $x < -\sqrt{a} \Rightarrow \sqrt{a} < x < -\sqrt{a} \Rightarrow$
$\sqrt{a} < -\sqrt{a}$, a contradiction
or
$\left(x - \sqrt{a}\right) < 0$ and $\left(x + \sqrt{a}\right) > 0 \Rightarrow$
$x > -\sqrt{a}$ and $x < \sqrt{a} \Rightarrow -\sqrt{a} < x < \sqrt{a}$
Thus, the solution set is $\left(-\sqrt{a}, \sqrt{a}\right)$.

90. $x^2 > a \Rightarrow x^2 - a > 0 \Rightarrow \left(x - \sqrt{a}\right)\left(x + \sqrt{a}\right) > 0$
$\left(x - \sqrt{a}\right)\left(x + \sqrt{a}\right) > 0 \Rightarrow$
$\left(x - \sqrt{a}\right) < 0$ and $\left(x + \sqrt{a}\right) < 0 \Rightarrow$
$x < \sqrt{a}$ and $x < -\sqrt{a} \Rightarrow x < -\sqrt{a}$
or
$\left(x - \sqrt{a}\right) > 0$ and $\left(x + \sqrt{a}\right) > 0 \Rightarrow$
$x > -\sqrt{a}$ and $x > \sqrt{a} \Rightarrow x > \sqrt{a}$
Thus, the solution set is
$\left(-\infty, -\sqrt{a}\right) \cup \left(\sqrt{a}, \infty\right)$.

In Exercises 91–102, answers may vary. Sample responses are given.

91. $|x - 4| < 3$ **92.** $|2x - 11| \le 5$

93. $|x - 4| \le 6$ **94.** $|x + 4| < 3$

95. $|x - 7| > 4$ **96.** $|x - 2| > 3$

97. $|2x - 5| \ge 15$ **98.** $|x + 2| \ge 1$

99. $\left|x - \dfrac{a+b}{2}\right| < \dfrac{a+b}{2} \Rightarrow |2x - a - b| < a + b$

100. $|2x - c - d| \le d - c$

101. $|2x - a - b| > b - a$

102. $|2x - c - d| \ge d - c$ **103.** $|x - 39| \le 31$

104. $|x - 1| \le 5$ and $|x| \ge 2$
Solve each inequality and then determine where the two solution sets intersect.
$|x - 1| \le 5 \Rightarrow -5 \le x - 1 \le 5 \Rightarrow -4 \le x \le 6$
$|x| \ge 2 \Rightarrow x \le -2$ or $x \ge 2$
The intersection of the two solution set is $[-4, -2] \cup [2, 6]$, so this is the solution set of the original problem.

105. $1 \le |x - 2| \le 3 \Rightarrow$
$1 \le x - 2 \le 3$ or $1 \le -(x - 2) \le 3$
$3 \le x \le 5$ $1 \le -x + 2 \le 3$
$-1 \le -x \le 1$
$1 \ge x \ge -1 \Rightarrow -1 \le x \le 1$
The union of the two solution sets is the solution set of the original inequality.
Solution set: $[-1, 1] \cup [3, 5]$

106. $|x - 1| + |x - 2| \le 4$
Solve the equation $|x - 1| + |x - 2| = 4$ to find the critical values for the inequality.
The points $x = 1$ and $x = 2$ are the points where the absolute value expressions equal zero. Since these expressions must be negative or positive for other x-values, then these points divide the number line into intervals each of which should be considered separately. Thus, we will consider the intervals $(-\infty, 1]$, $(1, 2]$, and $(2, \infty)$.

Interval	Test point	Sign of $x - 1$	Sign of $x - 2$
$(-\infty, 1]$	0	–	–
$(1, 2]$	$\frac{3}{2}$	+	–
$(2, \infty)$	3	+	+

On the first interval, both absolute-value expressions will have negative values, so change the signs on both of them when taking the bars off.

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(*continued*)

$$-(x-1)-(x-2)=4$$
$$-2x+3=4$$
$$-2x=1 \Rightarrow x=-\frac{1}{2}$$

Since $x=-\frac{1}{2}$ lies in the interval $(-\infty, 1]$, $x=-\frac{1}{2}$ is a valid solution.

On the second interval, $x-1$ is positive, so just take the bars off. But $x-2$ is negative, change the sign when taking the bars off.

$$(x-1)-(x-2)=4$$
$$1=4$$

Since this is a false statement, no values of x in the interval $(1, 2]$ are valid solutions of the equation.

On the third interval, the arguments of both absolute values expressions are positive, so just remove the absolute value bars.

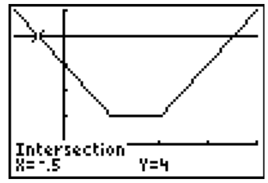
$$x-1+x-2=4$$
$$2x-3=4$$
$$2x=7 \Rightarrow x=\frac{7}{2}$$

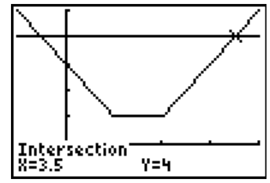
Since $\frac{7}{2}$ lies in the interval $(2,\infty)$, $x=\frac{7}{2}$ is a valid solution.

So $x=-\frac{1}{2}, 1, 2, \frac{7}{2}$ are critical values. Test values in the intervals $\left(-\infty, -\frac{1}{2}\right]$, $\left(-\frac{1}{2}, 1\right]$, $(1, 2]$, $\left(2, \frac{7}{2}\right]$, and $\left(\frac{7}{2}, \infty\right)$ to see where the original inequality is true.

Interval	Test point	Value of $\lvert x-1\rvert+\lvert x-2\rvert$	Value ≤ 4?
$\left(-\infty, -\frac{1}{2}\right]$	-1	5	No
$\left(-\frac{1}{2}, 1\right]$	0	3	Yes
$(1, 2]$	$\frac{3}{2}$	1	Yes
$\left(2, \frac{7}{2}\right]$	3	3	Yes
$\left(\frac{7}{2}, \infty\right)$	4	5	No

Thus, the solution set is $\left[-\frac{1}{2}, \frac{7}{2}\right]$. Verify this by graphing $Y_1=\lvert x-1\rvert+\lvert x-2\rvert$ and $Y_2=4$.





107. $\lvert x-2\rvert+\lvert x-4\rvert \ge 8$.

Solve the equation $\lvert x-2\rvert+\lvert x-4\rvert=8$ to find the critical values for the inequality.

The points $x=2$ and $x=4$ are the points where the absolute value expressions equal zero. Since these expressions must be negative or positive for other x-values, then these points divide the number line into intervals each of which should be considered separately. Thus, we will consider the intervals $(-\infty, 2]$, $(2, 4]$, and $(4,\infty)$.

Interval	Test point	Sign of $x-2$	Sign of $x-4$
$(-\infty, 2]$	0	$-$	$-$
$(2, 4]$	3	$+$	$-$
$(4,\infty)$	5	$+$	$+$

On the first interval, both absolute-value expressions will have negative values, so change the signs on both of them when taking the bars off.

$$-(x-2)+\left[-(x-4)\right]=8$$
$$-2x+6=8$$
$$-2x=2 \Rightarrow x=-1$$

Since $x=-1$ lies in the interval $(-\infty, 2]$, $x=-1$ is a valid solution.

On the second interval, $x-2$ is positive, so just take the bars off. But $x-4$ is negative, change the sign when taking the bars off.

$$(x-2)+\left[-(x-4)\right]=8$$
$$2=8$$

Since this is a false statement, no values of x in the interval $(2, 4]$ are valid solutions of the equation.

On the third interval, the arguments of both absolute values expressions are positive, so just remove the absolute value bars.

$$x-2+x-4=8$$
$$2x-6=8$$
$$2x=14 \Rightarrow x=7$$

Since 7 lies in the interval $(2,\infty)$, $x=7$ is a valid solution.

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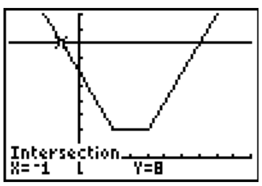
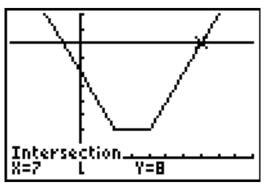
(*continued*)

So $x = -1, 2, 4,$ and 7 are critical values. Test values in the intervals $(-\infty, -1], [-1, 2],$ $[2, 4], [4, 7],$ and $[7, \infty)$ to see where the original inequality is true.

Interval	Test point	Value of $\lvert x-2\rvert + \lvert x-4\rvert$	Value ≥ 8?
$(-\infty, -1]$	−2	10	Yes
$[-1, 2]$	0	6	No
$[2, 4]$	3	2	No
$[4, 7]$	5	4	No
$[7, \infty)$	10	14	Yes

Thus, the solution set is $(-\infty, -1] \cup [7, \infty)$. Verify this by graphing $Y_1 = \lvert x-2\rvert + \lvert x-4\rvert$ and $Y_2 = 8$.





108. $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert \le 6$

Solve the equation $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert = 6$ to find the critical values for the inequality. The points $x = 1$, $x = 2$ and $x = 3$ are the points where the absolute value expressions equal zero. Since these expressions must be negative or positive for other x-values, then these points divide the number line into intervals each of which should be considered separately. Thus, we will consider the intervals $(-\infty, 1],$ $(1, 2], (2, 3],$ and $(3, \infty)$.

Interval	Test point	Sign of $x-1$	Sign of $x-2$	Sign of $x-3$
$(-\infty, 1]$	0	−	−	−
$(1, 2]$	$\frac{3}{2}$	+	−	−
$(2, 3]$	$\frac{5}{2}$	+	+	−
$(3, \infty)$	5	+	+	+

On the first interval, all three absolute-value expressions will have negative values, so change the signs on both of them when taking the bars off.

$$-(x-1) + [-(x-2)] + [-(x-3)] = 6$$
$$-3x + 6 = 6$$
$$-3x = 0 \Rightarrow x = 0$$

On the second interval, $x - 1$ is positive, so just take the bars off. But, $x - 2$ and $x - 3$ are both negative, so change the sign when taking the bars off.

$$(x-1) + [-(x-2)] + [-(x-3)] = 6$$
$$-x + 4 = 6$$
$$-x = 2 \Rightarrow x = -2$$

On the third interval, $x - 1$ and $x - 2$ are both positive, so just take the bars off; $x - 3$ is negative, so change the sign when taking the bars off.

$$(x-1) + (x-2) + [-(x-3)] = 6 \Rightarrow x = 6$$

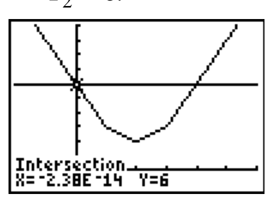
On the last interval, the arguments of all three absolute value expressions are positive, so just remove the bars.

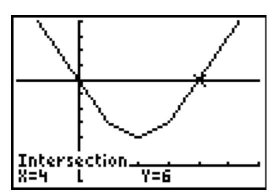
$$(x-1) + (x-2) + (x-3) = 6$$
$$3x - 6 = 6$$
$$3x = 12 \Rightarrow x = 4$$

So, $x = -2, 0, 4,$ and 6 are critical values. Test values in the intervals $(-\infty, -2],$ $[-2, 0], [0, 4], [4, 6],$ and $[6, \infty)$ to see where the original inequality is true.

Interval	Test point	Value of $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert$	Value ≤ 6?
$(-\infty, -2]$	−3	15	No
$[-2, 0]$	−1	9	No
$[0, 4]$	2	2	Yes
$[4, 6]$	5	9	No
$[6, \infty)$	10	24	No

Thus, the solution set is $[0, 4]$. Verify this by graphing $Y_1 = \lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert$ and $Y_2 = 6$.





109. $\dfrac{|x-1|-(x+2)}{x+2} \ge 0$.

The value of $x - 1$ changes from negative to positive at $x = 1$, where it is 0. So, we will consider two cases,

$$\begin{cases} \dfrac{(x-1)-(x+2)}{x+2} \ge 0 & \text{if } x \ge 1 \\ \dfrac{-(x-1)-(x+2)}{x+2} \ge 0 & \text{if } x < 1 \end{cases}$$

Test the case for $x \ge 1$.

$$\frac{(x-1)-(x+2)}{x+2} \ge 0 \Rightarrow \frac{-3}{x+2} \ge 0$$

$x + 2 = 0 \Rightarrow x = -2$

Since this test case is for $x \ge 1$, it is only necessary to test the interval $[1, \infty)$.

Interval	Test point	Value of $\frac{-3}{x+2}$	Result
$[1, \infty)$	2	$-\frac{3}{4}$	–

$-\frac{3}{4}$ is not ≥ 1, so there is no solution in this interval.

Now test the case $x < 1$.

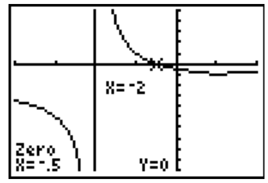
$$\frac{-(x-1)-(x+2)}{x+2} \ge 0 \Rightarrow \frac{-2x-1}{x+2} \ge 0$$

$-2x - 1 = 0 \Rightarrow x = -\frac{1}{2}$; $x + 2 = 0 \Rightarrow x = -2$

The intervals to be tested are $(-\infty, -2)$, $\left(-2, -\frac{1}{2}\right]$, and $\left[-\frac{1}{2}, 1\right]$.

Interval	Test point	Value of $\frac{-2x-1}{x+2}$	Result
$(-\infty, -2)$	–3	–5	–
$\left(-2, -\frac{1}{2}\right]$	–1	1	+
$\left[-\frac{1}{2}, 1\right]$	0	$-\frac{1}{2}$	–

The only interval where the value of $\frac{-2x-1}{x+2} \ge 0$ is $\left(-2, -\frac{1}{2}\right]$, so the solution set is $\left(-2, -\frac{1}{2}\right]$. Verify this by graphing $Y_1 = \dfrac{|x-1|-(x+2)}{x+2}$.



1.6 Critical Thinking/Discussion/Writing

110. To find what values make $\sqrt{(x-3)^2} = x - 3$ true, solve $x - 3 = 0 \Rightarrow x = 3,$ so we check the intervals $(-\infty, 3]$ and $[3, \infty)$ to see which makes the equation true. The equation is true for $[3, \infty)$.

111. To find what values make $\sqrt{\left(x^2 - 6x + 8\right)^2} = x^2 - 6x + 8$ true, solve $x^2 - 6x + 8 = 0 \Rightarrow (x-2)(x-4) = 0 \Rightarrow x = 2$ or $x = 4$. Then check the intervals $(-\infty, 2]$, $[2, 4]$, and $[4, \infty)$ to see which make the equation true. The equation is true for $(-\infty, 2] \cup [4, \infty)$.

112. To solve $|x-3|^2 - 7|x-3| + 10 = 0$, let $u = |x-3|$. So we have $u^2 - 7u + 10 = 0 \Rightarrow (u-5)(u-2) = 0 \Rightarrow u = 5$ or $u = 2$. Now solve for x.

$5 = |x-3| \Rightarrow -5 = x - 3$ or $5 = x - 3 \Rightarrow$ $x = -2$ or $x = 8$. $2 = |x-3| \Rightarrow -2 = x - 3$ or $2 = x - 3 \Rightarrow x = 1$ or $x = 5$. So the solution set is $\{-2, 1, 5, 8\}$.

1.6 Maintaining Skills

113. $\dfrac{2+5}{2} = \dfrac{7}{2} = 3.5$

114. $\dfrac{-3+7}{2} = \dfrac{4}{2} = 2$

115. $\dfrac{-3-7}{2} = \dfrac{-10}{2} = -5$

116. $\dfrac{(a-b)-(a+b)}{2} = \dfrac{a-b-a-b}{2} = \dfrac{-2b}{2} = -b$

117. $\sqrt{(5-2)^2 + (3-7)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

118. $\sqrt{(-8+3)^2+(-5-7)^2} = \sqrt{(-5)^2+(-12)^2}$
$= \sqrt{25+144} = \sqrt{169}$
$= 13$

119. $\sqrt{(2-5)^2+(8-6)^2} = \sqrt{(-3)^2+2^2} = \sqrt{9+4}$
$= \sqrt{13}$

120. $\sqrt{\left(\sqrt{3}-\sqrt{12}\right)^2+\left(\sqrt{2}+\sqrt{8}\right)^2}$
$= \sqrt{\left(\sqrt{3}-2\sqrt{3}\right)^2+\left(\sqrt{2}+2\sqrt{2}\right)^2}$
$= \sqrt{\left(-\sqrt{3}\right)^2+\left(3\sqrt{2}\right)^2} = \sqrt{3+9\cdot 2}$
$= \sqrt{3+18} = \sqrt{21}$

121. $x^2+4x+\underline{4} = (x+2)^2$

122. $x^2-6x+\underline{9} = (x-3)^2$

123. $x^2-5x+\left(\frac{-5}{2}\right)^2 = x^2-5x+\underline{\frac{25}{4}} = \left(x-\frac{5}{2}\right)^2$

124. $x^2+7x+\left(\frac{7}{2}\right)^2 = x^2+7x+\underline{\frac{49}{4}} = \left(x+\frac{7}{2}\right)^2$

125. $x^2+\frac{3}{2}x+\left(\frac{\frac{3}{2}}{2}\right)^2 = x^2+\frac{3}{2}x+\left(\frac{3}{4}\right)^2$
$= x^2+\frac{3}{2}x+\underline{\frac{9}{16}} = \left(x+\frac{3}{4}\right)^2$

126. $x^2-\frac{4}{5}x+\left(\frac{-\frac{4}{5}}{2}\right)^2 = x^2-\frac{4}{5}x+\left(-\frac{2}{5}\right)^2$
$= x^2-\frac{4}{5}x+\underline{\frac{4}{25}}$
$= \left(x-\frac{2}{5}\right)^2$

Chapter 1 Review Exercises

Basic Concepts and Skills

1. $5x-4=11 \Rightarrow 5x=15 \Rightarrow x=3$

2. $12x+7=31 \Rightarrow 12x=24 \Rightarrow x=2$

3. $3(2x-4)=9-(x+7) \Rightarrow 6x-12=2-x \Rightarrow$
$7x=14 \Rightarrow x=2$

4. $4(x+7)=40+2x \Rightarrow 4x+28=40+2x \Rightarrow$
$2x=12 \Rightarrow x=6$

5. $3x+8=3(x+2)+2 \Rightarrow 3x+8=3x+8$ This is an identity, so the solution set is $(-\infty,\infty)$.

6. $3x+8=3(x+1)+4 \Rightarrow 3x+8=3x+7$. This is not possible, so the solution set is $\varnothing$.

7. $x-(5x-2)=7(x-1)-2 \Rightarrow$
$-4x+2=7x-9 \Rightarrow -11x=-11 \Rightarrow x=1$

8. $7+4(3+y)=8(3y-1)+3 \Rightarrow$
$19+4y=24y-5 \Rightarrow 24=20y \Rightarrow \frac{6}{5}=y$

9. $\frac{2}{x+3}=\frac{5}{11x-1} \Rightarrow 2(11x-1)=5(x+3) \Rightarrow$
$22x-2=5x+15 \Rightarrow 17x=17 \Rightarrow x=1$

10. $\frac{7}{x+2}=\frac{3}{x-2} \Rightarrow 7(x-2)=3(x+2) \Rightarrow$
$7x-14=3x+6 \Rightarrow 4x=20 \Rightarrow x=5$

11. $\frac{y+5}{2}+\frac{y-1}{3}=\frac{7y+3}{8}+\frac{4}{3} \Rightarrow$
$12(y+5)+8(y-1)=3(7y+3)+8(4) \Rightarrow$
$20y+52=21y+41 \Rightarrow 11=y$

12. $\frac{y-3}{6}-\frac{y-4}{5}=-\frac{1}{6} \Rightarrow$
$5(y-3)-6(y-4)=5(-1) \Rightarrow$
$-y+9=-5 \Rightarrow y=14$

13. $|2x-3|=|4x+5| \Rightarrow 2x-3=4x+5 \Rightarrow$
$-2x=8 \Rightarrow x=-4$
or $2x-3=-(4x+5) \Rightarrow$
$2x-3=-4x-5 \Rightarrow 6x=-2 \Rightarrow x=-1/3$
The solution set is $\left\{-4,-\frac{1}{3}\right\}$.

14. $|5x-3|=|x+4| \Rightarrow 5x-3=x+4 \Rightarrow x=\frac{7}{4}$
or $5x-3=-(x+4) \Rightarrow 5x-3=-x-4 \Rightarrow$
$x=-\frac{1}{6}$. The solution set is $\left\{-\frac{1}{6},\frac{7}{4}\right\}$.

15. $|2x-1|=|2x+7|$
$2x-1=2x+7$ or $2x-1=-(2x+7)$
$-2=7$ False $\quad 2x-1=-2x-7$
$4x=-6 \Rightarrow x=-\frac{3}{2}$
Solution set: $\left\{-\frac{3}{2}\right\}$

16. $|x-1| = 2-x$

$x-1 = 2-x$ or $x-1 = -(2-x)$

$2x = 3 \qquad x-1 = -2+x$

$x = 3/2 \qquad -1 = -2$ False

Solution set: $\left\{\frac{3}{2}\right\}$

17. $|3x-2| = 2x+1$

$3x-2 = 2x+1$ or $3x-2 = -(2x+1)$

$x = 3 \qquad 3x-2 = -2x-1$

$5x = 1$

$x = \frac{1}{5}$

Solution set: $\left\{\frac{1}{5}, 3\right\}$

18. $|1-2x| = x+5$

$1-2x = x+5$ or $1-2x = -(x+5)$

$-3x = 4 \qquad 1-2x = -x-5$

$x = -\frac{4}{3} \qquad 6 = x$

Solution set: $\left\{-\frac{4}{3}, 6\right\}$

19. $p = k + gt \Rightarrow p - k = gt \Rightarrow \frac{p-k}{t} = g$

20. $RK = 4 + 3K \Rightarrow RK - 3K = 4 \Rightarrow$
$K(R-3) = 4 \Rightarrow K = \frac{4}{R-3}$

21. $T = \frac{2B}{B-1} \Rightarrow TB - T = 2B \Rightarrow TB - 2B = T \Rightarrow$
$B(T-2) = T \Rightarrow B = \frac{T}{T-2}$

22. $S = \frac{a}{1-r} \Rightarrow S - Sr = a \Rightarrow -Sr = a - S \Rightarrow$
$r = -\frac{a-S}{S} = \frac{S-a}{S}$

23. $x^2 - 7x = 0 \Rightarrow x(x-7) = 0 \Rightarrow$
$x = 0$ or $x = 7$

24. $x^2 - 32x = 0 \Rightarrow x(x-32) = 0 \Rightarrow$
$x = 0$ or $x = 32$

25. $x^2 - 3x - 10 = 0 \Rightarrow (x-5)(x+2) = 0 \Rightarrow$
$x = 5$ or $x = -2$

26. $2x^2 - 9x - 18 = 0 \Rightarrow (2x+3)(x-6) = 0 \Rightarrow$
$x = -\frac{3}{2}$ or $x = 6$

27. $(x-1)^2 = 2x^2 + 3x - 5 \Rightarrow$
$x^2 - 2x + 1 = 2x^2 + 3x - 5 \Rightarrow$
$0 = x^2 + 5x - 6 \Rightarrow 0 = (x+6)(x-1) \Rightarrow$
$x = -6$ or $x = 1$

28. $(x+2)^2 = x(3x+2) \Rightarrow$
$x^2 + 4x + 4 = 3x^2 + 2x \Rightarrow$
$0 = 2x^2 - 2x - 4 \Rightarrow 0 = x^2 - x - 2 \Rightarrow$
$0 = (x-2)(x+1) \Rightarrow x = 2$ or $x = -1$

29. $\frac{x^2}{4} + x = \frac{5}{4} \Rightarrow x^2 + 4x = 5 \Rightarrow$
$x^2 + 4x - 5 = 0 \Rightarrow (x+5)(x-1) = 0 \Rightarrow$
$x = -5$ or $x = 1$

30. $\frac{x^2}{4} + \frac{7}{16} = x \Rightarrow 4x^2 + 7 = 16x \Rightarrow$
$4x^2 - 16x + 7 = 0 \Rightarrow (2x-1)(2x-7) = 0 \Rightarrow$
$x = \frac{1}{2}$ or $x = \frac{7}{2}$

31. $3x(x+1) = 2x + 2 \Rightarrow 3x^2 + 3x = 2x + 2 \Rightarrow$
$3x^2 + x - 2 = 0 \Rightarrow (3x-2)(x+1) = 0 \Rightarrow$
$x = \frac{2}{3}$ or $x = -1$

32. $x^2 - x = 3(5-x) \Rightarrow x^2 - x = 15 - 3x \Rightarrow$
$x^2 + 2x - 15 = 0 \Rightarrow (x+5)(x-3) = 0 \Rightarrow$
$x = -5$ or $x = 3$

33. Use the quadratic formula to solve
$x^2 - 3x - 1 = 0$:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{13}}{2} = \frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

34. Use the quadratic formula to solve
$x^2 + 6x + 2 = 0$:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(2)}}{2(1)} = \frac{-6 \pm \sqrt{28}}{2}$$
$$= \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$$

35. $2x^2 + x - 1 = 0 \Rightarrow (2x-1)(x+1) = 0 \Rightarrow$
$x = \frac{1}{2}$ or $x = -1$

36. Use the quadratic formula to solve $x^2+4x+1=0$:

$$x=\frac{-4\pm\sqrt{4^2-4(1)(1)}}{2(1)}=\frac{-4\pm\sqrt{12}}{2}=\frac{-4\pm2\sqrt{3}}{2}=-2\pm\sqrt{3}$$

37. First factor 3 from $3x^2-12x-24=0$ to get $x^2-4x-8=0$. Now use the quadratic formula:

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-8)}}{2(1)}=\frac{4\pm\sqrt{48}}{2}=\frac{4\pm4\sqrt{3}}{2}=2\pm2\sqrt{3}$$

38. Use the quadratic formula to solve $2x^2+4x-3=0$:

$$x=\frac{-4\pm\sqrt{4^2-4(2)(-3)}}{2(2)}=\frac{-4\pm\sqrt{40}}{4}=\frac{-4\pm2\sqrt{10}}{4}=-1\pm\frac{\sqrt{10}}{2}$$

39. Use the quadratic formula to solve $2x^2-x-2=0$:

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-2)}}{2(2)}=\frac{1\pm\sqrt{17}}{4}=\frac{1}{4}\pm\frac{\sqrt{17}}{4}$$

40. Use the quadratic formula to solve $3x^2-5x+1=0$:

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(1)}}{2(3)}=\frac{5\pm\sqrt{13}}{6}=\frac{5}{6}\pm\frac{\sqrt{13}}{6}$$

41. Use the quadratic formula to solve $x^2-x+1=0$:

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}=\frac{1\pm\sqrt{-3}}{2}=\frac{1\pm i\sqrt{3}}{2}=\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$$

42. Use the quadratic formula to solve $3x^2+4x+3=0$:

$$x=\frac{-4\pm\sqrt{4^2-4(3)(3)}}{2(3)}=\frac{-4\pm\sqrt{-20}}{6}=\frac{-4\pm2i\sqrt{5}}{6}=-\frac{2}{3}\pm\frac{\sqrt{5}}{3}i$$

43. Use the quadratic formula to solve $x^2-6x+13=0$:

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}=\frac{6\pm\sqrt{-16}}{2}=\frac{6\pm4i}{2}=3\pm2i$$

44. Use the quadratic formula to solve $x^2-8x+20=0$:

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(20)}}{2(1)}=\frac{8\pm\sqrt{-16}}{2}=\frac{8\pm4i}{2}=4\pm2i$$

45. Use the quadratic formula to solve $4x^2-8x+13=0$:

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(4)(13)}}{2(4)}=\frac{8\pm\sqrt{-144}}{8}=\frac{8\pm12i}{8}=1\pm\frac{3}{2}i$$

46. Use the quadratic formula to solve $3x^2-4x+2=0$:

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(3)(2)}}{2(3)}=\frac{4\pm\sqrt{-8}}{6}=\frac{4\pm2i\sqrt{2}}{6}=\frac{2}{3}\pm\frac{\sqrt{2}}{3}i$$

47. The discriminant is $(-11)^2-4(3)(6)=49>0$, so there are 2 real unequal roots.

48. The discriminant is $(-14)^2-4(1)(49)=0$, so there is 1 real root.

49. The discriminant is $2^2-4(5)(1)=-16<0$, so there are 2 unequal complex roots.

50. The discriminant is $(0)^2-4(9)(-25)=900$, so there are 2 real unequal roots.

51. $\sqrt{x^2-16}=0\Rightarrow x^2-16=0\Rightarrow$ $(x+4)(x-4)=16\Rightarrow x=\pm4$

52. $\sqrt{x+6} = x \Rightarrow x+6 = x^2 \Rightarrow$
$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow$
$x = 3$ or $x = -2$
We reject –2 because the square root is not negative. So the solution set is {3}.

53. $\sqrt{4-7x} = \sqrt{2}x \Rightarrow 4 - 7x = 2x^2 \Rightarrow$
$2x^2 + 7x - 4 = 0 \Rightarrow (2x-1)(x+4) = 0 \Rightarrow$
$x = \frac{1}{2}$ or $x = -4$
If $x = -4$, then the equation becomes $\sqrt{4-7(-4)} = \sqrt{2}(-4) \Rightarrow \sqrt{32} = -4\sqrt{2}$, which is not true, so we reject that root. The solution set is $\left\{\frac{1}{2}\right\}$.

54. $t - 2\sqrt{t} + 1 = 0$. Let $u = \sqrt{t}$, so the equation becomes $u^2 - 2u + 1 = 0 \Rightarrow (u-1)^2 = 0 \Rightarrow$ $u = 1$. Now solve for t. $1 = \sqrt{t} \Rightarrow 1 = t$.

55. $y - 2\sqrt{y} - 3 = 0$. Let $u = \sqrt{y}$, so the equation becomes $u^2 - 2u - 3 = 0 \Rightarrow$
$(u-3)(u+1) = 0 \Rightarrow u = 3$ or $u = -1$.
Now solve for y. $3 = \sqrt{y} \Rightarrow 9 = y$ or $-1 = \sqrt{y}$ (reject this). The solution set is {9}.

56.
$$\begin{aligned}
\sqrt{3x+4} - \sqrt{x-3} &= 3\\
\sqrt{3x+4} &= 3 + \sqrt{x-3}\\
3x + 4 &= 9 + 6\sqrt{x-3} + x - 3\\
3x + 4 &= 6 + x + 6\sqrt{x-3}\\
2x - 2 &= 6\sqrt{x-3}\\
(2x-2)^2 &= (6\sqrt{x-3})^2\\
4x^2 - 8x + 4 &= 36(x-3)\\
4x^2 - 8x + 4 &= 36x - 108\\
4x^2 - 44x + 112 &= 0\\
4(x^2 - 11x + 28) &= 0\\
(x-7)(x-4) &= 0 \Rightarrow x = 7 \text{ or } x = 4
\end{aligned}$$
The solution set is $\{4, 7\}$.

57. $\sqrt{x} - 1 = \sqrt{5 + \sqrt{x}}$. Let $u = \sqrt{x}$. The equation becomes $u - 1 = \sqrt{5+u} \Rightarrow (u-1)^2 = 5 + u \Rightarrow$
$u^2 - 2u + 1 = 5 + u \Rightarrow u^2 - 3u - 4 \Rightarrow$
$(u-4)(u+1) = 0 \Rightarrow u = 4$ or $u = -1$.
Now solve for x.
$4 = \sqrt{x} \Rightarrow 16 = x$ or $-1 = \sqrt{x}$
(not possible). The solution set is {16}.

58. $\dfrac{1}{(1-x)^2} - \dfrac{7}{1-x} = -10$
$$\begin{aligned}
1 - 7(1-x) &= -10(1-x)^2\\
-6 + 7x &= -10(1 - 2x + x^2)\\
-6 + 7x &= -10 + 20x - 10x^2
\end{aligned}$$
$10x^2 - 13x + 4 = 0$
$(5x-4)(2x-1) = 0 \Rightarrow x = \frac{4}{5}$ or $x = \frac{1}{2}$
The solution set is $\{1/2, 4/5\}$.

59. $(7x+5)^2 + 2(7x+5) - 15 = 0$. Let $u = 7x + 5$. Then the equation becomes
$u^2 + 2u - 15 = 0 \Rightarrow$
$(u+5)(u-3) = 0 \Rightarrow u = -5$ or $u = 3$.
Now solve for x: $-5 = 7x + 5 \Rightarrow x = -\frac{10}{7}$ or
$3 = 7x + 5 \Rightarrow x = -\frac{2}{7}$.
The solution set is $\{-10/7, -2/7\}$.

60. $(x^2-1)^2 - 11(x^2-1) + 24 = 0$.
Let $u = x^2 - 1$. Then the equation becomes
$u^2 - 11u + 24 = 0 \Rightarrow (u-8)(u-3) = 0 \Rightarrow$
$u = 8$ or $u = 3$. Now solve for x:
$8 = x^2 - 1 \Rightarrow 9 = x^2 \Rightarrow \pm 3 = x$ or
$3 = x^2 - 1 \Rightarrow 4 = x^2 \Rightarrow \pm 2 = x$. The solution set is $\{-3, -2, 2, 3\}$.

61. $x^{2/3} + 3x^{1/3} - 4 = 0$. Let $u = x^{1/3}$. So the equation becomes $u^2 + 3u - 4 = 0 \Rightarrow$
$(u+4)(u-1) = 0 \Rightarrow u = -4$ or $u = 1$. Now solve for u: $-4 = x^{1/3} \Rightarrow -64 = x$ or
$1 = x^{1/3} \Rightarrow 1 = x$. The solution set is $\{-64, 1\}$.

62. $x^{-2/3} + x^{-1/3} - 6 = 0$
Let $u = x^{-1/3}$. Substituting, the equation becomes
$u^2 + u - 6 = 0 \Rightarrow (u+3)(u-2) = 0 \Rightarrow$
$u = -3$ or $u = 2$.
Now solve for u:
$-3 = x^{-1/3} \Rightarrow -\frac{1}{3} = x^{1/3} \Rightarrow x = -\frac{1}{27}$ or
$2 = x^{-1/3} \Rightarrow \frac{1}{2} = x^{1/3} \Rightarrow x = \frac{1}{8}$
Solution set: $\left\{-\frac{1}{27}, \frac{1}{8}\right\}$.

63. $\left(\sqrt{t}+5\right)^2-9\left(\sqrt{t}+5\right)+20=0.$
Let $u=\sqrt{t}+5$, so the equation becomes
$u^2-9u+20=0 \Rightarrow (u-5)(u-4)=0 \Rightarrow$
$u=5$ or $u=4$. Now solve for t. $5=\sqrt{t}+5 \Rightarrow$
$0=t$ or $4=\sqrt{t}+5 \Rightarrow -1=\sqrt{t}$. (reject this)
The solution set is $\{0\}$.

64. $3\left(\frac{y-1}{6}\right)^2-7\left(\frac{y-1}{6}\right)=0$. Let $u=\frac{y-1}{6}$. So the equation becomes $3u^2-7u=0 \Rightarrow$
$u(3u-7)=0 \Rightarrow u=0$ or $u=\frac{7}{3}$. Now solve for y: $0=\frac{y-1}{6} \Rightarrow y=1$ or $\frac{7}{3}=\frac{y-1}{6} \Rightarrow$
$42=3y-3 \Rightarrow 15=y$.
The solution set is $\{1, 15\}$.

65. $4x^4-37x^2+9=0$. Let $u=x^2$. So the equation becomes $4u^2-37u+9=0 \Rightarrow$
$(4u-1)(u-9)=0 \Rightarrow u=\frac{1}{4}$ or $u=9$. Now solve for x: $\frac{1}{4}=x^2 \Rightarrow x=\pm\frac{1}{2}$ or $9=x^2 \Rightarrow$
$x=\pm 3$. The solution set is $\left\{-3,-\frac{1}{2},\frac{1}{2},3\right\}$.

66. $\frac{1}{x}+\frac{1}{x-1}=\frac{5}{6} \Rightarrow 6(x-1)+6x=5x(x-1) \Rightarrow$
$12x-6=5x^2-5x \Rightarrow 5x^2-17x+6=0 \Rightarrow$
$(5x-2)(x-3)=0 \Rightarrow x=2/5$ or $x=3$
The solution set is $\left\{\frac{2}{5},3\right\}$.

67.
$$\frac{2x+1}{2x-1}=\frac{x-1}{x+1}$$
$$(2x+1)(x+1)=(2x-1)(x-1)$$
$$2x^2+3x+1=2x^2-3x+1$$
$$6x=0 \Rightarrow x=0$$
The solution set is $\{0\}$.

68.
$$6-\frac{2}{x}=\frac{4}{x-1}$$
$$6x(x-1)-2(x-1)=4x$$
$$6x^2-6x-2x+2=4x$$
$$6x^2-12x+2=0$$
Solve for x using the quadratic formula.
$$x=\frac{-(-12)\pm\sqrt{(-12)^2-4(6)(2)}}{2(6)}=\frac{12\pm\sqrt{96}}{12}$$
$$=\frac{12\pm 4\sqrt{6}}{12}=1+\frac{\sqrt{6}}{3} \text{ or } 1-\frac{\sqrt{6}}{3}$$
The solution set is $\left\{1-\frac{\sqrt{6}}{3},1+\frac{\sqrt{6}}{3}\right\}$.

69. $\left(\frac{7x}{x+1}\right)^2-3\left(\frac{7x}{x+1}\right)=18$. Let $u=\frac{7x}{x+1}$. So the equation becomes $u^2-3u=18 \Rightarrow$
$u^2-3u-18=0 \Rightarrow (u-6)(u+3)=0 \Rightarrow$
$u=6$ or $u=-3$. Now solve for x:
$-3=\frac{7x}{x+1} \Rightarrow -3x-3=7x \Rightarrow x=-\frac{3}{10}$ or
$6=\frac{7x}{x+1} \Rightarrow 6x+6=7x \Rightarrow x=6.$
The solution set is $\left\{-\frac{3}{10},6\right\}$.

70. $\left(\frac{4x^2-3}{x}\right)^2=1 \Rightarrow \frac{4x^2-3}{x}=1 \Rightarrow$
$4x^2-3=x \Rightarrow 4x^2-x-3=0 \Rightarrow$
$(4x+3)(x-1)=0 \Rightarrow x=-\frac{3}{4}$ or $x=1$ or
$\frac{4x^2-3}{x}=-1 \Rightarrow 4x^2-3=-x \Rightarrow$
$4x^2+x-3=0 \Rightarrow (4x-3)(x+1)=0 \Rightarrow$
$x=\frac{3}{4}$ or $x=-1$.
The solution set is $\left\{-1,-\frac{3}{4},\frac{3}{4},1\right\}$.

71. $x^2+2yx-3y^2=0$
$\left(x+3y\right)\left(x-y\right)=0$
$x+3y=0 \Rightarrow x=-3y$
$x-y=0 \Rightarrow x=y$
Solution set: $\{-3y, y\}$

72. $x^2+\left(y-2z\right)x-2yz=0$
$x^2+xy-2xz-2yz=0$
$x\left(x+y\right)-2z\left(x+y\right)=0$
$\left(x-2z\right)\left(x+y\right)=0$
$x-2z=0 \Rightarrow x=2z$
$x+y=0 \Rightarrow x=-y$
Solution set: $\{-y, 2z\}$

73. $x^2 + (3-2y)x + y^2 - 3y + 2 = 0$
Use the quadratic formula with $a = 1$, $b = 3 - 2y$ and $c = y^2 - 3y + 2$.

$$x = \frac{-(3-2y) \pm \sqrt{(3-2y)^2 - 4(1)(y^2 - 3y + 2)}}{2(1)}$$

$$x = \frac{2y - 3 \pm \sqrt{4y^2 - 12y + 9 - (4y^2 - 12y + 8)}}{2}$$

$$= \frac{(2y-3) \pm 1}{2} = \frac{2y-4}{2} = y - 2 \text{ or}$$

$$\frac{2y-2}{2} = y - 1$$

Solution set: $\{y - 2, y - 1\}$

74. $x^2 + (1-2y)x + y^2 - y - 2 = 0$
Use the quadratic formula with $a = 1$, $b = 1 - 2y$ and $c = y^2 - y - 2$.

$$x = \frac{-(1-2y) \pm \sqrt{(1-2y)^2 - 4(1)(y^2 - y - 2)}}{2(1)}$$

$$= \frac{2y - 1 \pm \sqrt{4y^2 - 4y + 1 - (4y^2 - 4y - 8)}}{2(1)}$$

$$= \frac{(2y-1) \pm \sqrt{9}}{2} = \frac{(2y-1) \pm 3}{2}$$

$$= \frac{2y-4}{2} = y - 2 \text{ or } \frac{2y+2}{2} = y + 1$$

Solution set: $\{y - 2, y + 1\}$

75. $x + 5 < 3 \Rightarrow x < -2$
The solution set is $(-\infty, -2)$.

76. $2x + 1 < 9 \Rightarrow 2x < 8 \Rightarrow x < 4$
The solution set is $(-\infty, 4)$.

77. $3(x-3) \le 8 \Rightarrow 3x - 9 \le 8 \Rightarrow 3x \le 17 \Rightarrow x \le \frac{17}{3}$
The solution set is $\left(-\infty, \frac{17}{3}\right]$.

78. $x + 5 \le 19 + 3x \Rightarrow -14 \le 2x \Rightarrow -7 \le x$
The solution set is $[-7, \infty)$.

79. $x + 2 \ge \frac{2}{3}x - 2x \Rightarrow 3x + 6 \ge 2x - 6x \Rightarrow$
$3x + 6 \ge -4x \Rightarrow 6 \ge -7x \Rightarrow -\frac{6}{7} \le x$
The solution set is $\left[-\frac{6}{7}, \infty\right)$.

80. $2x + 1 \ge \frac{5x-6}{3} \Rightarrow 6x + 3 \ge 5x - 6 \Rightarrow x \ge -9$
The solution set is $[-9, \infty)$.

81. $\frac{1}{6} > \frac{4-3x}{3} \Rightarrow 1 > 2(4-3x) \Rightarrow 1 > 8 - 6x \Rightarrow$
$-7 > -6x \Rightarrow \frac{7}{6} < x$
Solution set: $\left(\frac{7}{6}, \infty\right)$

82. $\frac{2}{5} > \frac{3-2x}{2} \Rightarrow 4 > 5(3-2x) \Rightarrow 4 > 15 - 10x \Rightarrow$
$-11 > -10x \Rightarrow \frac{11}{10} < x$
Solution set: $\left(\frac{11}{10}, \infty\right)$

83. $\frac{x-3}{3} - 2 \le \frac{x}{6} + \frac{1}{2} \Rightarrow 2(x-3) - 2(6) \le x + 3 \Rightarrow$
$2x - 6 - 12 \le x + 3 \Rightarrow 2x - 18 \le x + 3 \Rightarrow x \le 21$
Solution set: $(-\infty, 21]$

84. $\frac{x+1}{2} - 3 \le \frac{x}{4} + \frac{1}{2} \Rightarrow 2(x+1) - 3(4) \le x + 2 \Rightarrow$
$2x + 2 - 12 \le x + 2 \Rightarrow 2x - 10 \le x + 2 \Rightarrow$
$x \le 12$
Solution set: $(-\infty, 12]$

85.
$$\frac{3-2x}{4} + 1 > \frac{x-5}{3}$$
$$3(3-2x) + 12 > 4(x-5)$$
$$9 - 6x + 12 > 4x - 20$$
$$-6x + 21 > 4x - 20$$
$$-10x > -41 \Rightarrow x < \frac{41}{10}$$
Solution set: $\left(-\infty, \frac{41}{10}\right)$

86.
$$\frac{5-3x}{5} - 2 > \frac{2-x}{3}$$
$$3(5-3x) - 2(15) > 5(2-x)$$
$$15 - 9x - 30 > 10 - 5x$$
$$-9x - 15 > 10 - 5x$$
$$-4x > 25 \Rightarrow x < -\frac{25}{4}$$
Solution set: $\left(-\infty, -\frac{25}{4}\right)$

87. $3x-1<2$ or $11-2x<5$
$3x<3$ or $-2x<-6$
$x<1$ or $x>3$
Solution set: $(-\infty, 1)\cup(3, \infty)$

88. $3-2x>5$ or $15-3x<6$
$-2x>2$ or $-3x<-9$
$x<-1$ or $x>3$
Solution set: $(-\infty, -1)\cup(3, \infty)$

89. $4x-5<7$ and $7-3x<1$
$4x<12$ and $-3x<-6$
$x<3$ and $x>2$
Solution set: $(2, 3)$

90. $2x-1<3$ and $4-3x>1$
$2x<4$ and $-3x>-3$
$x<2$ and $x<1$
Solution set: $(-\infty, 1)$

91. $-3\le 2x+1<7 \Rightarrow -4\le 2x<6 \Rightarrow -2\le x<3$
Solution set: $[-2, 3)$

92. $\frac{1}{6}\le\frac{3x-4}{3}\le 4\Rightarrow 1\le 2(3x-4)\le 24\Rightarrow$
$1\le 6x-8\le 24\Rightarrow 9\le 6x\le 32\Rightarrow$
$\frac{3}{2}\le x\le\frac{16}{3}$
Solution set: $\left[\frac{3}{2}, \frac{16}{3}\right]$

93. $-3<3-2x\le 97\Rightarrow -6<-2x\le 94\Rightarrow$
$3>x\ge -47\Rightarrow -47\le x<3$
Solution set: $[-47, 3)$

94. $-\frac{1}{2}<\frac{4-3x}{3}\le\frac{1}{2}\Rightarrow -3<2(4-3x)\le 3\Rightarrow$
$-3<8-6x\le 3\Rightarrow -11<-6x\le -5$
$\frac{11}{6}>x\ge\frac{5}{6}\Rightarrow\frac{5}{6}\le x<\frac{11}{6}$
Solution set: $\left[\frac{5}{6}, \frac{11}{6}\right)$

95. $x^2+x-6\ge 0\Rightarrow (x+3)(x-2)\ge 0$
Solve the associated equation:
$(x+3)(x-2)=0\Rightarrow x=-3$ or $x=2$.
So, the intervals are $(-\infty, -3]$, $[-3, 2]$, and
and $[2,\infty)$.

Interval	Test point	Value of x^2+x-6	Result
$(-\infty, -3]$	–4	6	+
$[-3, 2]$	0	–6	–
$[2,\infty)$	3	6	+

The solution set is $(-\infty, -3]\cup[2,\infty)$.

96. $x^3-9x\le 0\Rightarrow x(x-3)(x+3)\le 0$
Solve the associated equation:
$x(x-3)(x+3)=0\Rightarrow x=0,\ x=3$ or $x=-3$.
So, the intervals are $(-\infty, -3]$, $[-3, 0]$,
$[0, 3]$, and $[3,\infty)$.

Interval	Test point	Value of x^3-9x	Result
$(-\infty, -3]$	–4	–28	–
$[-3, 0]$	–2	10	+
$[0, 3]$	1	–8	–
$[3,\infty)$	4	28	+

The solution set is $(-\infty, -3]\cup[0, 3]$.

97. $\frac{(x-1)(x+3)}{(x+2)(x+5)}\ge 0$
Set the numerator and denominator equal to zero and solve for x.
$(x-1)(x+3)=0\Rightarrow x=1, -3$
$(x+2)(x+5)=0\Rightarrow x=-2, -5$
The intervals are $(-\infty, -5)$, $(-5, -3]$,
$[-3, -2)$, $(-2, 1]$, and $(1, \infty)$.

Interval	Test point	Value of $\frac{(x-1)(x+3)}{(x+2)(x+5)}$	Result
$(-\infty, -5)$	–6	$\frac{21}{4}$	+
$(-5, -3]$	–4	$-\frac{5}{2}$	–
$[-3, -2)$	$-\frac{5}{2}$	$\frac{7}{5}$	+

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Interval	Test point	Value of $\frac{(x-1)(x+3)}{(x+2)(x+5)}$	Result
$(-2, 1]$	0	$-\frac{3}{10}$	–
$(1, \infty)$	2	$\frac{5}{28}$	+

The solution set is $(-\infty, -5)\cup[-3, -2)\cup(1, \infty)$.

98. $\frac{x^2+4x+3}{x^2+6x+8} \le 0.$
Set the numerator and denominator equal to zero and solve for *x*.
$x^2+4x+3=0 \Rightarrow (x+1)(x+3)=0 \Rightarrow$
$x=-1,\ x=-3$
$x^2+6x+8=0 \Rightarrow (x+2)(x+4)=0 \Rightarrow$
$x=-2,\ x=-4$
The intervals are $(-\infty, -4)$, $(-4, -3]$, $[-3, -2)$, $(-2, -1]$, and $[-1, \infty)$.

Interval	Test point	Value of $\frac{x^2+4x+3}{x^2+6x+8}$	Result
$(-\infty, -4)$	–5	$\frac{8}{3}$	+
$(-4, -3]$	$-\frac{7}{2}$	$-\frac{5}{3}$	–
$[-3, -2)$	$-\frac{5}{2}$	1	+
$(-2, -1]$	$-\frac{3}{2}$	$-\frac{3}{5}$	–
$[-1, \infty)$	0	$\frac{3}{8}$	+

The solution set is $(-4, -3]\cup(-2, -1]$.

99. $|3x+2| \le 7 \Rightarrow -7 \le 3x+2 \le 7 \Rightarrow$
$-9 \le 3x \le 5 \Rightarrow -3 \le x \le \frac{5}{3}$
The solution set is $\left[-3, \frac{5}{3}\right]$.

100. $|x-4| \ge 2 \Rightarrow x-4 \le -2$ or $x-4 \ge 2 \Rightarrow$
$x \le 2$ or $x \ge 6$
The solution set is $(-\infty, 2]\cup[6, \infty)$.

101. $4|x-2|+8>12 \Rightarrow 4|x-2|>4 \Rightarrow$
$|x-2|>1 \Rightarrow x-2<-1$ or $x-2>1 \Rightarrow$
$x<1$ or $x>3$
The solution set is $(-\infty, 1)\cup(3, \infty)$.

102. $3|x-1|+4<10 \Rightarrow 3|x-1|<6 \Rightarrow$
$|x-1|<2 \Rightarrow -2<x-1<2 \Rightarrow -1<x<3$
The solution set is $(-1, 3)$.

103. $\left|\frac{4-x}{5}\right| \ge 1 \Rightarrow \frac{4-x}{5} \le -1$ or $\frac{4-x}{5} \ge 1 \Rightarrow$
$4-x \le -5 \Rightarrow x \ge 9$ or $4-x \ge 5 \Rightarrow x \le -1$
The solution set is $(-\infty, -1]\cup[9, \infty)$.

104. $\left|\frac{1-x}{6}\right| < 1 \Rightarrow -1 < \frac{1-x}{6} < 1 \Rightarrow -6 < 1-x < 6 \Rightarrow$
$7 > x > -5$
The solution set is $(-5, 7)$.

105. $\left|\frac{x-1}{x+2}\right| \le 3 \Rightarrow -3 \le \frac{x-1}{x+2} \le 3$

$0 < \frac{x-1}{x+2}+3$	$\frac{x-1}{x+2}-3<0$
$0 < \frac{x-1+3(x+2)}{x+2}$	$\frac{x-1-3(x+2)}{x+2}<0$
$0 < \frac{4x+5}{x+2}$	$\frac{-2x-7}{x+2}<0$

$4x+5=0 \Rightarrow x=-\frac{5}{4};\ x+2=0 \Rightarrow x=-2$

Interval	Test point	Value of $\frac{4x+5}{x+2}$	Result
$(-\infty, -2)$	–4	$\frac{11}{2}$	+
$\left(-2, -\frac{5}{4}\right]$	$-\frac{3}{2}$	–2	–
$\left[-\frac{5}{4}, \infty\right)$	0	$\frac{5}{2}$	+

Note that the expression is undefined for $x=-2$. The solution set is for this part of the original inequality is $(-\infty, -2)\cup\left[-\frac{5}{4}, \infty\right)$.

$-2x-7=0 \Rightarrow x=-\frac{7}{2};\ x+2=0 \Rightarrow x=-2$

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Interval	Test point	Value of $\frac{-2x-7}{x+2}$	Result
$\left(-\infty, -\frac{7}{2}\right]$	−4	$-\frac{1}{2}$	−
$\left[-\frac{7}{2}, -2\right)$	−3	1	+
$(-2, \infty)$	0	$-\frac{7}{2}$	−

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $\left(-\infty, -\frac{7}{2}\right] \cup (-2, \infty)$.

Both inequalities are true on $\left(-\infty, -\frac{7}{2}\right]$ and $\left[-\frac{5}{4}, \infty\right)$, so the solution set is $\left(-\infty, -\frac{7}{2}\right] \cup \left[-\frac{5}{4}, \infty\right)$.

106. $\left|\frac{x+3}{x-5}\right| < 4 \Rightarrow -4 < \frac{x+3}{x-5} < 4$

$$0 < \frac{x+3}{x-5} + 4 \qquad \frac{x+3}{x-5} - 4 < 0$$
$$0 < \frac{x+3+4(x-5)}{x-5} \qquad \frac{x+3-4(x-5)}{x-5} < 0$$
$$0 < \frac{5x-17}{x-5} \qquad \frac{-3x+23}{x-5} < 0$$

$5x - 17 = 0 \Rightarrow x = \frac{17}{5}$; $x - 5 = 0 \Rightarrow x = 5$

Interval	Test point	Value of $\frac{5x-17}{x-5}$	Result
$\left(-\infty, \frac{17}{5}\right)$	0	$\frac{17}{5}$	+
$\left(\frac{17}{5}, 5\right)$	4	−3	−
$(5, \infty)$	6	13	+

Note that the expression is undefined for $x = 5$. The solution set is for this part of the original inequality is $\left(-\infty, \frac{17}{5}\right) \cup (5, \infty)$.

$-3x + 23 = 0 \Rightarrow x = \frac{23}{3}$; $x - 5 = 0 \Rightarrow x = 5$

Interval	Test point	Value of $\frac{-3x+23}{x-5}$	Result
$(-\infty, 5)$	0	$-\frac{23}{5}$	−
$\left(5, \frac{23}{3}\right)$	6	5	+
$\left(\frac{23}{3}, \infty\right)$	10	$-\frac{7}{5}$	−

Note that the expression is undefined for $x = 5$. The solution set is for this part of the original inequality is $(-\infty, 5) \cup \left(\frac{23}{3}, \infty\right)$.

Both inequalities are true on $\left(-\infty, \frac{17}{5}\right)$ and $\left(\frac{23}{3}, \infty\right)$, so the solution set is $\left(-\infty, \frac{17}{5}\right) \cup \left(\frac{23}{3}, \infty\right)$.

107. $\left|\frac{2x-3}{x+2}\right| \ge 2 \Rightarrow \frac{2x-3}{x+2} \le -2$ or $\frac{2x-3}{x+2} \ge 2$

$$\frac{2x-3}{x+2} \le -2 \qquad \frac{2x-3}{x+2} \ge 2$$
$$\frac{2x-3}{x+2} + 2 \le 0 \qquad \frac{2x-3}{x+2} - 2 \ge 0$$
$$\frac{2x-3+2(x+2)}{x+2} \le 0 \qquad \frac{2x-3-2(x+2)}{x+2} \ge 0$$
$$\frac{4x+1}{x+2} \le 0 \qquad \frac{-7}{x+2} \ge 0$$

$4x + 1 = 0 \Rightarrow x = -\frac{1}{4}$; $x + 2 = 0 \Rightarrow x = -2$

Interval	Test point	Value of $\frac{4x+1}{x+2}$	Result
$(-\infty, -2)$	−3	11	+
$\left(-2, -\frac{1}{4}\right]$	−1	−3	−
$\left[-\frac{1}{4}, \infty\right)$	0	$\frac{1}{2}$	+

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $\left(-2, -\frac{1}{4}\right]$.

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Interval	Test point	Value of $\frac{-7}{x+2}$	Result
$(-\infty, -2)$	−3	$\frac{7}{5}$	+
$(-2, \infty)$	0	$-\frac{7}{2}$	−

Note that the expression is undefined for $x = -2$. The solution set is for this part of the original inequality is $(-\infty, -2)$. Since the original inequality is an "or" inequality, the solution set of the original inequality is the union of the two solution sets.

The solution set is $(-\infty, -2) \cup \left(-2, -\frac{1}{4}\right]$.

108. $\left|\frac{x+1}{2x-5}\right| > 3 \Rightarrow \frac{x+1}{2x-5} < -3$ or $\frac{x+1}{2x-5} > 3$

$$\frac{x+1}{2x-5} < -3 \qquad \frac{x+1}{2x-5} > 3$$
$$\frac{x+1}{2x-5} + 3 < 0 \qquad \frac{x+1}{2x-5} - 3 > 0$$
$$\frac{x+1+3(2x-5)}{2x-5} < 0 \qquad \frac{x+1-3(2x-5)}{2x-5} > 0$$
$$\frac{7x-14}{2x-5} < 0 \qquad \frac{-5x+16}{2x-5} > 0$$

$7x - 14 = 0 \Rightarrow x = 2;\ 2x - 5 = 0 \Rightarrow x = \frac{5}{2}$

Interval	Test point	Value of $\frac{7x-14}{2x-5}$	Result
$(-\infty, 2)$	0	$\frac{14}{5}$	+
$\left(2, \frac{5}{2}\right)$	$\frac{9}{4}$	$-\frac{7}{2}$	−
$\left(\frac{5}{2}, \infty\right)$	3	7	+

Note that the expression is undefined for $x = \frac{5}{2}$. The solution set is for this part of the original inequality is $\left(2, \frac{5}{2}\right)$.

$-5x + 16 = 0 \Rightarrow x = \frac{16}{5};\ 2x - 5 = 0 \Rightarrow x = \frac{5}{2}$

Interval	Test point	Value of $\frac{-5x+16}{2x-5}$	Result
$\left(-\infty, \frac{5}{2}\right)$	0	$-\frac{16}{5}$	−
$\left(\frac{5}{2}, \frac{16}{5}\right)$	3	1	+
$\left(\frac{16}{5}, \infty\right)$	4	$-\frac{4}{3}$	−

Note that the expression is undefined for $x = \frac{5}{2}$. The solution set is for this part of the original inequality is $\left(\frac{5}{2}, \frac{16}{5}\right)$. Since the original inequality is an "or" inequality, the solution set of the original inequality is the union of the two solution sets.

The solution set is $\left(2, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \frac{16}{5}\right)$.

Applying the Concepts

109. $C = 2\pi r \Rightarrow 22 = 2\pi r \Rightarrow \frac{11}{\pi} = r$

The radius is $11/\pi$ cm.

110. $P = 2l + 2w \Rightarrow 18 = 2(5) + 2w \Rightarrow 4 = w$. The width of the rectangle is 4 inches.

111. $A = \frac{1}{2}h(b_1 + b_2) \Rightarrow 32 = \frac{1}{2}(8)(5 + b_2) \Rightarrow$
$32 = 4(5 + b_2) \Rightarrow 8 = 5 + b_2 \Rightarrow 3 = b_2$
The other base is 3 meters.

112. $I = prt \Rightarrow 354.20 = 4(.07)p \Rightarrow$
$354.2 = 0.28p \Rightarrow 1265.$
\$1265 must be deposited.

113. $V = lwh \Rightarrow 4212 = 27(12)h \Rightarrow 13 = h$
The box is 13 cm high.

114. Let l = the current liabilities. Then
$2.7 = \frac{256,500}{l} \Rightarrow l = \frac{256,500}{2.7} \Rightarrow l = 95,000$
The current liabilities are \$95,000.

115. $V = \pi r^2 h \Rightarrow 8750\pi = 5^2 \pi h \Rightarrow 350 = h$
The height is 350 cm.

116. Let x = the selling price. Then
$x - 0.15x = 2210 \Rightarrow 0.85x = 2210 \Rightarrow$
$x = 2600.$ The selling price is \$2600.

117. Let x = measure or the base angle.
Then 40 + 2x = the measure of the third angle. So,
$x+x+40+2x=180 \Rightarrow 4x+40=180 \Rightarrow$
$x=35.$
The three angles are 35°, 35°, and 110°.

118. Let x = distance traveled across flat terrain. Then 600 – x = distance traveled across hilly terrain. The time for each is 6 hours. The rate across the flat terrain is $x/6$ and the rate over hilly terrain is $(600-x)/6$. So, we have
$\frac{600-x}{6}=\frac{x}{6}-20 \Rightarrow 600-x=x-120 \Rightarrow$
$x=360.$ The distance traveled over flat terrain is 360 miles at 60 mph, and the distance traveled over hilly terrain is 240 miles at 40 mph.

119. Let x = amount invested at 6%.
Then 30,000 – x = amount invested at 8%. So, we have $0.06x+0.08(30,000-x)=2160 \Rightarrow$
$-0.02x+2400=2160 \Rightarrow -0.02x=-240 \Rightarrow$
$x=12,000$. \$12,000 was invested at 6%, and \$18,000 was invested at 8%.

120. Let x = the monthly note for the 2 year lease. Then 250 + x = the monthly note for the 1 1/2 year lease. The total income for the 2 year lease is 24x. The total income for the 1 1/2 year lease is 18(250 + x). So, we have
$24x+18(250+x)=21,300 \Rightarrow$
$24x+4500+18x=21,300 \Rightarrow$
$42x=16,800 \Rightarrow x=400.$
The note for the 2 year lease is \$400, and the note for the 1 1/2 year lease is \$650.

121. Let x = amount of 4 1/2% solution.
Then 10 – x = the amount of 12% solution. So, we have $0.045x+0.12(10-x)=0.06(10) \Rightarrow$
$-0.075x+1.2=0.6 \Rightarrow -0.075x=-0.6 \Rightarrow$
$x=8$. There are 8 liters of the 4 1/2% solution and 2 liters of the 12% solution.

122. Let x = the speed of the first car.
Then 5 + x = the speed of the second car. So, we have
$3x+3(x+5)=495 \Rightarrow 6x+15=495 \Rightarrow$
$6x=480 \Rightarrow x=80$. The first car traveled at 80 kph and the second car traveled at 85 kph.

123. Let x = the number of people at the party. Each person shook x – 1 hands. So, there are $x(x - 1)$ handshakes. However, each handshake is counted twice (if A shakes hands with B, that is the same as B shaking hands with A).
So, we have $\frac{x(x-1)}{2}=28 \Rightarrow x^2-x=56 \Rightarrow$
$x^2-x-56=0 \Rightarrow (x-8)(x+7)=0 \Rightarrow$
$x=8$ or $x=-7$. We reject the negative answer. There were 8 people at the party.

124. Let x = the first number. Then x – 7 = the second number.
$x(x-7)=408 \Rightarrow x^2-7x-408=0 \Rightarrow$
$(x-24)(x+17)=0 \Rightarrow x=24$ or $x=-17$
Reject −17 since the numbers are given to be positive. Thus, the numbers are 24 and 17.

125. Let x = the number of shares that Lavina bought. She spent \$18,040 for the stock, or $18,040/x$ per share.
She sold x – 20 shares at $\frac{18,040}{x}+18$ per share for a total of \$20,000. Therefore,
$$(x-20)\left(\frac{18,040}{x}+18\right)=20,000$$
$$\frac{18,040+18x}{x}=\frac{20,000}{x-20}$$
$$(x-20)(18,040+18x)=20,000x$$
$$18x^2+17,680x-360,800=20,000x$$
$$18x^2-2320x-360,800=0$$
$$2(9x+820)(x-220)=0$$
$9x+820=0 \Rightarrow x=-\frac{820}{9}$ (reject this)
$x-220=0 \Rightarrow x=220$
Lavina bought 220 shares of stock.

126. Let x = Joann's original speed.
Then x + 10 = her new speed. So,

	Rate	Distance	Time
First part of trip	x	15	$\frac{15}{x}$
Second part of trip	$x + 10$	20	$\frac{20}{x+10}$

The total time is 1 hour, so
$$\frac{15}{x}+\frac{20}{x+10}=1$$
$$15(x+10)+20x=x(x+10)$$
$$15x+150+20x=x^2+10x$$
$$x^2-25x-150=0$$
$(x+5)(x-30)=0 \Rightarrow x=-5$ or $x=30$
Reject the negative solution. Therefore, Joann's original speed was 30 miles per hour.

127. Let x = the number of horses in the herd. Then $x/4 =$ the number of horses in the forest, and $2\sqrt{x} =$ the number of horses in the mountains.

$\frac{x}{4} + 2\sqrt{x} + 15 = x \Rightarrow x + 8\sqrt{x} + 60 = 4x \Rightarrow$

$8\sqrt{x} = 3x - 60 \Rightarrow 64x = (3x - 60)^2 \Rightarrow$

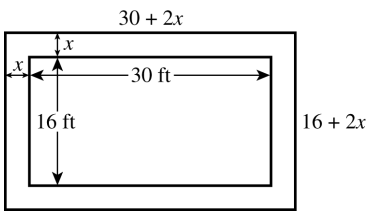
$64x = 9x^2 - 360x + 3600 \Rightarrow$

$9x^2 - 424x + 3600 = 0 \Rightarrow$

$(x - 36)(9x - 100) = 0 \Rightarrow x = 36$ or $x = 100/9$

Reject the fractional solution. There were 36 horses in the herd.

128. Let x = the width of the path.



$$(30 + 2x)(16 + 2x) - (30)(16) = 312$$
$$480 + 92x + 4x^2 - 480 = 312$$
$$4x^2 + 92x - 312 = 0$$
$$4(x - 3)(x + 26) = 0 \Rightarrow x = 3 \text{ or}$$
$$x = -26 \text{ (reject this)}$$

The path is 3 feet wide.

129. Let x = the original number of members going on the trip. Then $x + 4$ = the final number of members going on the trip. The cost per member for the original number going on the trip was $\frac{324}{x}$, and the cost for the final number going on the trip is $\frac{324}{x} - 0.9$. Therefore,

$$\frac{324}{x+4} = \frac{324}{x} - 0.9$$
$$324x = 324(x + 4) - 0.9x(x + 4)$$
$$324x = 324x + 1296 - 0.9x^2 - 3.6x$$
$$0 = -0.9x^2 - 3.6x + 1296$$
$$0 = x^2 + 4x - 1440$$
$$0 = (x - 36)(x + 40) \Rightarrow x = 36 \text{ or}$$

$x = -40$ (reject this). Thus, 36 people originally signed up for the trip, and 40 people went on the trips.

Chapter 1 Practice Test A

1. $5x - 9 = 3x - 5 \Rightarrow 2x = 4 \Rightarrow x = 2$

2. $\frac{7}{24} = \frac{x}{8} + \frac{1}{6} \Rightarrow 7 = 3x + 4 \Rightarrow x = 1$

3. $\frac{1}{x-2} - 5 = \frac{1}{x+2} \Rightarrow \frac{1 - 5x + 10}{x-2} = \frac{1}{x+2} \Rightarrow$

$\frac{-5x+11}{x-2} = \frac{1}{x+2} \Rightarrow$

$(-5x + 11)(x + 2) = x - 2$

$-5x^2 + x + 22 = x - 2$

$-5x^2 + 24 = 0 \Rightarrow -5x^2 = -24 \Rightarrow$

$x^2 = \frac{24}{5} \Rightarrow x = \pm\frac{2\sqrt{6}}{\sqrt{5}} = \pm\frac{2\sqrt{30}}{5}$

4. Let x = the length of the rectangle. Then $x - 3$ = the width of the rectangle.

$x(x - 3) = 54 \Rightarrow x^2 - 3x - 54 = 0 \Rightarrow$

$(x + 6)(x - 9) = 0 \Rightarrow x = -6$ or $x = 9$

We reject the negative answer. The rectangle is 9 cm by 6 cm.

5. $x^2 + 36 = -13x \Rightarrow x^2 + 13x + 36 = 0 \Rightarrow$

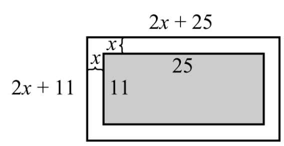
$(x + 9)(x + 4) = 0 \Rightarrow x = -9$ or $x = -4$

6. Let x = the amount to be invested at 8%. Then the total amount invested is $x + 8200$.

$$0.06(8200) + 0.08x = 0.07(8200 + x)$$
$$492 + 0.08x = 574 + 0.07x$$
$$0.01x = 82 \Rightarrow x = 8200$$

Fran must invest \$8200 at 7%.

7. Let x = the width of the border. Then the length of the border is $2x + 25$, and the width of the border is $2x + 11$.



$$(2x + 25)(2x + 11) = 351$$
$$4x^2 + 72x + 275 = 351$$
$$4x^2 + 72x - 76 = 0$$
$$4(x + 19)(x - 1) = 0 \Rightarrow x = -19 \text{ or } x = 1$$

The border is 1 inch wide.

8.
$$-6x - 15 = (2x + 5)^2$$
$$-6x - 15 = 4x^2 + 20x + 25$$
$$4x^2 + 26x + 40 = 0$$
$$2(2x + 5)(x + 4) = 0 \Rightarrow x = -\frac{5}{2} \text{ or } x = -4$$

The solution set is $\{-4, -5/2\}$.

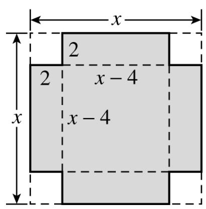
9. To find the constant term, find 1/2 of 2/3 = 1/3 and then square the answer: $(1/3)^2 = 1/9$.

The trinomial is $x^2 + \frac{2}{3}x + \underline{\frac{1}{9}}$, which factors into $\left(x + \frac{1}{3}\right)^2$.

10. $x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)}$

$= \dfrac{5 \pm \sqrt{37}}{6} = \dfrac{5}{6} \pm \dfrac{\sqrt{37}}{6}$

11. Let x = the length of the side of the original piece of cardboard. Then $x - 4$ = the length of the side of the box.



$2(x-4)^2 = 50 \Rightarrow$
$(x-4)^2 = 25 \Rightarrow$
$x - 4 = 5 \Rightarrow x = 9$
(Reject the negative solution.) The side of the original cardboard square is 9 inches.

12. $x = \dfrac{-12 \pm \sqrt{12^2 - 4(1)(33)}}{2(1)}$

$= \dfrac{-12 \pm \sqrt{12}}{2} = \dfrac{-12 \pm 2\sqrt{3}}{2} = -6 \pm \sqrt{3}$

13. $3x^4 - 75x^2 = 0 \Rightarrow 3x^2(x^2 - 25) = 0 \Rightarrow$
$3x^2(x-5)(x+5) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0$
or $x - 5 = 0 \Rightarrow x = 5$
or $x + 5 = 0 \Rightarrow x = -5$
The solution set is {–5, 0, 5}.

14. Let $u = \sqrt{x}$. The equation becomes
$3u^2 - 2 - 5u = 0 \Rightarrow 3u^2 - 5u - 2 = 0 \Rightarrow$
$(3u+1)(u-2) = 0 \Rightarrow u = -\frac{1}{3}$ or $u = 2$

Now solve for x: $2 = \sqrt{x} \Rightarrow x = 4$ (We reject the negative solution). The solution set is {4}.

15. $\left|\frac{1}{3}x + 5\right| = \left|\frac{2}{3}x + 7\right| \Rightarrow \frac{1}{3}x + 5 = \frac{2}{3}x + 7$
or $\frac{1}{3}x + 5 = -\left(\frac{2}{3}x + 7\right)$.
$\frac{1}{3}x + 5 = \frac{2}{3}x + 7 \Rightarrow x + 15 = 2x + 21 \Rightarrow x = -6.$
$\frac{1}{3}x + 5 = -\left(\frac{2}{3}x + 7\right) \Rightarrow \frac{1}{3}x + 5 = -\frac{2}{3}x - 7 \Rightarrow$
$x + 15 = -2x - 21 \Rightarrow 3x = -36 \Rightarrow x = -12$
The solution set is {–12, –6}.

16. $\frac{x}{2} - 5 \ge \frac{4x}{9} \Rightarrow 9x - 90 \ge 8x \Rightarrow x \ge 90$
The solution set is $[90, \infty)$.

17. $-4 < 2x - 3 < 4 \Rightarrow -1 < 2x < 7 \Rightarrow$
$-\frac{1}{2} < x < \frac{7}{2}$
The solution set is $\left(-\frac{1}{2}, \frac{7}{2}\right)$.

18. $\left|\frac{2}{3}x - 1\right| - 2 > \frac{1}{3} \Rightarrow \left|\frac{2}{3}x - 1\right| > \frac{7}{3} \Rightarrow$
$\frac{2}{3}x - 1 < -\frac{7}{3}$ or $\frac{2}{3}x - 1 > \frac{7}{3}$
$\frac{2}{3}x - 1 < -\frac{7}{3} \Rightarrow 2x - 3 < -7 \Rightarrow 2x < -4 \Rightarrow$
$x < -2$
$\frac{2}{3}x - 1 > \frac{7}{3} \Rightarrow 2x - 3 > 7 \Rightarrow 2x > 10 \Rightarrow x > 5$
The solution set is $(-\infty, -2) \cup (5, \infty)$.

19. $\frac{2}{5}y - 13 \le -\left(7 + \frac{13}{5}y\right)$
$\frac{2}{5}y - 13 \le -7 - \frac{13}{5}y$
$2y - 65 \le -35 - 13y \Rightarrow 15y \le 30 \Rightarrow y \le 2$
The solution set is $(-\infty, 2]$.

20. $0 \le 5x - 2 \le 8 \Rightarrow 2 \le 5x \le 10 \Rightarrow \frac{2}{5} \le x \le 2$
The solution set is $\left[\frac{2}{5}, 2\right]$.

Chapter 1 Practice Test B

1. $2x - 2 = 5x + 34 \Rightarrow -3x = 36 \Rightarrow x = -12.$
The answer is D.

2. $\frac{z}{2} = 2z + 35 \Rightarrow z = 4z + 70 \Rightarrow -3z = 70 \Rightarrow$
$z = -\frac{70}{3}$. The answer is D.

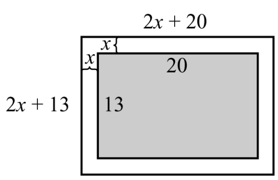
3. $\frac{1}{t-2} - \frac{1}{2} = \frac{-2t}{4t-1} \Rightarrow$
$2(4t-1) - (t-2)(4t-1) = -2t(2)(t-2) \Rightarrow$
$8t - 2 - 4t^2 + 9t - 2 = -4t^2 + 8t \Rightarrow$
$9t - 4 = 0 \Rightarrow 9t = 4 \Rightarrow t = \frac{4}{9}.$
The answer is A.

4. Let w = the width of the rectangle.
Then $w + 4$ = the length of the rectangle.
$w(w+4) = 77 \Rightarrow w^2 + 4w - 77 = 0 \Rightarrow$
$(w+11)(w-7) = 0 \Rightarrow w = -11$ or $w = 7$
We reject the negative solution. The rectangle is 7 cm by 11 cm. The answer is C.

5. $x^2 + 12 = -7x \Rightarrow x^2 + 7x + 12 = 0 \Rightarrow$
$(x+4)(x+3) = 0 \Rightarrow x = -4$ or $x = -3$
The answer is B.

6. Let x = the amount to be invested at 12%.
Then the total amount invested is $7500 + x$.
$0.07(7500) + 0.12x = 0.10(7500 + x)$
$525 + 0.12x = 750 + 0.10x$
$0.02x = 225 \Rightarrow x = 11,250$
Rena must invest $11,250 at 12%.
The answer is A.

7. Let x = the width of the border. Then the length of the border is $2x + 20$, and the width of the border is $2x + 13$.



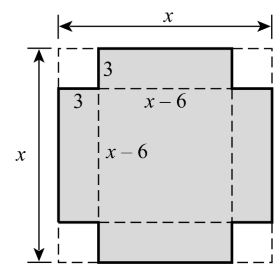
$(2x+20)(2x+13) = 368$
$4x^2 + 66x + 260 = 368$
$4x^2 + 66x - 108 = 0$
$2(2x-3)(x+18) = 0 \Rightarrow x = \frac{3}{2}$ or $x = -18$
The border is 1.5 inch wide. The answer is D.

8. $-6x - 2 = (3x+1)^2$
$-6x - 2 = 9x^2 + 6x + 1$
$9x^2 + 12x + 3 = 0$
$3x^2 + 4x + 1 = 0$
$(3x+1)(x+1) = 0 \Rightarrow x = -\frac{1}{3}$ or $x = -1$
The answer is D.

9. To find the constant term, find 1/2 of 1/6 = 1/12 and then square the answer: $(1/12)^2 = 1/144$.
The trinomial is $x^2 + \frac{1}{6}x + \frac{1}{144}$, which factors into $\left(x + \frac{1}{12}\right)^2$. The answer is B.

10. $x = \frac{-10 \pm \sqrt{10^2 - 4(7)(2)}}{2(7)}$
$= \frac{-10 \pm \sqrt{44}}{14} = \frac{-10 \pm 2\sqrt{11}}{14} = -\frac{5}{7} \pm \frac{\sqrt{11}}{7}$
The answer is A.

11. Let x = the length of the side of the original piece of cardboard. Then $x - 6$ = the length of the side of the box.



$3(x-6)^2 = 675 \Rightarrow (x-6)^2 = 225 \Rightarrow$
$x - 6 = 15 \Rightarrow x = 21$ (Note that we reject the negative solution.) The side of the original cardboard square is 21 inches.
The answer is B.

12. $x = \frac{-14 \pm \sqrt{14^2 - 4(1)(38)}}{2(1)}$
$= \frac{-14 \pm \sqrt{44}}{2} = \frac{-14 \pm 2\sqrt{11}}{2}$
$= -7 \pm \sqrt{11}$
The answer is D.

13. $5x^4 - 45x^2 = 0 \Rightarrow 5x^2(x^2 - 9) = 0 \Rightarrow$
$5x^2(x-3)(x+3) = 0 \Rightarrow 5x^2 = 0 \Rightarrow x = 0$
or $x - 3 = 0 \Rightarrow x = 3$
or $x + 3 = 0 \Rightarrow x = -3$
The solution set is {–3, 0, 3}.
The answer is A.

14. Let $u = \sqrt{x}$. The equation becomes
$u^2 - 2048 - 32u = 0 \Rightarrow$
$u^2 - 32u - 2048 = 0 \Rightarrow$
$(u-64)(u+32) = 0 \Rightarrow u = 64$ or $u = -32$
Now solve for x: $64 = \sqrt{x} \Rightarrow x = 4096$ (We reject the negative solution). The answer is A.

15. $\left|\frac{1}{2}x+2\right| = \left|\frac{3}{4}x-2\right| \Rightarrow \frac{1}{2}x+2 = \frac{3}{4}x-2$
or $\frac{1}{2}x+2 = -\left(\frac{3}{4}x-2\right)$.
$\frac{1}{2}x+2 = \frac{3}{4}x-2 \Rightarrow 2x+8 = 3x-8 \Rightarrow x = 16$
$\frac{1}{2}x+2 = -\left(\frac{3}{4}x-2\right) \Rightarrow \frac{1}{2}x+2 = -\frac{3}{4}x+2 \Rightarrow$
$2x+8 = -3x+8 \Rightarrow 5x = 0 \Rightarrow x = 0$
The answer is D.

16. $\frac{x}{6} - \frac{1}{3} \le \frac{x}{3} + 1 \Rightarrow x-2 \le 2x+6 \Rightarrow -8 \le x$
The answer is D.

17. $-13 \le -3x+2 < -4 \Rightarrow -15 \le -3x < -6 \Rightarrow$
$5 \ge x > 2$
The answer is B.

18. $8 + \left|1-\frac{x}{2}\right| \ge 10 \Rightarrow \left|1-\frac{x}{2}\right| \ge 2 \Rightarrow$
$1-\frac{x}{2} \le -2 \Rightarrow -\frac{x}{2} \le -3 \Rightarrow x \ge 6$ or
$1-\frac{x}{2} \ge 2 \Rightarrow -\frac{x}{2} \ge 1 \Rightarrow x \le -2$
The solution set is $(-\infty, -2] \cup [6, \infty)$.
The answer is C.

19. $\frac{2}{3}x - 2 < \frac{5}{3}x \Rightarrow 2x-6 < 5x \Rightarrow -6 < 3x \Rightarrow$
$-2 < x$
The solution set is $(-2, \infty)$. The answer is A.

20. $0 \le 7x-1 \le 13 \Rightarrow 1 \le 7x \le 14 \Rightarrow \frac{1}{7} \le x \le 2$
The solution set is $\left[\frac{1}{7}, 2\right]$. The answer is A.