Chapter 4 Exponential and Logarithmic Functions

4.1 Exponential Functions

4.1 Practice Problems

1. $f(x) = \left(\frac{1}{4}\right)^x$

$f(2) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$f(0) = \left(\frac{1}{4}\right)^0 = 1$

$f(-1) = \left(\frac{1}{4}\right)^{-1} = 4$

$f\left(\frac{5}{2}\right) = \left(\frac{1}{4}\right)^{5/2} = \left(\sqrt{\frac{1}{4}}\right)^5 = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

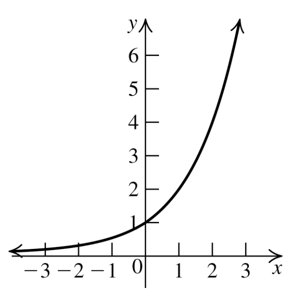
$f\left(-\frac{3}{2}\right) = \left(\frac{1}{4}\right)^{-3/2} = 4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

2. a. $3^{\sqrt{8}} \cdot 3^{\sqrt{2}} = 3^{\sqrt{8}+\sqrt{2}} = 3^{2\sqrt{2}+\sqrt{2}} = 3^{3\sqrt{2}} = 27^{\sqrt{2}}$

b. $\left(a^{\sqrt{8}}\right)^{\sqrt{2}} = a^{\sqrt{8}\cdot\sqrt{2}} = a^{\sqrt{16}} = a^4$

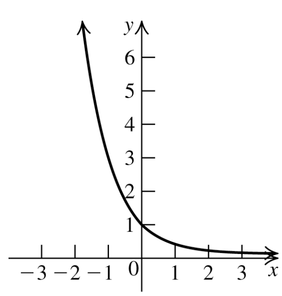
3.

x	2^x
–3	1/8
–2	1/4
–1	1/2
0	1
1	2
2	4
3	8



4.

x	$(1/3)^x$
–3	27
–2	9
–1	3
0	1
1	1/3
2	1/9
3	1/27



5. a. (0, 1) and (2, 49)

$f(0) = 1 \Rightarrow 1 = c \cdot a^0 = c \cdot 1 = c$

Therefore,

$f(x) = 1 \cdot a^x \Rightarrow 49 = 1 \cdot a^x \Rightarrow$

$49 = a^2 \Rightarrow a = 7$

So, $f(x) = 7^x$.

b. (–2, 16) and $\left(3, \frac{1}{2}\right)$

$16 = ca^{-2}$

$\frac{1}{2} = ca^3$

$\frac{16}{1/2} = \frac{ca^{-2}}{ca^3}$

$32 = a^{-5} \Rightarrow a = \frac{1}{2}$

$16 = c\left(\frac{1}{2}\right)^{-2} \Rightarrow 16 = 4c \Rightarrow 4 = c$

So, $f(x) = 4\left(\frac{1}{2}\right)^x$.

6. $A = P + I = P + Prt$
$= 10,000 + 10,000(0.075)(2)$
$= 10,000 + 1500 = 11,500$

There will be \$11,500 in the account.

7. a. $A = P(1+r)^t = 8000(1+0.075)^5$
$\approx 11,485.03$

There will be \$11,485.03 in the account.

b. Interest $= A - P = 11,485.03 - 8000$
$= 3485.03$

She will receive \$3485.03

8. The formula for compound interest is

$A = P\left(1+\frac{r}{n}\right)^{nt}$.

For each of the following, $t = 1$.

(i) Annual compounding ($n = 1$)

$A = \$5000\left(1+\frac{0.065}{1}\right)^1 = \$5325.00$

(ii) Semiannual compounding ($n = 2$)

$A = \$5000\left(1+\frac{0.065}{2}\right)^2 \approx \$5330.28$

(iii) Quarterly compounding ($n = 4$)

$$A = \$5000\left(1+\frac{0.065}{4}\right)^4 \approx \$5333.01$$

(iv) Monthly compounding ($n = 12$)

$$A = \$5000\left(1+\frac{0.065}{12}\right)^{12} \approx \$5334.86$$

(v) Daily compounding ($n = 365$)

$$A = \$5000\left(1+\frac{0.065}{365}\right)^{365} \approx \$5335.76$$

9. $$20,000 = 9000\left(1+\frac{r}{12}\right)^{12\cdot 8}$$

$$\left(1+\frac{r}{12}\right)^{96} = \frac{20,000}{9000} = \frac{20}{9}$$

$$1+\frac{r}{12} = \left(\frac{20}{9}\right)^{1/96}$$

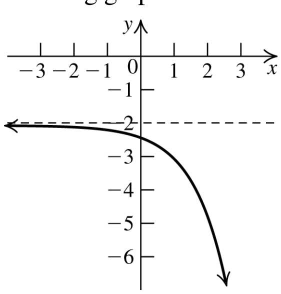
$$\frac{r}{12} = \left(\frac{20}{9}\right)^{1/96} - 1$$

$$r = 12\left[\left(\frac{20}{9}\right)^{1/96} - 1\right] \approx .10023$$

Carmen needs an interest rate of about 10.023%.

10. $A = Pe^{rt} = \$9000e^{(0.06)(8.25)} \approx \$14,764.48$

11. Shift the graph of $y = e^x$ one unit right, then reflect the graph about the x-axis. Shift the resulting graph two units down.



12. Use the exponential growth/decay formula $A(t) = A_0e^{kt}$

a. $A(30) = 6.08e^{0.016(30)} \approx 9.8257$

The model predicts that if the rate of growth is 1.6% per year, there will be about 9.83 billion people in the world in the year 2030.

b. $A(-10) = 6.08e^{0.016(-10)} \approx 5.1810$

The model predicts that if the rate of growth is 1.6% per year, there were about 5.18 billion people in the world in the year 1990.

13. $A(t) = A_0e^{-kt}$

$A(6) = 22,000e^{(-0.18)(6)} \approx \$7471.10$

4.1 Basic Concepts and Skills

1. For the exponential function $f(x) = ca^x, a > 0,\ a \neq 1$, the domain is $\underline{(-\infty, \infty)}$ and for $c > 0$, the range is $\underline{(0, \infty)}$.

2. The graph of $f(x) = 3^x$ has y-intercept $\underline{1}$ and has $\underline{\text{no}}$ x-intercept.

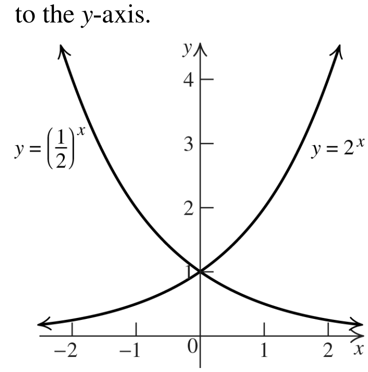
3. The horizontal asymptote of the graph of $y = \left(\frac{1}{3}\right)^x$ is $\underline{\text{the } x\text{-axis}}$.

4. The exponential function $f(x) = a^x$ is increasing if $\underline{a > 1}$ and is decreasing if $\underline{0 < a < 1}$.

5. The formula for compound interest at rate r compounded n times per year is $A = \underline{P\left(1+\frac{r}{n}\right)^{nt}}$.

6. The formula continuous compound interest is $A = \underline{Pe^{rt}}$.

7. False. The graphs are symmetric with respect to the y-axis.



8. False. This is true for $a > 1$ only.

9. True

10. False. The graph of $y = e^x + 1$ is obtained by shifting the graph of $y = e^x$ *vertically* one unit.

11. Not an exponential function. The base is not a constant.

12. Exponential function. The base is a constant, 4.

13. Exponential function. The base is a constant, 1/2.

14. Not an exponential function. The base is not a positive constant.

15. Not an exponential function. The base is not a constant.

16. Not an exponential function. The exponent is not a variable.

17. Not an exponential function. The base is not a positive constant.

18. Exponential function. The base is a constant, 1.8.

19. $f(0) = 5^{0-1} = 5^{-1} = \frac{1}{5}$

20. $f(-2) = -2^{-2+1} = -2^{-1} = -\frac{1}{2}$

21. $g(3.2) = 3^{1-3.2} = 3^{-2.2} = 0.0892$
$g(-1.2) = 3^{1-(-1.2)} = 3^{2.2} = 11.2116$

22. $g(2.8) = \left(\frac{1}{2}\right)^{2.8+1} = \left(\frac{1}{2}\right)^{3.8} = 0.0718$
$g(-3.5) = \left(\frac{1}{2}\right)^{-3.5+1} = \left(\frac{1}{2}\right)^{-2.5} = 5.6569$

23. $h(1.5) = \left(\frac{2}{3}\right)^{2(1.5)-1} = \left(\frac{2}{3}\right)^{2} = \frac{4}{9}$
$h(-2.5) = \left(\frac{2}{3}\right)^{2(-2.5)-1} = \left(\frac{2}{3}\right)^{-6} = \frac{729}{64}$

24. $f(2) = 3 - 5^2 = -22$
$f(-1) = 3 - 5^{-1} = 2.8$

25. $3^{\sqrt{2}} \cdot 3^{\sqrt{2}} = 3^{\sqrt{2}+\sqrt{2}} = 3^{2\sqrt{2}}$

26. $7^{\sqrt{3}} \cdot 49^{\sqrt{12}} = 7^{\sqrt{3}} \cdot 7^{2\sqrt{12}} = 7^{\sqrt{3}+2\sqrt{12}}$
$= 7^{\sqrt{3}+4\sqrt{3}} = 7^{5\sqrt{3}}$

27. $8^{\pi} \div 4^{\pi} = \left(2^3\right)^{\pi} \div \left(2^2\right)^{\pi} = 2^{3\pi} \div 2^{2\pi}$
$= 2^{3\pi-2\pi} = 2^{\pi}$

28. $9^{\sqrt{5}} \div 27^{\sqrt{5}} = \left(3^2\right)^{\sqrt{5}} \div \left(3^3\right)^{\sqrt{5}} = 3^{2\sqrt{5}} \div 3^{3\sqrt{5}}$
$= 3^{2\sqrt{5}-3\sqrt{5}} = 3^{-\sqrt{5}}$

29. $\left(3^{\sqrt{2}}\right)^{\sqrt{3}} = 3^{\sqrt{2}\cdot\sqrt{3}} = 3^{\sqrt{6}}$

30. $\left(2^{\sqrt{3}}\right)^{\sqrt{5}} = 2^{\sqrt{3}\cdot\sqrt{5}} = 2^{\sqrt{15}}$

31. $\left(a^{\sqrt{3}}\right)^{\sqrt{12}} = a^{\sqrt{3}\cdot\sqrt{12}} = a^{\sqrt{36}} = a^6$

32. $a^{\sqrt{2}} \cdot \left(a^2\right)^{\sqrt{8}} = a^{\sqrt{2}} \cdot a^{2\sqrt{8}} = a^{\sqrt{2}+2\sqrt{8}}$
$= a^{\sqrt{2}+4\sqrt{2}} = a^{5\sqrt{2}}$

33. a. (0, 1) and (2, 16)
$f(0) = 1 \Rightarrow 1 = ca^0 \Rightarrow 1 = c$
$f(2) = 16 \Rightarrow 16 = 1 \cdot a^2 \Rightarrow a = 4$
$f(x) = 4^x$

b. (0, 1) and $\left(-2, \frac{1}{9}\right)$
$f(0) = 1 \Rightarrow 1 = ca^0 \Rightarrow 1 = c$
$f(-2) = \frac{1}{9} \Rightarrow \frac{1}{9} = 1 \cdot a^{-2} \Rightarrow a = 3$
$f(x) = 3^x$

34. a. (0, 3) and (2, 12)
$f(0) = 3 \Rightarrow 3 = ca^0 \Rightarrow 3 = c$
$f(2) = 12 \Rightarrow 12 = 3 \cdot a^2 \Rightarrow 4 = a^2 \Rightarrow a = 2$
$f(x) = 3 \cdot 2^x$

b. (0, 5) and (1, 15)
$f(0) = 5 \Rightarrow 5 = ca^0 \Rightarrow 5 = c$
$f(1) = 15 \Rightarrow 15 = 5 \cdot a^1 \Rightarrow a = 3$
$f(x) = 5 \cdot 3^x$

35. a. (1, 1) and (2, 5)

$f(1) = 1 \Rightarrow 1 = ca^1$

$f(2) = 5 \Rightarrow 5 = ca^2$

$\frac{1}{5} = \frac{ca^1}{ca^2} \Rightarrow \frac{1}{5} = a^{-1} \Rightarrow 5 = a$

$f(1) = 1 \Rightarrow 1 = c \cdot 5^1 \Rightarrow c = \frac{1}{5}$

$f(x) = \frac{1}{5} \cdot (5)^x$

b. (1, 1) and $\left(2, \frac{1}{5}\right)$

$f(1) = 1 \Rightarrow 1 = ca^1$

$f(2) = \frac{1}{5} \Rightarrow \frac{1}{5} = ca^2$

$\frac{1}{1/5} = \frac{ca^1}{ca^2} \Rightarrow 5 = a^{-1} \Rightarrow \frac{1}{5} = a$

$f(1) = 1 \Rightarrow 1 = c\left(\frac{1}{5}\right)^1 \Rightarrow c = 5$

$f(x) = 5 \cdot \left(\frac{1}{5}\right)^x$

36. a. (1, 5) and (2, 125)

$f(1) = 5 \Rightarrow 5 = ca^1$

$f(2) = 125 \Rightarrow 125 = ca^2$

$\frac{5}{125} = \frac{ca^1}{ca^2} \Rightarrow \frac{1}{25} = a^{-1} \Rightarrow 25 = a$

$f(1) = 5 \Rightarrow 5 = c \cdot 25^1 \Rightarrow c = \frac{1}{5}$

$f(x) = \frac{1}{5} \cdot (25)^x$

b. (−1, 4) and (1, 16)

$f(-1) = 4 \Rightarrow 4 = ca^{-1}$

$f(1) = 16 \Rightarrow 16 = ca^1$

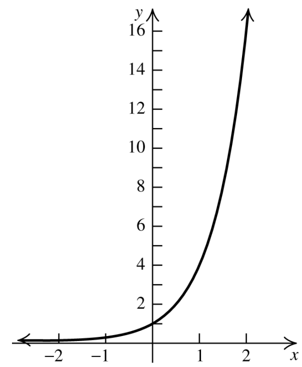
$\frac{4}{16} = \frac{ca^{-1}}{ca^1} \Rightarrow \frac{1}{4} = a^{-2} \Rightarrow 2 = a$

$f(1) = 16 \Rightarrow 16 = c \cdot 2^1 \Rightarrow c = 8$

$f(x) = 8 \cdot (2)^x$

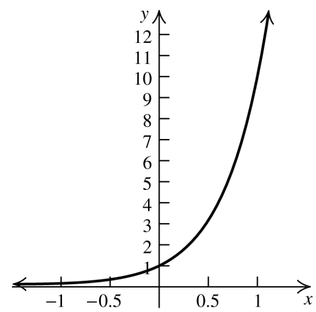
37.

x	4^x
–2	1/16
–1	1/4
0	1
1	4
2	16



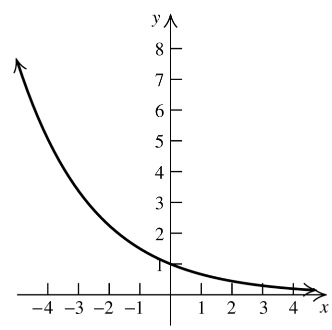
38.

x	10^x
–1	1/10
–0.5	≈ 0.32
0	1
0.5	≈ 3.2
1	10



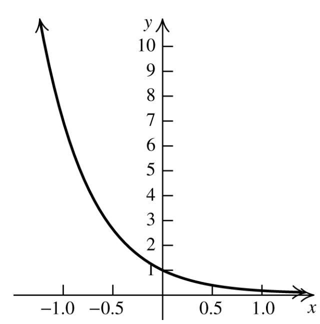
39.

x	$(3/2)^{-x}$
–4	81/16
–2	9/4
–1	3/2
0	1
1	2/3
2	4/9
4	16/81



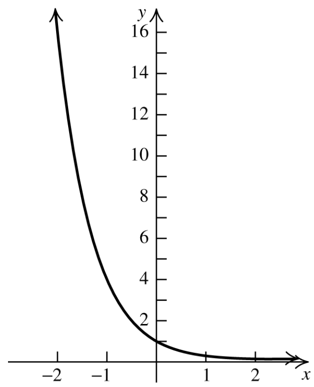
40.

x	7^{-x}
–2	49
–1	7
0	1
1	1/7
2	1/49



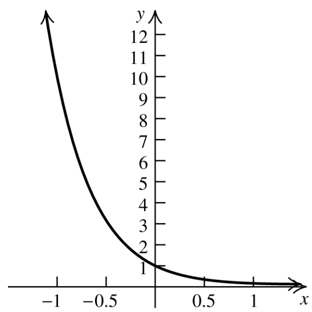
41.

x	$(1/4)^x$
–2	16
–1	4
0	1
1	1/4
2	1/16



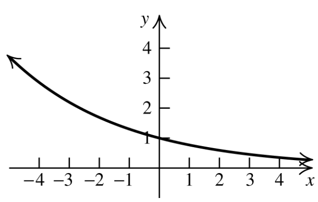
42.

x	$(1/10)^x$
–2	100
–1	10
0	1
1	0.1
2	0.01



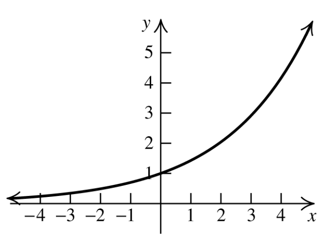
43.

x	1.3^{-x}
–4	≈ 2.86
–3	≈ 2.2
0	1
2	≈ 0.59
4	≈ 0.35



44.

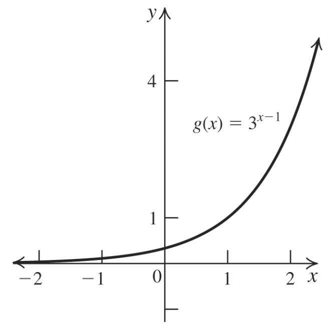
x	0.7^{-x}
–4	≈ 0.24
–2	0.49
0	1
2	≈ 2.04
4	≈ 4.16



45. c **46.** b **47.** a **48.** d

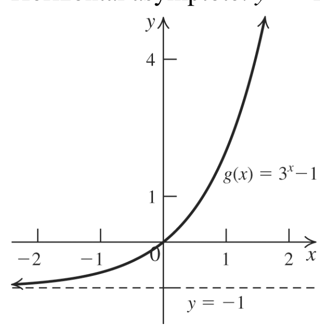
49. $g(x) = 3^{x-1}$

Shift the graph of $f(x) = 3^x$ one unit right.
Domain: $(-\infty, \infty)$; Range: $(0, \infty)$
Horizontal asymptote: $y = 0$



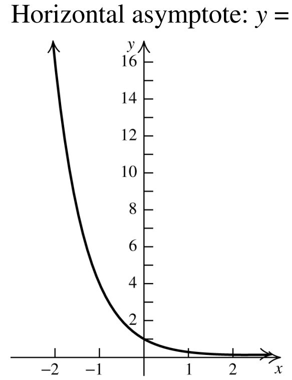
50. $g(x) = 3^x - 1$

Shift the graph of $f(x) = 3^x$ one unit down.
Domain: $(-\infty, \infty)$; Range: $(-1, \infty)$
Horizontal asymptote: $y = -1$



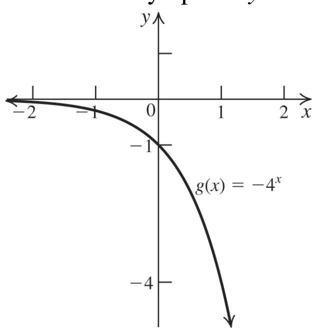
51. $g(x) = 4^{-x}$

Reflect the graph of $f(x) = 4^x$ about the y-axis.
Domain: $(-\infty, \infty)$; Range: $(0, \infty)$
Horizontal asymptote: $y = 0$



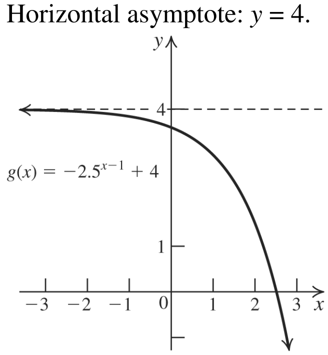
52. $g(x) = -4^x$

Reflect the graph of $f(x) = 4^x$ about the x-axis.
Domain: $(-\infty, \infty)$; Range: $(-\infty, 0)$
Horizontal asymptote: $y = 0$



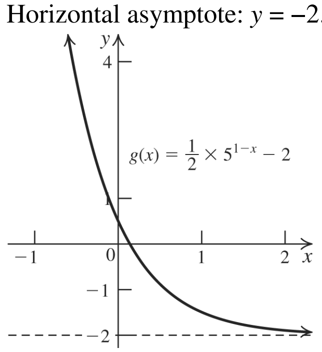
53. $g(x) = -2 \cdot 5^{x-1} + 4$

Shift the graph of $f(x) = 5^x$ one unit to the right, reflect it about the x-axis, stretch vertically by a factor of 2, then shift four units up. Domain: $(-\infty, \infty)$; Range: $(-\infty, 4)$.
Horizontal asymptote: $y = 4$.



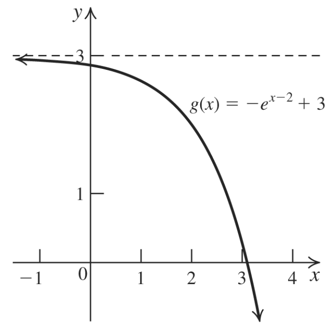
54. $g(x) = \frac{1}{2} \cdot 5^{1-x} - 2$

Reflect the graph of $f(x) = 5^x$ about the y-axis, then shift the graph one unit to the right, compress vertically by a factor of 2, then shift two units down.
Domain: $(-\infty, \infty)$; Range: $(-2, \infty)$.
Horizontal asymptote: $y = -2$.



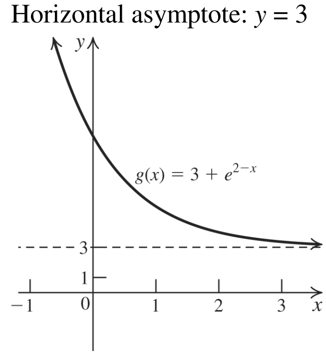
55. $g(x) = -e^{x-2} + 3$

Shift the graph of $f(x) = e^x$ two units to the right, then reflect it about the x-axis, then shift the graph three units up.
Domain: $(-\infty, \infty)$; Range: $(-\infty, 3)$
Horizontal asymptote: $y = 3$



56. $g(x) = 3 + e^{2-x}$

Reflect the graph of $f(x) = e^x$ about the x-axis, then shift it two units to the right, then shift the graph three units up.
Domain: $(-\infty, \infty)$; Range: $(3, \infty)$
Horizontal asymptote: $y = 3$



57. The graph passes through the point (0, 3) and (1, 3.5).

$f(0) = 3 \Rightarrow 3 = a^0 + b \Rightarrow 3 = 1 + b \Rightarrow b = 2$
$f(1) = 3.5 \Rightarrow 3.5 = a^1 + 2 \Rightarrow 1.5 = a$
$f(x) = 1.5^x + 2$
$f(2) = 1.5^2 + 2 = 4.25$

58. The graph passes through $(-2, 6)$ and (0, 3).

$f(0) = 3 \Rightarrow 3 = a^0 + b \Rightarrow 3 = 1 + b \Rightarrow b = 2$
$f(-2) = 6 \Rightarrow 6 = a^{-2} + 2 \Rightarrow 4 = a^{-2} \Rightarrow$
$\frac{1}{a^2} = 4 \Rightarrow a = \frac{1}{2}$
$f(x) = \left(\frac{1}{2}\right)^x + 2 \Rightarrow f(2) = \left(\frac{1}{2}\right)^2 + 2 = \frac{9}{4}$

59. The graph passes through (−2, 7) and (−1, 1).
$f(-2) = 7 \Rightarrow 7 = a^{-2} + b \quad (1)$
$f(-1) = 1 \Rightarrow 1 = a^{-1} + b \quad (2)$
Subtract (2) from (1).
$6 = a^{-2} - a^{-1} \Rightarrow 6 = \frac{1}{a^2} - \frac{1}{a} \Rightarrow$
$6a^2 = 1 - a \Rightarrow 6a^2 + a - 1 = 0 \Rightarrow$
$(2a+1)(3a-1) = 0 \Rightarrow a = -\frac{1}{2}, \frac{1}{3}$
If $a = -\frac{1}{2}$, then
$1 = \left(-\frac{1}{2}\right)^{-1} + b \Rightarrow 1 = -2 + b \Rightarrow b = 3.$
If $a = \frac{1}{3}$, then
$1 = \left(\frac{1}{3}\right)^{-1} + b \Rightarrow 1 = 3 + b \Rightarrow b = -2.$
The horizontal asymptote of the given graph is $y = -2$, so the equation of the graph is
$f(x) = \left(\frac{1}{3}\right)^x - 2.$
$f(2) = \left(\frac{1}{3}\right)^2 - 2 = \frac{1}{9} - 2 = -\frac{17}{9}$

60. The graph passes through (−1, 2.5) and (1, 1).
$f(-1) = 2.5 \Rightarrow 2.5 = a^{-1} + b \quad (1)$
$f(1) = 1 \Rightarrow 1 = a^1 + b \Rightarrow 1 = a + b \quad (2)$
Subtract (2) from (1).
$1.5 = a^{-1} - a \Rightarrow 1.5 = \frac{1}{a} - a \Rightarrow$
$1.5a = 1 - a^2 \Rightarrow a^2 + \frac{3}{2}a - 1 = 0 \Rightarrow$
$2a^2 + 3a - 2 = 0 \Rightarrow (a+2)(2a-1) = 0 \Rightarrow$
$a = -2, \frac{1}{2}$
If $a = -2$, then $1 = -2 + b \Rightarrow 3 = b.$
If $a = \frac{1}{2}$, then $1 = \frac{1}{2} + b \Rightarrow \frac{1}{2} = b.$
The horizontal asymptote of the given graph is $y = 1/2$, so the equation of the graph is
$f(x) = \left(\frac{1}{2}\right)^x + \frac{1}{2}.$
$f(2) = \left(\frac{1}{2}\right)^2 + \frac{1}{2} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

61. $y = 2^{x+2} + 5$

62. $y = 3^{-(x-3)}$ or $y = 3^{-x+3}$

63. $y = 2\left(\frac{1}{2}\right)^x - 5$ **64.** $y = -2^{-x} + 3$

65. $I = Prt = \$5000 \cdot 0.1 \cdot 5 = \$2500$

66. $I = Prt = \$10,000 \cdot 0.05 \cdot 10 = \$5000$

67. $I = Prt = \$7800 \cdot 0.06875 \cdot 10.75 = \$5764.69$

68. $I = Prt = \$8670 \cdot 0.04125 \cdot 6\frac{2}{3} = \$2384.25$

69. a. $A = 3500\left(1 + \frac{0.065}{1}\right)^{13} = \$7936.21$

b. interest = $\$7936.21 - \$3500 = \$4436.21$

70. a. $A = 6240\left(1 + \frac{0.075}{12}\right)^{12(12)} = \$15,305$

b. interest = $\$15,305 - \$6240 = \$9065$

71. a. $A = 7500e^{0.05(10)} = \$12,365.41$

b. interest = $\$12,365.41 - \$7500 = \$4865.41$

72. a. $A = 8000\left(1 + \frac{0.065}{365}\right)^{365(15)} = \$21,207.50$

b. interest =
$\$21,207.50 - \$8000 = \$13,207.50$

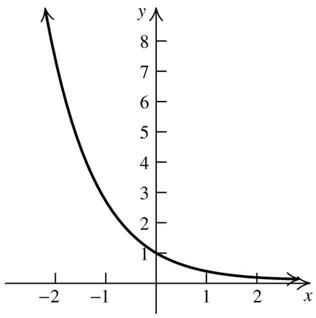
73. $10,000 = P\left(1 + \frac{0.08}{1}\right)^{10} = P(1.08)^{10} \Rightarrow$
$P = \$4631.93$

74. $10,000 = P\left(1 + \frac{0.08}{4}\right)^{4(10)} = P(1.02)^{40} \Rightarrow$
$P = \$4528.90$

75. $10,000 = P\left(1 + \frac{0.08}{365}\right)^{365(10)} \Rightarrow P = \$4493.68$

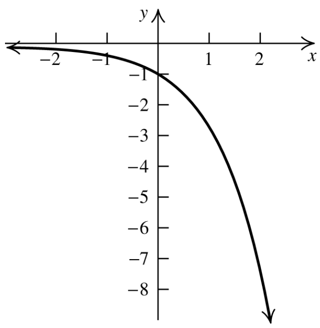
76. $10,000 = Pe^{0.08(10)} \Rightarrow P = \$4493.29$

77. Reflect the graph of $y = e^x$ about the y-axis.



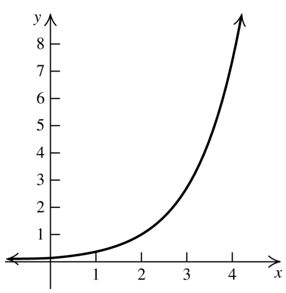
Horizontal asymptote: $y = 0$

78. Reflect the graph of $y = e^x$ about the x-axis.



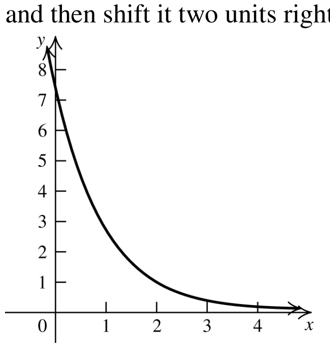
Horizontal asymptote: $y = 0$

79. Shift the graph of $y = e^x$ two units right.



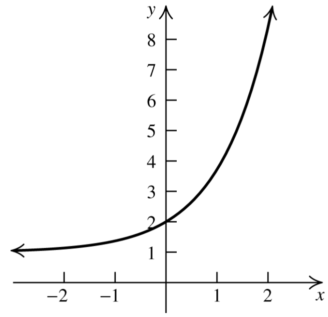
Horizontal asymptote: $y = 0$

80. Reflect the graph of $y = e^x$ about the y-axis, and then shift it two units right.



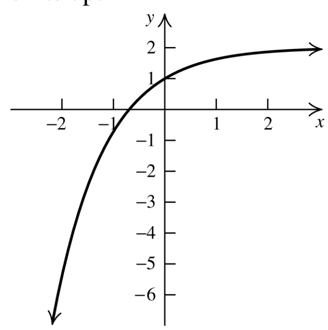
Horizontal asymptote: $y = 0$

81. Shift the graph of $y = e^x$ one unit up.



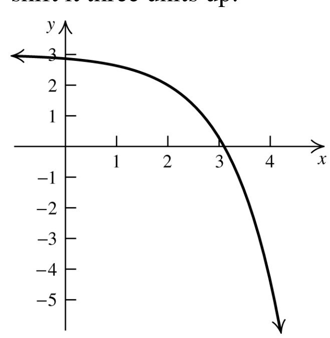
Horizontal asymptote: $y = 1$

82. Reflect the graph of $y = e^x$ about the y-axis, then reflect it about the x-axis, and shift it two units up.



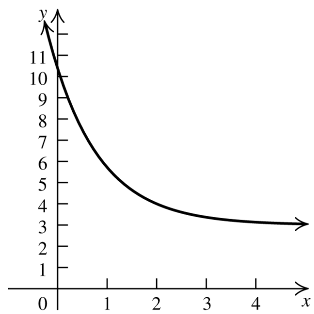
Horizontal asymptote: $y = 2$

83. Shift the graph of $y = e^x$ two units right, reflect the graph about the x-axis, and then shift it three units up.



Horizontal asymptote: $y = 3$

84. Shift the graph of $y = e^x$ two units right, reflect the graph about the y-axis, and then shift it three units up.



Horizontal asymptote: $y = 3$

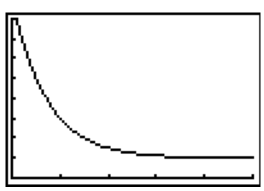
4.1 Applying the Concepts

85. a. **(i)** $T = 200 \cdot 4^{-0.1(2)} + 25 = 176.6°\text{C}$

(ii) $T = 200 \cdot 4^{-0.1(3.5)} + 25 = 148.1°\text{C}$

b. $125 = 200 \cdot 4^{-0.1t} + 25 \Rightarrow 100 = 200 \cdot 4^{-0.1t} \Rightarrow$
$\frac{1}{2} = 4^{-0.1t} \Rightarrow 2^{-1} = 2^{-0.2t} \Rightarrow -1 = -0.2t \Rightarrow$
$t = 5$ hours

c. As $t \to \infty$, $T \to 25$. Verify graphically.



[0, 50, 10] by [0, 200, 25]

86. a. **(i)** Because t = the number of years since 2010, $t = 0$.
$p(0) = 1.6 \cdot 2^{0.1047(0)} = 1600$ thousand
$= 1{,}600{,}000$

(ii) $t = 8$, so $p(8) = 1.6 \cdot 2^{0.1047(8)}$
≈ 2859 thousand $= 2{,}859{,}000$.

b. $x = 2025 - 2010 = 15$,
$p(15) = 1.6 \cdot 2^{0.1047(15)} \approx 4752.1$ thousand
$= 4{,}752{,}100$

87. $A = 190{,}000\left(1 + \frac{0.03}{1}\right)^{1(5)} \approx \$220{,}262$

88. $80{,}000 = P\left(1 + \frac{0.07}{4}\right)^{4(21)} \Rightarrow P = \$18{,}629.40$

89. $A = 80{,}000\left(1 - \frac{0.15}{1}\right)^{1(5)} \approx \$35{,}496.43$

90. a. $I = 24(0.06)(380) = \$547.20$. The total investment is $\$547.20 + \$24 = \$571.20$

b. $A = 24\left(1 + \frac{0.06}{1}\right)^{1(380)} \approx \$99{,}183{,}639{,}920$

c. $A = 24\left(1 + \frac{0.06}{12}\right)^{(380)(12)}$
$\approx \$180{,}905{,}950{,}100$

d. $A = 24e^{0.06(380)} \approx \$191{,}480{,}886{,}300$

91. Assume that interest is compounded annually.
$22{,}000{,}000 = 5000\left(1 + \frac{r}{1}\right)^{1(54)} \Rightarrow$
$4400 = (1 + r)^{54} \Rightarrow 4400^{1/54} = 1 + r \Rightarrow$
$1.1681 \approx 1 + r \Rightarrow r \approx 0.1681 \approx 16.81\%$

92. $15{,}000 = 12{,}000e^{10k} \Rightarrow \frac{5}{4} = e^{10k} \Rightarrow$
$e^k = \left(\frac{5}{4}\right)^{1/10}$.
$A = 15{,}000e^{10k} = 15{,}000\left(\frac{5}{4}\right) = 18{,}750$

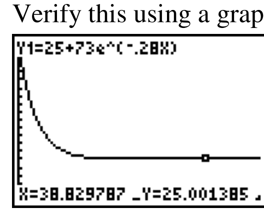
93. $A = 10e^{-0.43(10)} \approx 0.1357 \text{ mm}^2$

94. a. $T = 25 + 73e^{-0.28(0)} = 98°\text{C}$

b. $T = 25 + 73e^{-0.28(10)} \approx 29.44°\text{C}$

c. $T = 25 + 73e^{-0.28(20)} \approx 25.27°\text{C}$

d. As $t \to \infty$, $e^{-0.28t} \to 0 \Rightarrow T \to 25°\text{C}$.
Verify this using a graphing calculator.



95. $95{,}600 = 60{,}000\left(1 + \frac{r}{4}\right)^{4(12)} \Rightarrow$
$\frac{239}{150} = \left(1 + \frac{r}{4}\right)^{48} \Rightarrow \sqrt[48]{\frac{239}{150}} = 1 + \frac{r}{4} \Rightarrow$
$\sqrt[48]{\frac{239}{150}} - 1 = \frac{r}{4} \Rightarrow 4\left(\sqrt[48]{\frac{239}{150}} - 1\right) \approx 0.039 = r$
The interest rate on the bond was about 3.9%.

96. Let P represent the amount invested. Then

(i) 9.7% compounded annually for one year would yield

$$A = P\left(1+\frac{0.097}{1}\right)^{1(1)} = 1.097P.$$

(ii) 9.6% compounded monthly for one year would yield

$$A = P\left(1+\frac{0.096}{12}\right)^{1(12)} \approx 1.1003P.$$

(iii) 9.5% compounded monthly for one year would yield $A = Pe^{0.095(1)} \approx 1.1000P.$

The 9.6% investment compound monthly is the best deal.

97. The number of pieces of paper is 2^x, where x is the number of tears. The height of the paper is $0.015 \cdot 2^x$.

a. $0.015 \cdot 2^{30} \approx 16{,}106{,}127$ cm

b. $0.015 \cdot 2^{40} \approx 16{,}492{,}674{,}420$ cm

c. $0.015 \cdot 2^{50} \approx 16{,}888{,}498{,}600{,}000$ cm $= 1.689 \times 10^{13}$ cm

98. Because the jar is full in 60 minutes, and the number of bacteria doubles every minute, it will be half full one minute earlier, or in 59 minutes.

4.1 Beyond the Basics

99. a.
$$\frac{f(x+h)-f(x)}{h} = \frac{e^{x+h}-e^x}{h} = \frac{e^x(e^h-1)}{h} = e^x\,\frac{(e^h-1)}{h}$$

b. $f(x+y) = e^{x+y} = e^x e^y = f(x)f(y)$

c. $f(-x) = e^{-x} = \dfrac{1}{e^x} = \dfrac{1}{f(x)}$

100. a.
$$f(x)+g(x) = \left(3^x+3^{-x}\right)+\left(3^x-3^{-x}\right) = 2\cdot 3^x$$

b.
$$f(x)-g(x) = \left(3^x+3^{-x}\right)-\left(3^x-3^{-x}\right) = 2\cdot 3^{-x}$$

c.
$$\begin{aligned}&\left[f(x)\right]^2-\left[g(x)\right]^2\\ &= \left[\left(3^x+3^{-x}\right)\right]^2-\left[\left(3^x-3^{-x}\right)\right]^2\\ &= \left(3^{2x}+3^{-2x}+2\right)-\left(3^{2x}+3^{-2x}-2\right)=4\end{aligned}$$

d.
$$\begin{aligned}&\left[f(x)\right]^2+\left[g(x)\right]^2\\ &= \left[\left(3^x+3^{-x}\right)\right]^2+\left[\left(3^x-3^{-x}\right)\right]^2\\ &= \left(3^{2x}+3^{-2x}+2\right)+\left(3^{2x}+3^{-2x}-2\right)\\ &= 2\left(3^{2x}+3^{-2x}\right)\end{aligned}$$

101.
$$S_5 = 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!} = \frac{163}{60} \approx 2.71667$$
$$S_{10} = S_5+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}+\frac{1}{9!}+\frac{1}{10!} \approx 2.7182818$$
$$S_{15} = S_{10}+\frac{1}{11!}+\frac{1}{12!}+\frac{1}{13!}+\frac{1}{14!}+\frac{1}{15!} \approx 2.718281828$$

The sum approaches the value of e.

102. a. $f(-x) = \dfrac{e^{-x}+e^x}{2} = f(x) \Rightarrow \cosh x$ is even.

b. $f(-x) = \dfrac{e^{-x}-e^x}{2} = -f(x) \Rightarrow \sin x$ is odd.

c.
$$\cosh x+\sinh x = \frac{e^x+e^{-x}}{2}+\frac{e^x-e^{-x}}{2} = \frac{2e^x}{2} = e^x$$

d.
$$\begin{aligned}&(\cosh x)^2-(\sinh x)^2\\ &= \left(\frac{e^x+e^{-x}}{2}\right)^2-\left(\frac{e^x-e^{-x}}{2}\right)\\ &= \frac{e^{2x}+2e^{x-x}+e^{-2x}}{4}-\frac{e^{2x}-2e^{x-x}+e^{-2x}}{4}\\ &= \frac{e^{2x}+2+e^{-2x}}{4}-\frac{e^{2x}-2+e^{-2x}}{4} = \frac{4}{4} = 1\end{aligned}$$

103.
$$A = P\left(1+\frac{r}{m}\right)^{m(1)} = P(1+y) \Rightarrow$$
$$\left(1+\frac{r}{m}\right)^m = 1+y \Rightarrow y = \left(1+\frac{r}{m}\right)^m - 1$$

104. a. $\dfrac{A(t+1)-A(t)}{A(t)} = \dfrac{Pe^{k(t+1)} - Pe^{kt}}{Pe^{kt}}$
$= \dfrac{e^{k(t+1)} - e^{kt}}{e^{kt}}$
$= \dfrac{e^{kt}e^{k} - e^{kt}}{e^{kt}} = e^{k} - 1$

b. $\dfrac{A(t+h)-A(t)}{A(t)} = \dfrac{Pe^{k(t+h)} - Pe^{kt}}{Pe^{kt}}$
$= \dfrac{e^{k(t+h)} - e^{kt}}{e^{kt}}$
$= \dfrac{e^{kt}e^{kh} - e^{kt}}{e^{kt}} = e^{kh} - 1$

105. $y = e^x \to y = e^{x-1} \to y = e^{2x-1} \to y = 3e^{2x-1}$

106. $y = e^x \to y = e^{x+2} \to y = e^{3x+2} \to$
$y = 2e^{3x+2} \to y = -2e^{3x+2} \to y = -2e^{3x+2} - 5$

107. $y = e^x \to y = e^{2+x} \to y = e^{2+3x} \to$
$y = e^{2-3x} \to y = 5e^{2-3x} \to y = 5e^{2-3x} + 4$

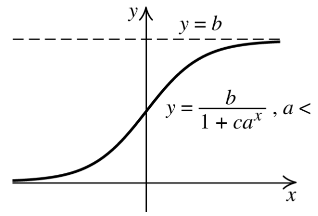
108. $y = e^x \to y = e^{3+x} \to y = e^{3+4x} \to$
$y = 2e^{3+4x} \to y = -2e^{3+4x} \to y = -2e^{3-4x} \to$
$y = -2e^{3-4x} - 1$

4.1 Critical Thinking/Discussion/Writing

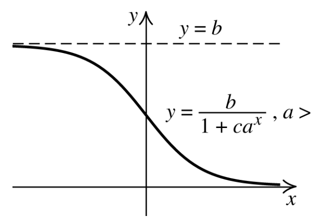
109. The base a cannot be 1 because this makes the function a constant, $f(x) = 1$. Similarly, the base a cannot be 0 because this becomes $f(x) = 0^x = 0$, a constant. We rule out negative bases so that the domain can include all real numbers. For example, a cannot be −3 because $f\left(\frac{1}{2}\right) = (-3)^{1/2} = \sqrt{-3}$ is not a real number.

110. $g(x)$ is defined for the set of integers. For example, $g(-2) = (-3)^{-2} = \dfrac{1}{9}$ and $g(3) = (-3)^3 = -27.$

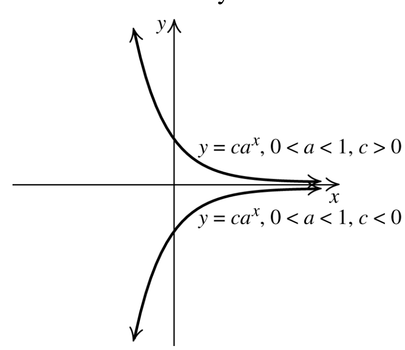
111. If $0 < a < 1$, then, for $x < 0$, the denominator becomes very large, so $y = \dfrac{b}{1+ca^x}$ approaches 0. For $x \ge 0$, ca^x approaches 0, so the denominator $1+ca^x$ approaches 1 and $y = \dfrac{b}{1+ca^x}$ approaches b.

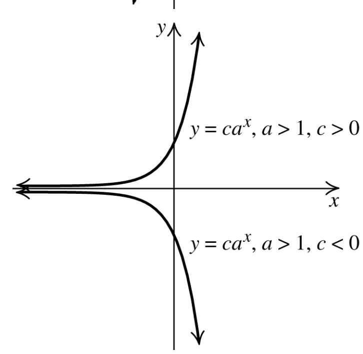


If $a > 1$, then, for $x < 0$, ca^x approaches 0, so the denominator $1+ca^x$ approaches 1 and $y = \dfrac{b}{1+ca^x}$ approaches b. For $x \ge 0$, the denominator becomes very large, so $y = \dfrac{b}{1+ca^x}$ approaches 0.

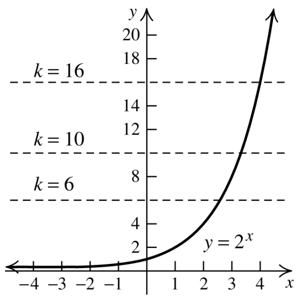


112. There are four possibilities, 0 < a < 1 with c < 0, 0 < a < 1 with c > 0, a > 1 with c < 0, and a > 1 with c > 0. They are illustrated below

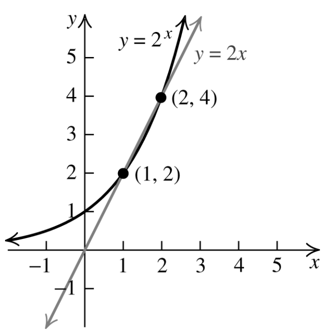




113. The function $y = 2^x$ is an increasing function, so it is one-to-one. The horizontal line $y = k$ for $k > 0$ intersects the graph of $y = 2^x$ in exactly one point, so there is exactly one solution for each value of k.



114. The graphs of $y = 2^x$ and $y = x$ intersect in exactly two points: (1, 2) and (2, 4). So $2^x = 2x \Rightarrow x = 1$ or 2.



4.1 Maintaining Skills

115. $10^0 = 1$

116. $10^{-1} = \frac{1}{10} = 0.1$

117. $(-8)^{1/3} = \sqrt[3]{-8} = -2$

118. $25^{1/2} = \sqrt{25} = 5$

119. $\left(\frac{1}{7}\right)^{-2} = 7^2 = 49$

120. $\left(\frac{1}{9}\right)^{-1/2} = 9^{1/2} = \sqrt{9} = 3$

121. $18^{1/2} = \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$

122. $\left(\frac{1}{12}\right)^{-1/2} = 12^{1/2} = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

123. $f(x) = y = 3x + 4$

Interchange x and y, then solve for y.

$x = 3y + 4 \Rightarrow x - 4 = 3y \Rightarrow$

$\frac{x-4}{3} = y = f^{-1}(x)$

The domain and range of f^{-1} are $(-\infty, \infty)$.

124. $f(x) = y = \frac{1}{2}x - 5$

Interchange x and y, then solve for y.

$x = \frac{1}{2}y - 5 \Rightarrow x + 5 = \frac{1}{2}y \Rightarrow 2(x+5) = y \Rightarrow$

$y = f^{-1}(x) = 2x + 10$

The domain and range of f^{-1} are $(-\infty, \infty)$.

125. $f(x) = y = \sqrt{x},\ x \ge 0$

Interchange x and y, then solve for y.

$x = \sqrt{y} \Rightarrow x^2 = y = f^{-1}(x),\ x \ge 0$

The domain and range of f^{-1} are $[0, \infty)$.

126. $f(x) = y = x^2 + 1,\ x \ge 0$

Interchange x and y, then solve for y.

$x = y^2 + 1 \Rightarrow x - 1 = y^2 \Rightarrow$

$y = f^{-1}(x) = \sqrt{x-1},\ x \ge 1$

The domain of f^{-1} is $[1, \infty)$.

The range of f^{-1} is $[0, \infty)$.

127. $f(x) = y = \frac{1}{x-1},\ x \ne 1$

Interchange x and y, then solve for y.

$x = \frac{1}{y-1} \Rightarrow x(y-1) = 1 \Rightarrow y - 1 = \frac{1}{x} \Rightarrow$

$y = f^{-1}(x) = 1 + \frac{1}{x},\ x \ne 0$

The domain of f^{-1} is $(-\infty, 0) \cup (0, \infty)$. The range of f^{-1} is $(-\infty, 1) \cup (1, \infty)$.

128. $f(x) = y = \sqrt[3]{x}$

Interchange x and y, then solve for y.

$x = \sqrt[3]{y} \Rightarrow x^3 = y = f^{-1}(x)$

The domain and range of f^{-1} are $(-\infty, \infty)$.

4.2 Logarithmic Functions

4.2 Practice Problems

1. a. $2^{10} = 1024$ is equivalent to $\log_2 1024 = 10$.

b. $9^{-1/2} = \frac{1}{3}$ is equivalent to $\log_9\left(\frac{1}{3}\right) = -\frac{1}{2}$.

c. $p = a^q$ is equivalent to $\log_a p = q$

2. a. $\log_2 64 = 6$ is equivalent to $2^6 = 64$.

b. $\log_v u = w$ is equivalent to $v^w = u$.

3. a. $\log_3 9 = y \Rightarrow 3^y = 9 \Rightarrow 3^y = 3^2 \Rightarrow y = 2$
Thus, $\log_3 9 = 2$.

b. $\log_9 \frac{1}{3} = y \Rightarrow 9^y = \frac{1}{3} \Rightarrow 3^{2y} = 3^{-1} \Rightarrow y = -\frac{1}{2}$
Thus, $\log_9 \frac{1}{3} = -\frac{1}{2}$.

c. $\log_{1/2} 32 = y \Rightarrow \left(\frac{1}{2}\right)^y = 32 \Rightarrow 2^{-y} = 2^5 \Rightarrow$
$y = -5$
Thus, $\log_{1/2} 32 = -5$.

4. a. $\log_5 1 = 0$ **b.** $\log_3 3^5 = 5$

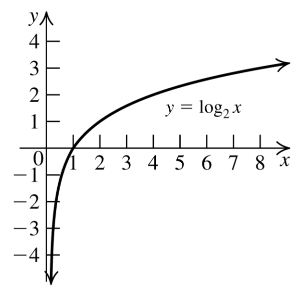
c. $7^{\log_7 5} = 5$

5. Since the domain of the logarithmic function is $(0, \infty)$, the expression $\sqrt{1-x}$ must be positive. $\sqrt{1-x}$ is defined for $(-\infty, 1)$, so the domain of $\log_{10}\sqrt{1-x}$ is $(-\infty, 1)$.

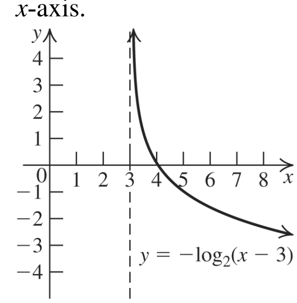
6. Create a table of values to find ordered pairs on the graph of $y = \log_2 x$.

x	$y = \log_2 x$	(x, y)
$\frac{1}{8}$	$2^{-3} = \frac{1}{8} \Rightarrow y = \log_2 \frac{1}{8} = -3$	$\left(\frac{1}{8}, -3\right)$
$\frac{1}{4}$	$2^{-2} = \frac{1}{4} \Rightarrow y = \log_2 \frac{1}{4} = -2$	$\left(\frac{1}{4}, -2\right)$
$\frac{1}{2}$	$2^{-1} = \frac{1}{2} \Rightarrow y = \log_2 \frac{1}{2} = -1$	$\left(\frac{1}{2}, -1\right)$
1	$2^0 = 1 \Rightarrow y = \log_2 1 = 0$	(1, 0)
2	$2^1 = 2 \Rightarrow y = \log_2 2 = 1$	(2, 1)
4	$2^2 = 4 \Rightarrow y = \log_2 4 = 2$	(4, 2)
8	$2^3 = 8 \Rightarrow y = \log_2 8 = 3$	(8, 3)

Plot the ordered pairs and connect them with a smooth curve.



7. Shift the graph of $y = \log_2 x$ three units right, then reflect the resulting graph about the x-axis.



8. $P_2 = \log 3 - \log 2 \approx 0.176$
This means that about 17.6% of the data is expected to have 2 as the first digit.

9. a. $y = \ln\frac{1}{e} \Rightarrow e^y = \frac{1}{e} \Rightarrow e^y = e^{-1} \Rightarrow y = -1$
Thus, $\ln\frac{1}{e} = -1$.

b. Using a calculator, we have $\ln 2 \approx 0.693$.

10. a. If P dollars are invested, then the amount $A = 3P$.
$A = Pe^{rt} \Rightarrow 3P = Pe^{0.065t} \Rightarrow 3 = e^{0.065t} \Rightarrow$
$\ln 3 = 0.065t \Rightarrow t = \frac{\ln 3}{0.065} \approx 16.9$
It will take approximately 17 years to triple your money.

b. $A = Pe^{rt} \Rightarrow 3P = Pe^{5r} \Rightarrow 3 = e^{5r} \Rightarrow$
$\ln 3 = 5r \Rightarrow t = \frac{\ln 3}{5} \approx 0.2197$
The investment will triple in 5 years at the rate of 21.97%.

11. Start with equation (2) in Example 11 in the text.
$T = 72 + 108e^{-0.1495317t}$
This equation gives the rate of cooling in a 72ºF room when the starting temperature is 180ºF. Since the final temperature is 120ºF, replace T with 120 and solve for t.

$$120 = 72 + 108e^{-0.1495317t}$$
$$48 = 108e^{-0.1495317t}$$
$$\frac{4}{9} = e^{-0.1495317t}$$
$$\ln\left(\frac{4}{9}\right) = -0.1495317t$$
$$-0.8109302 = -0.1495317t \Rightarrow 5.423 = t$$

The employees should wait about 5.423 (realistically 5.5 minutes) to deliver the coffee at 120ºF.

4.2 Basic Concepts and Skills

1. The domain of the function $y = \log_a x$ is $\underline{(0, \infty)}$ and it range is $\underline{(-\infty, \infty)}$.

2. The logarithmic form $y = \log_a x$ is equivalent to the exponential form $\underline{a^y = x}$.

3. The logarithm with base 10 is called the <u>common</u> logarithm, and the logarithm with base *e* is called the <u>natural</u> logarithm.

4. $a^{\log_a x} = \underline{x}$ and $\log_a a^x = \underline{x}$.

5. False

6. True

7. $\log_5 25 = 2$

8. $\log_{49}\left(\frac{1}{7}\right) = -\frac{1}{2}$

9. $\log_{1/16} 4 = -\frac{1}{2}$

10. $\log_{a^2} a^4 = 2$

11. $\log_{10} 1 = 0$

12. $\log_{10} 10{,}000 = 4$

13. $\log_{10} 0.1 = -1$

14. $\log_3 5 = x$

15. $a^2 + 2 = 7 \Rightarrow a^2 = 5 \Rightarrow \log_a 5 = 2$

16. $\log_a \pi = e$

17. $2a^3 - 3 = 10 \Rightarrow 2a^3 = 13 \Rightarrow a^3 = \frac{13}{2} \Rightarrow$
$\log_a\left(\frac{13}{2}\right) = 3$

18. $5 \cdot 2^{ct} = 11 \Rightarrow 2^{ct} = \frac{11}{5} \Rightarrow \log_2\left(\frac{11}{5}\right) = ct$

19. $2^5 = 32$

20. $7^2 = 49$

21. $10^2 = 100$

22. $10^1 = 10$

23. $10^0 = 1$

24. $a^0 = 1$

25. $10^{-2} = 0.01$

26. $\left(\frac{1}{5}\right)^{-1} = 5$

27. $3\log_8 2 = 1 \Rightarrow \log_8 2 = \frac{1}{3} \Rightarrow 8^{1/3} = 2$

28. $1 + \log 1000 = 4 \Rightarrow \log 1000 = 3 \Rightarrow 10^3 = 1000$

29. $e^x = 2$

30. $e^a = \pi$

31. $\log_5 125 = 3$ because $5^3 = 125$

32. $\log_9 81 = 2$ because $9^2 = 81$

33. $\log 10{,}000 = 4$ because $10^4 = 10{,}000$

34. $\log_3 \frac{1}{3} = -1$ because $3^{-1} = \frac{1}{3}$

35. $\log_2 \frac{1}{8} = -3$ because $2^{-3} = \frac{1}{8}$

36. $\log_4 \frac{1}{64} = -3$ because $4^{-3} = \frac{1}{64}$

37. $\log_3 \sqrt{27} = \frac{3}{2}$ because $3^{3/2} = \sqrt{27}$

38. $\log_{27} 3 = \frac{1}{3}$ because $27^{1/3} = 3$

39. $\log_{16} 2 = \frac{1}{4}$ because $16^{1/4} = 2$

40. $\log_5 \sqrt{125} = \frac{3}{2}$ because $5^{3/2} = \sqrt{125}$

41. $\log_3 1 = 0$

42. $\log_{1/2} 1 = 0$

43. $\log_7 7 = 1$

44. $\log_{1/9} \frac{1}{9} = 1$

45. $\log_6 6^7 = 7$

46. $\log_{1/2}\left(\frac{1}{2}\right)^5 = 5$

47. $3^{\log_3 5} = 5$

48. $7^{\log_7\left(\frac{1}{2}\right)} = \frac{1}{2}$

49. $2^{\log_2 7} + \log_5 5^{-3} = 7 + (-3) = 4$

50. $3^{\log_3 5} - \log_2 2^{-3} = 5 - (-3) = 8$

51. $4^{\log_4 6} - \log_4 4^{-2} = 6 - (-2) = 8$

52. $10^{\log x} - e^{\ln y} = x - y$

53. Since the domain of the logarithmic function is $(0, \infty)$, the expression $x + 1$ must be positive. This occurs in the interval $(-1, \infty)$, so the domain of $\log_2(x+1)$ is $(-1, \infty)$.

54. Since the domain of the logarithmic function is $(0, \infty)$, the expression $x - 8$ must be positive. This occurs in the interval $(8, \infty)$, so the domain of $\log_3(x-8)$ is $(8, \infty)$.

55. Since the domain of the logarithmic function is $(0, \infty)$, the expression $\sqrt{x-1}$ must be positive. This occurs in the interval $(1, \infty)$, so the domain of $\log_3\sqrt{x-1}$ is $(1, \infty)$.

56. Since the domain of the logarithmic function is $(0, \infty)$, the expression $\sqrt{3-x}$ must be positive. This occurs in the interval $(-\infty, 3)$, so the domain of $\log_4\sqrt{3-x}$ is $(-\infty, 3)$.

57. Since the domain of the logarithmic function is $(0, \infty)$, the expressions $(x - 2)$ and $(2x - 1)$ must be positive. This occurs in the interval $(2, \infty)$ for $x - 2$ and in the interval $\left(\frac{1}{2}, \infty\right)$ for $(2x - 1)$. The intersection of the two intervals is $(2, \infty)$, so the domain of f is $(2, \infty)$.

58. Since the domain of the logarithmic function is $(0, \infty)$, the expressions $\sqrt{x+5}$ and $(x + 1)$ must be positive. This occurs in the interval $(-5, \infty)$ for $\sqrt{x+5}$ and in the interval $(-1, \infty)$ for $x + 1$. The intersection of the two intervals is $(-1, \infty)$, so the domain of g is $(-1, \infty)$.

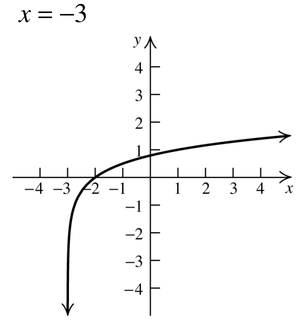
59. Since the domain of the logarithmic function is $(0, \infty)$, the expressions $(x - 1)$ and $(2 - x)$ must be positive. This occurs in the interval $(1, \infty)$ for $x - 2$ and in the interval $(-\infty, 2)$ for $(2 - x)$. The intersection of the two intervals is $(1, 2)$, so the domain of h is $(1, 2)$.

60. Since the domain of the logarithmic function is $(0, \infty)$, the expressions $(x - 3)$ and $(2 - x)$ must be positive. This occurs in the interval $(3, \infty)$ for $x - 2$ and in the interval $(-\infty, 2)$ for $(2 - x)$. The intersection of the two intervals is $\varnothing$, so the domain of f is $\varnothing$.

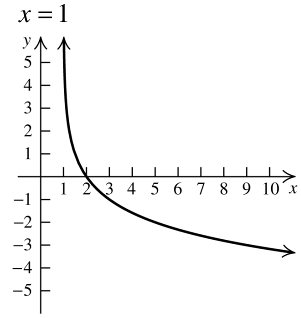
61. a. F **b.** A
c. D **d.** B
e. E **f.** C

62. a. $f(x) = \log_2 x$ **b.** $f(x) = \log_3 x$
c. $f(x) = \log_{1/2} x$ **d.** $f(x) = \log_4 x$

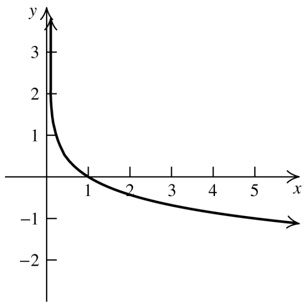
63. Shift the graph of $y = \log_4 x$ three units left.
Domain: $(-3, \infty)$; range: $(-\infty, \infty)$; asymptote: $x = -3$



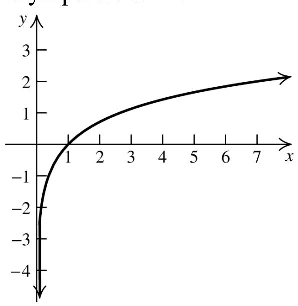
64. Shift the graph of $y = \log_{1/2} x$ one unit right.
Domain: $(1, \infty)$; range: $(-\infty, \infty)$; asymptote: $x = 1$



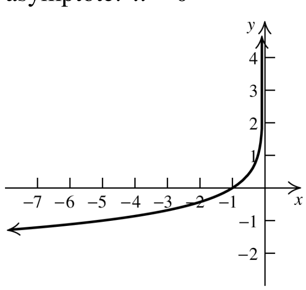
65. Reflect the graph of $y = \log_5 x$ about the x-axis. Domain: $(0, \infty)$; range: $(-\infty, \infty)$; asymptote: $x = 0$



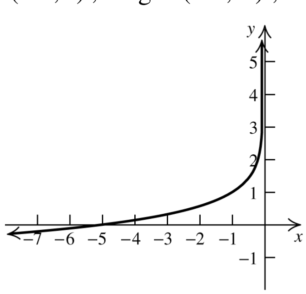
66. Stretch the graph of $y = \log_7 x$ vertically by a factor of 2. Domain: $(0, \infty)$; range: $(-\infty, \infty)$; asymptote: $x = 0$



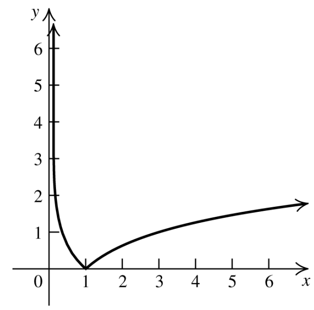
67. Reflect the graph of $y = \log_{1/5} x$ about the y-axis. Domain: $(-\infty, 0)$; range: $(-\infty, \infty)$; asymptote: $x = 0$



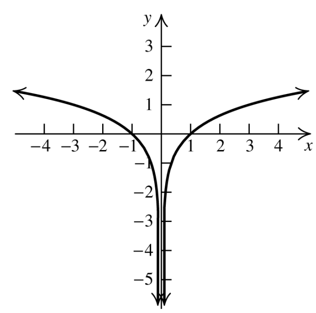
68. Reflect the graph of $y = \log_{1/5} x$ in the y-axis, and then shift it up one unit. Domain: $(-\infty, 0)$; range: $(-\infty, \infty)$; asymptote: $x = 0$



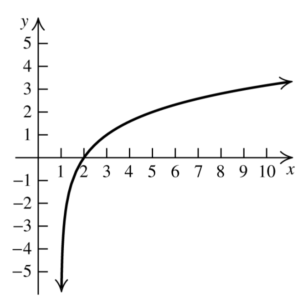
69. On $(0,1)$, reflect the graph of $y = \log_3 x$ about the x-axis. Domain: $(0, \infty)$; range: $[0, \infty)$; asymptote: $x = 0$



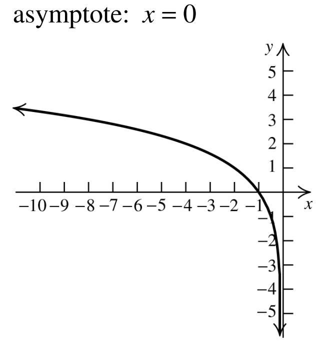
70. On $(-\infty, 0)$, reflect the graph of $y = \log_3 x$ about the y-axis. Domain: $(-\infty, 0) \cup (0, \infty)$; range: $(-\infty, \infty)$; asymptote: $x = 0$



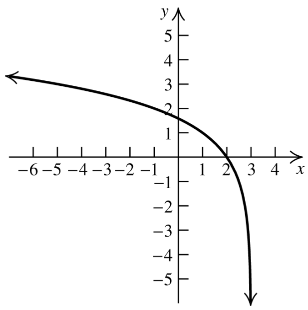
71. Shift the graph of $y = \log_2 x$ one unit right. Domain: $(1, \infty)$; range: $(-\infty, \infty)$; asymptote: $x = 1$



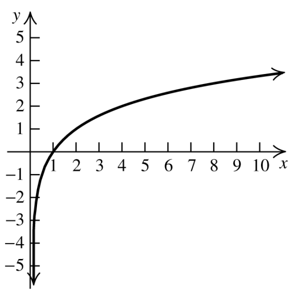
72. Reflect the graph of $y = \log_2 x$ about the y-axis. Domain: $(-\infty, 0)$; range: $(-\infty, \infty)$; asymptote: $x = 0$



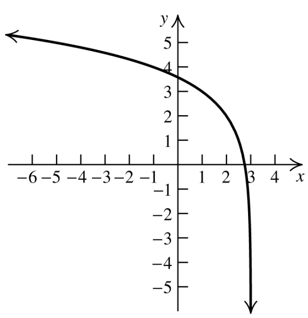
73. Shift the graph of $y = \log_2 x$ three units right and then reflect it about the *y*-axis.
Domain: $(-\infty, 3)$; range: $(-\infty, \infty)$;
asymptote: $x = 3$



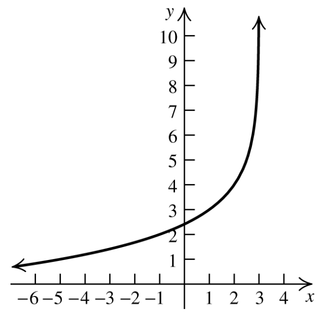
74. No transformation. Domain: $(0, \infty)$;
range: $(-\infty, \infty)$; asymptote: $x = 0$



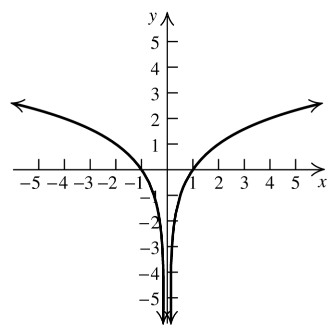
75. Shift the graph of $y = \log_2 x$ three units right, then reflect it about the *y*-axis, and then shift it 2 units up. Domain: $(-\infty, 3)$; range: $(-\infty, \infty)$;
asymptote: $x = 3$



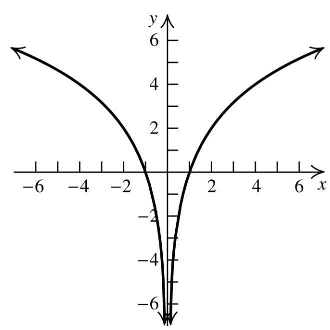
76. Shift the graph of $y = \log_2 x$ three units right, reflect it about the *y*-axis, reflect the resulting graph in the *x*-axis, and then shift it four units up. Domain: $(-\infty, 3)$; range: $(-\infty, \infty)$;
asymptote: $x = 3$



77. On $(-\infty, 0)$, reflect the graph of $y = \log_2 x$ about the *y*-axis. Domain: $(-\infty, 0) \cup (0, \infty)$;
range: $(-\infty, \infty)$; asymptote: $x = 0$



78. On $(-\infty, 0)$, reflect the graph of $y = \log_2 x$ about the *y*-axis. On $(-\infty, 0) \cup (0, \infty)$, stretch the graph vertically by a factor of *x*.
Domain: $(-\infty, 0) \cup (0, \infty)$; range: $(-\infty, \infty)$;
asymptote: $x = 0$



79. $\log_4(\log_3 81) = \log_4 4 = 1$ because $3^4 = 81$ and $4^1 = 4$

80. $\log_4[\log_3(\log_2 8)] = \log_4[\log_3 3] = \log_4 1 = 0$ because $2^3 = 8, 3^1 = 3,$ and $4^0 = 1$.

81. $\log_{\sqrt{2}} 2 = 2$ because $\sqrt{2}^2 = 2$.

82. $\log_{2\sqrt{2}} 8 = 2$ because $\left(2\sqrt{2}\right)^2 = 8$.

83. $\log_{\sqrt{2}} 4 = 4$ because $\left(\sqrt{2}\right)^4 = 4$.

84. $\log_{\sqrt{3}} 27 = 6$ because $\left(\sqrt{3}\right)^6 = 27.$

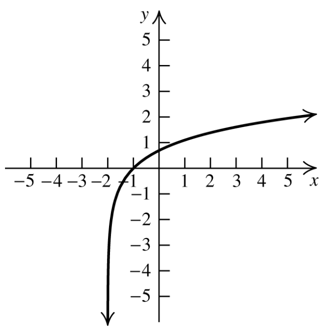
85. $\log x = 2 \Rightarrow 10^2 = 100 = x$

86. $\log(x-1) = 1 \Rightarrow 10^1 = x - 1 \Rightarrow x = 11$

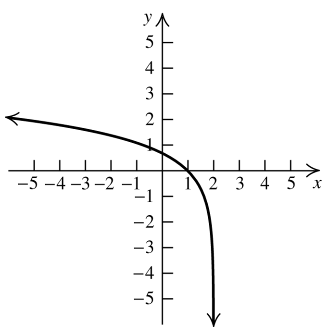
87. $\ln x = 1 \Rightarrow e^1 = e = x$

88. $\ln x = 0 \Rightarrow e^0 = 1 = x$

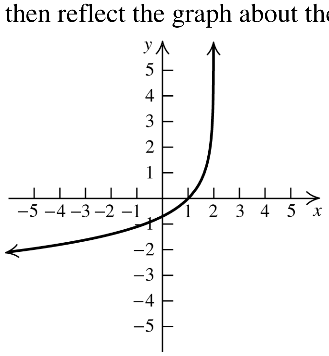
89. Shift the graph of $y = \ln x$ two units left.



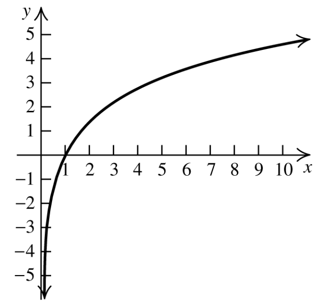
90. Shift the graph of $y = \ln x$ two units right, then reflect the graph about the y-axis.



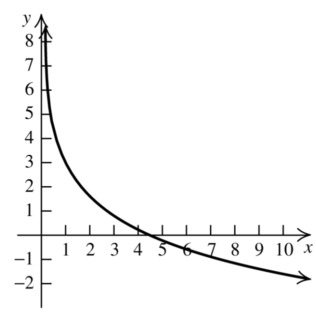
91. Shift the of $y = \ln x$ graph two units right, then reflect the graph about the y-axis, and then reflect the graph about the x-axis.



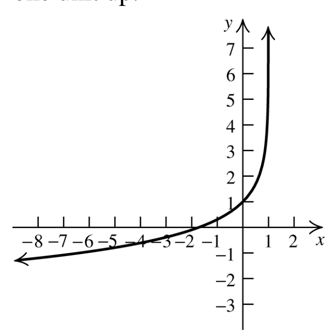
92. Stretch the graph of $y = \ln x$ vertically by a factor of 2.



93. Stretch the graph of $y = \ln x$ vertically by a factor of 2, reflect the resulting graph about the x-axis, and shift it three units up.



94. Shift the graph of $y = \ln x$ one unit right, reflect the graph about the y-axis, reflect the graph about the x-axis, then shift the graph one unit up.



4.2 Applying the Concepts

95. $2P = Pe^{0.08t} \Rightarrow 2 = e^{0.08t} \Rightarrow \ln 2 = 0.08t \Rightarrow$
$t \approx 8.66$ years

96. $120{,}000 = 10{,}000e^{0.1t} \Rightarrow 12 = e^{0.1t} \Rightarrow$
$\ln 12 = 0.1t \Rightarrow t \approx 24.85$ years

97. $2P = Pe^{6k} \Rightarrow 2 = e^{6k} \Rightarrow \ln 2 = 6k \Rightarrow$
$k \approx 0.1155 = 11.55\%$

98. $50{,}000 = 8000e^{25k} \Rightarrow 6.25 = e^{25k} \Rightarrow$
$\ln 6.25 = 25k \Rightarrow k \approx 0.0733 = 7.33\%$

99. a. The population in 2010 was 109.7% of the population in 2000.
$308 = 1.097A_0 \Rightarrow A_0 \approx 280.7657247$
The population in 2000 was about 280.8 million.

b. $308 = 280.7657247e^{10r} \Rightarrow$
$\frac{308}{280.7657247} = e^{10r} \Rightarrow$
$\ln\left(\frac{308}{280.7657247}\right) = 10r \Rightarrow$
$r = \frac{\ln\left(\frac{308}{280.7657247}\right)}{10} \approx 0.009257 \approx 0.93\%$

c. $400 = 308e^{0.0092t} \Rightarrow \frac{400}{308} = e^{0.0092t} \Rightarrow$
$\ln\left(\frac{400}{308}\right) = 0.0092t \Rightarrow t = \frac{\ln\left(\frac{400}{308}\right)}{0.0092} \approx 28.4$
years after 2010, or in the year 2039.

100. a. The population is 2010 was 112.33% of the population in 2000.
$35 = 1.1233A_0 \Rightarrow A_0 \approx 31.5819461$
The population was about 31.2 million in 2000.

b. $35 = 31.2e^{10r} \Rightarrow \frac{35}{31.5819461} = e^{10r} \Rightarrow$
$\ln\left(\frac{35}{31.5819461}\right) = 10r \Rightarrow$
$r = \frac{\ln \frac{35}{31.5819461}}{10} \approx 0.011627 \approx 1.16\%$

c. $70 = 35e^{0.011627t} \Rightarrow 2 = e^{0.011627t} \Rightarrow$
$\ln 2 = 0.011627t \Rightarrow$
$t = \frac{\ln 2}{0.011627} \approx 59.614$ yr or in the year 2070.

d. $50 = 35e^{0.011627t} \Rightarrow \frac{50}{35} = e^{0.011627t} \Rightarrow$
$\ln\left(\frac{50}{35}\right) = 0.011627t \Rightarrow$
$t = \frac{\ln\left(\frac{50}{35}\right)}{0.011627} \approx 30.676 \approx 30.7$ yr or in the year 2040.

101. Find k using $T = 50, T_0 = 75, T_s = 20,$ and $t = 1$ minute.
$50 = 20 + (75 - 20)e^{-k(1)} \Rightarrow \frac{30}{55} = e^{-k} \Rightarrow$
$\ln \frac{30}{55} = -k \Rightarrow 0.6061 \approx k$

a. **(i)** $T = 20 + (75 - 20)e^{-0.6061(5)} \Rightarrow$
$T \approx 22.66°\text{F}$

(ii) $T = 20 + (75 - 20)e^{-0.6061(10)} \Rightarrow$
$T \approx 20.13°\text{F}$

(iii) $T = 20 + (75 - 20)e^{-0.6061(60)} \Rightarrow$
$T \approx 20°\text{F}$

b. $22 = 20 + (75 - 20)e^{-0.6061t} \Rightarrow$
$\frac{2}{55} = e^{-0.6061t} \Rightarrow \ln \frac{2}{55} = -0.6061t \Rightarrow$
$t \approx 5.5$ minutes

102. Find k using $T = 40, T_0 = 32, T_s = 85,$ and $t = 30$ minutes.
$40 = 85 + (32 - 85)e^{-30k} \Rightarrow \frac{45}{53} = e^{-30k} \Rightarrow$
$\ln \frac{45}{53} = -30k \Rightarrow k \approx 0.005454$
$50 = 85 + (32 - 85)e^{-0.005454t} \Rightarrow$
$\ln \frac{35}{53} = -0.005454t \Rightarrow t \approx 76$ min

103. Find k using $T = 160, T_0 = 50, T_s = 400,$ and $t = 10$ minutes.
$160 = 400 + (50 - 400)e^{-10k} \Rightarrow \frac{240}{350} = e^{-10k} \Rightarrow$
$\ln \frac{24}{35} = -10k \Rightarrow k \approx 0.03773$
Use this value of k to find the time given $T = 220, T_0 = 50,$ and $T_s = 400.$
$220 = 400 + (50 - 400)e^{-0.03773t} \Rightarrow$
$\frac{18}{35} = e^{-0.03773t} \Rightarrow \ln \frac{18}{35} = -0.03773t \Rightarrow$
$t \approx 17.6$ minutes

104. Find k using $T = 84.8, T_0 = 85.7, T_s = 55,$ and $t = 30$ minutes.
$84.8 = 55 + (85.7 - 55)e^{-k(30)} \Rightarrow$
$29.8 = 30.7e^{-k(30)} \Rightarrow \ln \frac{29.8}{30.7} = -30k \Rightarrow$
$k \approx 0.000992$

(*continued on next page*)

(*continued*)

Now use this value of k to find the number of minutes that passed until the body was found.

$85.7 = 55 + (98.6 - 55)e^{-0.000992t} \Rightarrow$

$\frac{30.7}{43.6} = e^{-0.000992t} \Rightarrow \ln\frac{30.7}{43.6} = -0.001t \Rightarrow$

$t \approx 353$ minutes

The murder occurred 353 minutes before 2:10 A.M., or at 8:17 P.M., assuming that the temperature of the live body was 98.6°F.

105. a. $A(1) = 0.8(0.04) = 0.032$

Using the model $A(t) = A_0e^{rt}$, we have

$0.032 = 0.04e^{r} \Rightarrow 0.8 = e^{r} \Rightarrow r = \ln 0.8$

So, in one year (12 months), the concentration of the contaminant will be

$A(12) = 0.04e^{12\ln 0.8} = .0027 = 0.27\%$

b. $0.0001 \le 0.04e^{\ln 0.8t} \Rightarrow 0.0025 \le e^{\ln 0.8t} \Rightarrow$

$\ln 0.0025 \le \ln 0.8t \Rightarrow t \le 26.85$ months

106. a. $T(1) = \frac{3}{4}(16,000) = 12,000$

Using the model $T(n) = T_0e^{rn}$, we have

$12,000 = 16,000e^{r} \Rightarrow \frac{3}{4} = e^{r} \Rightarrow r = \ln(3/4)$

So $T(n) = 16,000e^{\ln(3/4)n} = 16,000(3/4)^{n}$.

b. $T(12) = 16,000e^{\ln(3/4)\cdot 12} = 506.82\text{ m}^3$

c. $0.2(16,000) = 16,000e^{\ln(3/4)t} \Rightarrow$

$0.2 = e^{\ln(3/4)t} \Rightarrow \ln 0.2 = \ln(3/4)t \Rightarrow$

$t \approx 5.6$ years

107. a. $3000 = 1500e^{3r} \Rightarrow \ln 2 = 3r \Rightarrow$

$r = \frac{\ln 2}{3} \approx 0.231$

Thus, the function is $P \approx 1500e^{0.231t}$. Alternatively, since the population doubles every three years, we have the geometric progression progression given by

$P = 1500 \cdot 2^{t/3}$.

b. $P = 1500e^{0.231(7)} \approx 7557$

There will be about 7557 sheep in the herd seven years from now.

c. $15,000 = 1500e^{0.231t} \Rightarrow 10 = e^{0.231t} \Rightarrow$

$\ln 10 = 0.231t \Rightarrow t = \frac{\ln 10}{0.231} \approx 10$

The herd will have 15,000 sheep about 10 years from now.

108. a. In two years, there will be $8/9$ the number of sharks than there are now.

$\frac{8}{9} = e^{2r} \Rightarrow \ln\left(\frac{8}{9}\right) = 2r \Rightarrow$

$r = \frac{\ln\left(\frac{8}{9}\right)}{2} \approx -0.059$

Thus, the function is $P = 150e^{-0.059t}$.

b. $150e^{-0.059(4)} \approx 118$

There will be about 118 sharks four years from now.

c. $35 = 150e^{-0.059t} \Rightarrow \frac{35}{150} = e^{-0.059t} \Rightarrow$

$\ln\left(\frac{35}{150}\right) = -0.059t \Rightarrow t = \frac{\ln\left(\frac{35}{150}\right)}{-0.059} \approx 24.7$

There will be 35 sharks in about 24.7 years.

109. a. $P_3 = \log 4 - \log 3 = \log\left(\frac{4}{3}\right) \approx 0.125$

About 12.5% of the data can be expected to have 3 as the first digit.

b. $P_1 = \log 2 - \log 1 \approx 0.3010$
$P_2 = \log 3 - \log 2 \approx 0.1761$
$P_3 = \log 4 - \log 3 \approx 0.1249$
$P_4 = \log 5 - \log 4 \approx 0.0969$
$P_5 = \log 6 - \log 5 \approx 0.0792$
$P_6 = \log 7 - \log 6 \approx 0.0669$
$P_7 = \log 8 - \log 7 \approx 0.0580$
$P_8 = \log 9 - \log 8 \approx 0.0512$
$P_9 = \log 10 - \log 9 \approx 0.0458$

$P_1 + P_2 + P_3 + \cdots P_8 + P_9 = 1$

This means that one of the digits $1 \ldots 9$ will appear as the first digit.

110. a. 2, 3, 5, 7, 11, 13, 17, 19

$\pi(20) = 8$

b. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

$\pi(50) = 15$

$\frac{50}{\ln 50} \approx 13$

c. $\pi(1{,}000{,}000) \approx \dfrac{1{,}000{,}000}{\ln 1{,}000{,}000} \approx 72{,}382$

4.2 Beyond the Basics

111. Since the domain of the logarithmic function is $(0,\infty)$, the expression $\dfrac{x-2}{x+1}$ must be positive. This occurs in the interval $(-\infty,-1)\cup(2,\infty)$, so the domain of $\log_3\left(\dfrac{x-2}{x+1}\right)$ is $(-\infty,-1)\cup(2,\infty)$.

112. Since the domain of the logarithmic function is $(0,\infty)$, the expression $\dfrac{x+3}{x-2}$ must be positive. This occurs in the interval $(-\infty,-3)\cup(2,\infty)$, so the domain of $\log_3\left(\dfrac{x+3}{x-2}\right)$ is $(-\infty,-3)\cup(2,\infty)$.

113. a. $h(x)=\log_3 x$ and $g(x)=\log_2 x$. So, $f(x)=g(h(x))$. The domain of $h(x)$ is $(0,\infty) \Rightarrow$ the domain of $f(x)$ is $(1,\infty)$.

b. $h(x)=\ln(x-1)$ and $g(x)=\log x$. So, $f(x)=g(h(x))$. The domain of $h(x)$ is $(1,\infty) \Rightarrow$ the domain of $f(x)$ is $(2,\infty)$.

c. $h(x)=\log(x-1)$ and $g(x)=\ln x$. So, $f(x)=g(h(x))$. The domain of $h(x)$ is $(1,\infty) \Rightarrow$ the domain of $f(x)$ is $(2,\infty)$.

d. $h(x)=\log(x-1)$ and $g(x)=\log x$. So, $f(x)=g(g(h(x)))$. The domain of $h(x)$ is $(1,\infty)$. The domain of $g(h(x))$ is $(2,\infty)$. (Note that $\log(x-1)=0 \Rightarrow x=2$.) So, the domain of $g(g(h(x)))$ is $(11,\infty)$.

114. a. $y=\log_2(\log_3 x)$. Switch the variables and then solve for y:
$x=\log_2(\log_3 y) \Rightarrow 2^x=\log_3 y \Rightarrow 3^{2^x}=y$

b. $y=\log(\ln(x-1))$. Switch the variables and then solve for y:
$x=\log(\ln(y-1)) \Rightarrow 10^x=\ln(y-1) \Rightarrow$
$e^{10^x}=y-1 \Rightarrow e^{10^x}+1=y$

c. $y=\ln(\log(x-1))$. Switch the variables and then solve for y: $x=\ln(\log(y-1)) \Rightarrow$
$e^x=\log(y-1) \Rightarrow 10^{e^x}=y-1 \Rightarrow 10^{e^x}+1=y$

d. $y=\log(\log(\log(x-1)))$. Switch the variables and then solve for y:
$x=\log(\log(\log(y-1))) \Rightarrow$
$10^x=\log(\log(y-1)) \Rightarrow$
$10^{10^x}=\log(y-1) \Rightarrow 10^{10^{10^x}}+1=y$

115. $y=\log x \to y=\log(x-2) \to$
$y=\log\left(\frac{1}{2}x-2\right) \to y=3\log\left(\frac{1}{2}x-2\right) \to$
$y=-3\log\left(\frac{1}{2}x-2\right) \to y=-3\log\left(\frac{1}{2}x-2\right)+4$

116. $y=\log x \to y=\log(2+x) \to$
$y=4\log(2+x) \to y=4\log(2+3x) \to$
$y=4\log(2-3x) \to y=-4\log(2-3x) \to$
$y=-4\log(2-3x)+1$

117. a. $P=100{,}000e^{-0.07(20)}=\$24{,}659.69$

b. $50{,}000=75{,}000e^{-10r} \Rightarrow \frac{2}{3}=e^{-10r} \Rightarrow$
$\ln(2/3)=-10r \Rightarrow r\approx 0.0405=4.05\%$

118. a. Using the simple interest formula, we find that $50{,}000=P(0.05)(1) \Rightarrow P=1{,}000{,}000$. So we need \$1,000,000 in order to earn \$50,000 per year. To determine the amount in the account at age 40, we have
$1{,}000{,}000=Pe^{(0.05\cdot 25)} \Rightarrow P\approx \$286{,}504.80$.

b. Using the simple interest formula, we find that $50{,}000=P(0.08)(1) \Rightarrow P=\$625{,}000$. So we need \$625,000 in order to earn \$50,000 per year. To determine the amount in the account at age 40, we have
$625{,}000=Pe^{(0.08\cdot 25)} \Rightarrow P\approx \$84{,}584.55$.

c. Using the simple interest formula, we find that $50{,}000=P(0.1)(1) \Rightarrow P=500{,}000$. So we need \$500,000 in order to earn \$50,000 per year. To determine the amount in the account at age 40, we have
$500{,}000=Pe^{(0.1\cdot 25)} \Rightarrow P\approx \$41{,}042.50$.

4.2 Critical Thinking/Discussion/Writing

119. $2^{\log_2 3}-3^{\log_3 2}=3-2=1$

120. $\log_3 4 = \log_3\left(2^2\right) = 2\log_3 2$

$\log_2 9 = \log_2\left(3^2\right) = 2\log_2 3$

Let $x = \log_3 2$ and let $y = \log_2 3$.

$$\begin{aligned}&\left(\log_3 4 + \log_2 9\right)^2 - \left(\log_3 4 - \log_2 9\right)^2\\ &= (2x+2y)^2 - (2x-2y)^2\\ &= \left[(2x+2y)+(2x-2y)\right]\cdot\\ &\qquad\left[(2x+2y)-(2x-2y)\right]\\ &= (4x)(4y) = 16xy\end{aligned}$$

Note that

$x = \log_3 2 \Rightarrow 3^x = 2$ and $y = \log_2 3 \Rightarrow 2^y = 3$

$3^{xy} = \left(3^x\right)^y = 2^y = 3 \Rightarrow 3^{xy} = 3 \Rightarrow xy = 1$

Alternatively,

$xy = \log_3 2 \cdot \log_2 3 = \dfrac{\log 2}{\log 3}\cdot\dfrac{\log 3}{\log 2} = 1.$

Therefore, $16xy = 16\cdot 1 = 16,$ so

$\left(\log_3 4 + \log_2 9\right)^2 - \left(\log_3 4 - \log_2 9\right)^2 = 16.$

121. $\log_3\left[\log_4\left(\log_2 x\right)\right] = 0 \Rightarrow \log_4\left(\log_2 x\right) = 3^0 \Rightarrow$

$\log_4\left(\log_2 x\right) = 1 \Rightarrow \log_2 x = 4 \Rightarrow x = 2^4 = 16$

122. a. $f(x) = \left|\log x\right| = \begin{cases} -\log x & \text{if } 0 < x < 1 \\ \log x & \text{if } x \ge 1\end{cases}$

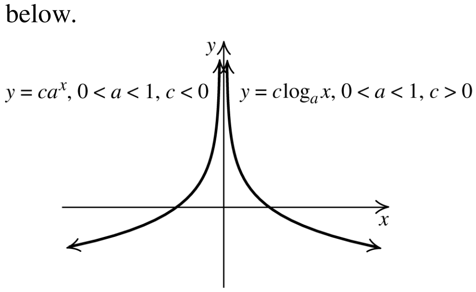
b. $g(x) = \left|\ln(x-1)\right| + \left|\ln(x-2)\right|$

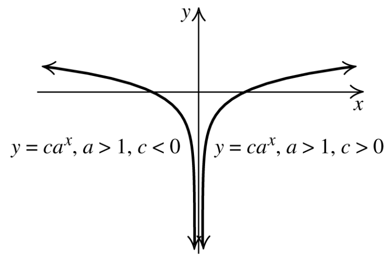
$= \begin{cases} \ln(x-1) - \ln(x-2) & \text{if } 2 < x < 3 \\ \ln(x-1) + \ln(x-2) & \text{if } x \ge 3\end{cases}$

123. a. Yes, the statement is always true.

b. The increasing property is used.

124. There are four possibilities, $0 < a < 1$ with $c < 0$, $0 < a < 1$ with $c > 0$, $a > 1$ with $c < 0$, and $a > 1$ with $c > 0$. They are illustrated below.





4.2 Maintaining Skills

125. $a^2 \cdot a^7 = a^9$

126. $\left(a^2\right)^3 = a^{2\cdot 3} = a^6$

127. $\sqrt{a^8} = \left(a^8\right)^{1/2} = a^{8\cdot(1/2)} = a^4$

128. $\sqrt[3]{a^6} = \left(a^6\right)^{1/3} = a^{6(1/3)} = a^2$

129. $\left(\dfrac{243}{32}\right)^{4/5} = \left[\left(\dfrac{243}{32}\right)^{1/5}\right]^4 = \left(\dfrac{3}{2}\right)^4$

130. $\sqrt[5]{32^{-3}} = \sqrt[5]{\left(\dfrac{1}{32}\right)^3} = \left(\dfrac{1}{32}\right)^{3(1/5)} = \left[\left(\dfrac{1}{32}\right)^{(1/5)}\right]^3$

$= \left(\dfrac{1}{2}\right)^3$

131. $\left(4.7\times 10^7\right)\left(8.1\times 10^5\right) = (4.7\times 8.1)\left(10^7\times 10^5\right)$

$= 38.07\times 10^{12}$

$= 3.807\times 10^{13}$

132. $\dfrac{7.2\times 10^6}{2.4\times 10^{-3}} = \dfrac{7.2}{2.4}\times\dfrac{10^6}{10^{-3}} = 3\times 10^9$

133. $\log_3 81 = 4$ because $3^4 = 81.$

$\log_3 3 + \log_3 27 = 1 + 3 = 4$ because $\log_3 3 = 1$ and $3^3 = 27.$ Therefore,

$\log_3 81 = \log_3 3 + \log_3 27.$

134. $\log_2 8 = 3$ because $2^3 = 8.$

$\log_2 128 - \log_2 16 = 7 - 4 = 3$ because $2^7 = 128$ and $2^4 = 16.$

135. $\log_2 16 = 4$ because $2^4 = 16.$

$2\log_2 4 = 2\cdot 2 = 4$ because $2^2 = 4.$

Therefore, $\log_2 16 = 2\log_2 4.$

136. $\log_4 64 = 3$ because $4^3 = 64$.
$\frac{\log_2 64}{\log_2 4} = \frac{6}{2} = 3$ because $2^6 = 64$ and
$2^2 = 4$. Therefore, $\log_4 64 = \frac{\log_2 64}{\log_2 4}$.

4.3 Rules of Logarithms

4.3 Practice Problems

1. Given $\log_5 z = 3$ and $\log_5 y = 2$

a. $\log_5 (y/z) = \log_5 y - \log_5 z = 2 - 3 = -1$

b. $\log_5 (y^2 z^3) = \log_5 (y^2) + \log_5 (z^3)$
$= 2\log_5 y + 3\log_5 z$
$= 2 \cdot 2 + 3 \cdot 3 = 13$

2. a. $\ln \frac{2x-1}{x+4} = \ln(2x-1) - \ln(x+4)$

b. $\log \sqrt{\frac{4xy}{z}} = \log\left(\frac{4xy}{z}\right)^{1/2} = \frac{1}{2}\log\left(\frac{4xy}{z}\right)$
$= \frac{1}{2}(\log 4xy - \log z)$
$= \frac{1}{2}(\log 4 + \log x + \log y - \log z)$
$= \frac{1}{2}\left(\log\left(2^2\right) + \log x + \log y - \log z\right)$
$= \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y - \frac{1}{2}\log z$

3. $\frac{1}{2}\left[\log(x+1) + \log(x-1)\right]$
$= \frac{1}{2}\left[\log((x+1)(x-1))\right]$
$= \frac{1}{2}\log\left(x^2-1\right) = \log\left(x^2-1\right)^{1/2}$
$= \log\sqrt{x^2-1}$

4. $K = 234^{567}$
$\log K = \log\left(234^{567}\right)$
$\log K = 567 \log 234 \approx 1343.345391$
Since log K lies between the integers 1343 and 1344, the number K requires 1344 digits to the left of the decimal point. By definition of the common logarithm, we have
$K = 10^{1343.345391} = 10^{0.345391} \times 10^{1343}$
$\approx 2.215088 \times 10^{1343}$

5. $\log_3 15 = \frac{\log 15}{\log 3} \approx 2.46497$

6. a., b., Substitute (3, 3) and (9, 1) in the equation $y = c + b\log x$ to obtain
$3 = c + b\log 3$ (1)
$1 = c + b\log 9$ (2)
Subtract equation (1) from equation (2) and solve the resulting equation for b.
$-2 = b\log 9 - b\log 3 = b(\log 9 - \log 3)$
$= b\log\frac{9}{3} = b\log 3$
$b = -\frac{2}{\log 3}$
Substitute the value for b into equation (1) and solve for c.
$3 = c + \left(-\frac{2}{\log 3}\right)\log 3 \Rightarrow 3 = c - 2 \Rightarrow c = 5$
Substituting the values for b and c into $y = c + b\log x$ gives
$y = 5 - \frac{2}{\log 3}(\log x) = 5 - 2\left(\frac{\log x}{\log 3}\right)$
$= 5 - 2\log_3 x$

7. $\ln\left(\frac{A(t)}{A_0}\right) = kt \Rightarrow \ln\left(\frac{66}{100}\right) = 15k \Rightarrow$
$\ln(0.66) = 15k \Rightarrow k = \frac{\ln(0.66)}{15}$
To find the half-life, we use the formula
$h = -\frac{\ln 2}{k} = -\frac{\ln 2}{\ln(0.66)/15} \approx 25.$
The half-life of strontium-90 is about 25 years.

8. King Tut died in 1346 B.C., so the object was made in 1540 B.C., so the time elapsed between when the object was made and 1960 is 1540 + 1960 = 3500 years. The decay function for carbon-14 in exponential form is $A(t) = A_0 e^{-0.0001216t}$. (See example 7in the text.) Let x = the percent of the original amount of carbon-14 in the object remaining after t years. Then
$xA_0 = A_0 e^{-0.0001216t} \Rightarrow x = e^{-0.0001216t}$.
t = 3500, so
$x = e^{-0.0001216(3500)} \approx 0.6534 \approx 65.34\%$.

4.3 Basic Concepts and Skills

1. $\log_a MN = \underline{\log_a M} + \underline{\log_a N}$.

2. $\underline{\log_a \frac{M}{N}} = \log_a M - \log_a N$.

3. $\log_a M^r = \underline{r \log_a M}$.

4. The change-of-base formula using base e is $\log_a M = \underline{\dfrac{\ln M}{\ln a}}$.

5. False. There is no rule for the logarithm of a sum. $\log_a u + \log_a v = \log_a(uv)$.

6. True. $\log \dfrac{x}{10} = \log x - \log 10 = \log x - 1$.

7. $\log 6 = \log(2 \cdot 3) = \log 2 + \log 3 = 0.3 + 0.48 = 0.78$

8. $\log 4 = \log(2 \cdot 2) = \log 2 + \log 2 = 0.3 + 0.3 = 0.6$

9. $\log 5 = \log\left(\dfrac{10}{2}\right) = \log 10 - \log 2 = 1 - 0.3 = 0.7$

10. $\log(3x) = \log 3 + \log x = 0.48 + 2 = 2.48$

11. $\log\left(\dfrac{2}{x}\right) = \log 2 - \log x = 0.3 - 2 = -1.7$

12. $\log x^2 = 2\log x = 2(2) = 4$

13. $\log(2x^2 y) = \log 2 + 2\log x + \log y = 0.3 + 2(2) + 3 = 7.3$

14. $\log(xy^3) = \log x + 3\log y = 2 + 3(3) = 11$

15. $\log \sqrt[3]{x^2 y^4} = \log(x^{2/3} y^{4/3}) = \dfrac{2}{3}\log x + \dfrac{4}{3}\log y = \dfrac{2}{3}(2) + \dfrac{4}{3}(3) = \dfrac{16}{3}$

16. $\log(\log x^2) = \log(2\log x) = \log(2 \cdot 2) = \log 2 + \log 2 = 0.3 + 0.3 = 0.6$

17. $\log \sqrt[3]{48} = \log(2^4 \cdot 3)^{1/3} = \dfrac{4}{3}\log 2 + \dfrac{1}{3}\log 3 = \dfrac{4}{3}(0.3) + \dfrac{1}{3}(0.48) = 0.56$

18. $\log_2 3 = \dfrac{\log 3}{\log 2} = \dfrac{0.48}{0.3} = 1.6$

19. $\ln[x(x-1)] = \ln x + \ln(x-1)$

20. $\ln \dfrac{x(x+1)}{(x-1)^2} = \ln(x(x+1)) - \ln(x-1)^2 = \ln x + \ln(x+1) - 2\ln(x-1)$

21. $\log_a \sqrt{x}\, y^3 = \log_a \sqrt{x} + \log_a y^3 = \log_a x^{1/2} + \log_a y^3 = \dfrac{1}{2}\log_a x + 3\log_a y$

22. $\log_a \dfrac{3x^2}{\sqrt{y}} = \log_a 3x^2 - \log_a \sqrt{y} = \log_a 3 + \log_a x^2 - \log_a y^{1/2} = \log_a 3 + 2\log_a x - \dfrac{1}{2}\log_a y$

23. $\log_a \sqrt[3]{\dfrac{x}{y}} = \log_a \left(\dfrac{x}{y}\right)^{1/3} = \dfrac{1}{3}\log_a\left(\dfrac{x}{y}\right) = \dfrac{1}{3}\log_a x - \dfrac{1}{3}\log_a y$

24. $\log_a \sqrt[3]{\dfrac{x^2}{y^5}} = \log_a \left(\dfrac{x^2}{y^5}\right)^{1/3} = \dfrac{1}{3}\log_a\left(\dfrac{x^2}{y^5}\right) = \dfrac{1}{3}\log_a x^2 - \dfrac{1}{3}\log_a y^5 = \dfrac{2}{3}\log_a x - \dfrac{5}{3}\log_a y$

25. $\log_2 \sqrt[4]{\dfrac{xy^2}{8}} = \log_2 \left(\dfrac{xy^2}{8}\right)^{1/4} = \dfrac{1}{4}\log_2\left(\dfrac{xy^2}{8}\right)$
$= \dfrac{1}{4}\log_2\left(xy^2\right) - \dfrac{1}{4}\log_2 8$
$= \dfrac{1}{4}\log_2 x + \dfrac{1}{4}\log_2 y^2 - \dfrac{1}{4}(3)$
$= \dfrac{1}{4}\log_2 x + 2 \cdot \dfrac{1}{4}\log_2 y - \dfrac{3}{4}$
$= \dfrac{1}{4}\log_2 x + \dfrac{1}{2}\log_2 y - \dfrac{3}{4}$

26. $\log \sqrt[3]{\dfrac{x^2 y}{100}} = \log\left(\dfrac{x^2 y}{100}\right)^{1/3} = \dfrac{1}{3}\log\left(\dfrac{x^2 y}{100}\right)$
$= \dfrac{1}{3}\log\left(x^2 y\right) - \dfrac{1}{3}\log 100$
$= \dfrac{1}{3}\log x^2 + \dfrac{1}{3}\log y - \dfrac{1}{3}(2)$
$= \dfrac{2}{3}\log x + \dfrac{1}{3}\log y - \dfrac{2}{3}$

27. $\log \dfrac{\sqrt{x^2+1}}{x+3} = \log \sqrt{x^2+1} - \log(x+3) = \dfrac{1}{2}\log(x^2+1) - \log(x+3)$

28. $\log_4\left(\dfrac{x^2-9}{x^2-6x+8}\right)^{2/3} = \dfrac{2}{3}\log_4(x^2-9)-\dfrac{2}{3}\log_4(x^2-6x+8)$

$= \left[\dfrac{2}{3}\log_4((x-3)(x+3))\right]-\left[\dfrac{2}{3}\log_4((x-2)(x-4))\right]$

$= \left[\dfrac{2}{3}\log_4(x-3)+\dfrac{2}{3}\log_4(x+3)\right]-\dfrac{2}{3}\left(\log_4(x-2)+\log_4(x-4)\right)$

$= \dfrac{2}{3}\log_4(x-3)+\dfrac{2}{3}\log_4(x+3)-\dfrac{2}{3}\log_4(x-2)-\dfrac{2}{3}\log_4(x-4)$

29. $\log_b x^2y^3z = 2\log_b x + 3\log_b y + \log_b z$

30. $\log_b \sqrt{xyz} = \log_b (xyz)^{1/2} = \dfrac{1}{2}\log_b x + \dfrac{1}{2}\log_b y + \dfrac{1}{2}\log_b z$

31. $\ln\left(\dfrac{x\sqrt{x-1}}{x^2+2}\right) = \ln\left(x\sqrt{x-1}\right)-\ln\left(x^2+2\right) = \ln x + \ln\left((x-1)^{1/2}\right)-\ln\left(x^2+2\right) = \ln x + \dfrac{1}{2}\ln(x-1)-\ln\left(x^2+2\right)$

32. $\ln\left(\dfrac{\sqrt{x-2}\sqrt[3]{x+1}}{x^2+3}\right) = \ln\left(\sqrt{x-2}\sqrt[3]{x+1}\right)-\ln\left(x^2+3\right) = \ln\left((x-2)^{1/2}\right)+\ln\left((x+1)^{1/3}\right)-\ln\left(x^2+3\right)$

$= \dfrac{1}{2}\ln(x-2)+\dfrac{1}{3}\ln(x+1)-\ln\left(x^2+3\right)$

33. $\ln\left(\dfrac{(x+1)^2}{(x-3)\sqrt{x+4}}\right) = \ln\left((x+1)^2\right)-\ln\left((x-3)\sqrt{x+4}\right) = 2\ln(x+1)-\left(\ln(x-3)+\ln(x+4)^{1/2}\right)$

$= 2\ln(x+1)-\ln(x-3)-\dfrac{1}{2}\ln(x+4)$

34. $\ln\left(\dfrac{2x+3}{(x+4)^2(x-3)^4}\right) = \ln(2x+3)-\ln\left((x+4)^2(x-3)^4\right) = \ln(2x+3)-\left(\ln\left((x+4)^2\right)+\ln\left((x-3)^4\right)\right)$

$= \ln(2x+3)-2\ln(x+4)-4\ln(x-3)$

35. $\ln\left((x+1)\sqrt{\dfrac{x^2+2}{x^2+5}}\right) = \ln(x+1)+\ln\left(\dfrac{x^2+2}{x^2+5}\right)^{1/2} = \ln(x+1)+\dfrac{1}{2}\left(\ln\left(x^2+2\right)-\ln\left(x^2+5\right)\right)$

$= \ln(x+1)+\dfrac{1}{2}\ln\left(x^2+2\right)-\dfrac{1}{2}\ln\left(x^2+5\right)$

36. $\ln\left(\dfrac{\sqrt[3]{2x+1}(x+1)}{(x-1)^2(3x+2)}\right) = \ln\left(\sqrt[3]{2x+1}(x+1)\right)-\ln\left((x-1)^2(3x+2)\right)$

$= \dfrac{1}{3}\ln(2x+1)+\ln(x+1)-\left[2\ln(x-1)+\ln(3x+2)\right]$

$= \dfrac{1}{3}\ln(2x+1)+\ln(x+1)-2\ln(x-1)-\ln(3x+2)$

37. $\ln\left(\dfrac{x^3(3x+1)^4}{\sqrt{x^2+1}(x+2)^{-5}(x-3)^2}\right) = \ln\left(\dfrac{x^3(3x+1)^4(x+2)^5}{\left(x^2+1\right)^{1/2}(x-3)^2}\right)$

$= \ln\left[x^3(3x+1)^4(x+2)^5\right]-\ln\left[\left(x^2+1\right)^{1/2}(x-3)^2\right]$

$= 3\ln x + 4\ln(3x+1)+5\ln(x+2)-\dfrac{1}{2}\ln\left(x^2+1\right)-2\ln(x-3)$

38. $$\ln\left(\frac{(x+1)^{1/2}\left(x^2-2\right)^{2/5}}{(2x-1)^{3/2}\left(x^2+2\right)^{-4/5}}\right)=\ln\left(\frac{(x+1)^{1/2}\left(x^2-2\right)^{2/5}\left(x^2+2\right)^{4/5}}{(2x-1)^{3/2}}\right)$$
$$=\ln\left[(x+1)^{1/2}\left(x^2-2\right)^{2/5}\left(x^2+2\right)^{4/5}\right]-\ln\left[(2x-1)^{3/2}\right]$$
$$=\frac{1}{2}\ln(x+1)+\frac{2}{5}\ln\left(x^2-2\right)+\frac{4}{5}\ln\left(x^2+2\right)-\frac{3}{2}\ln(2x-1)$$

39. $\log_2 x+\log_2 7=\log_2(7x)$

40. $\log_2 x-\log_2 3=\log_2\left(\frac{x}{3}\right)$

41. $\frac{1}{2}\log x-\log y+\log z=\log\left(\frac{z\sqrt{x}}{y}\right)$

42. $\frac{1}{2}(\log x+\log y)=\log(xy)^{1/2}=\log\sqrt{xy}$

43. $$\frac{1}{5}(\log_2 z+2\log_2 y)=\log_2\left(y^2z\right)^{1/5}$$
$$=\log_2\sqrt[5]{y^2z}$$

44. $$\frac{1}{3}(\log x-2\log y+3\log z)$$
$$=\frac{1}{3}\log\left(\frac{xz^3}{y^2}\right)=\log\left(\frac{xz^3}{y^2}\right)^{1/3}=\log\sqrt[3]{\frac{xz^3}{y^2}}$$

45. $\ln x+2\ln y+3\ln z=\ln(xy^2z^3)$

46. $2\ln x-3\ln y+4\ln z=\ln\left(\frac{x^2z^4}{y^3}\right)$

47. $$2\ln x-\frac{1}{2}\ln(x^2+1)=\ln x^2-\ln\sqrt{x^2+1}$$
$$=\ln\left(\frac{x^2}{\sqrt{x^2+1}}\right)$$

48. $$2\ln x+\frac{1}{2}\ln(x^2-1)-\frac{1}{2}\ln(x^2+1)$$
$$=\ln\left(\frac{x^2\sqrt{x^2-1}}{\sqrt{x^2+1}}\right)$$

49. $$K=e^{500}$$
$$\log K=\log\left(e^{500}\right)=500\log e=217.147241\Rightarrow$$
$$K=10^{217.147241}=10^{0.147241}\times10^{217}$$
$$\approx1.4036\times10^{217}$$

50. $$K=\pi^{650}$$
$$\log K=\log\left(\pi^{650}\right)=650\log\pi$$
$$=323.1474173\Rightarrow$$
$$K=10^{323.1474173}=10^{0.1474173}\times10^{323}$$
$$\approx1.4042\times10^{323}$$

51. $$K=324^{756}$$
$$\log K=\log\left(324^{756}\right)=756\log324$$
$$=1897.972028\Rightarrow$$
$$K=10^{1897.972028}=10^{0.972028}\times10^{1897}$$
$$\approx9.3762\times10^{1897}$$

52. $$K=723^{416}$$
$$\log K=\log\left(723^{416}\right)=416\log723$$
$$=1189.401532\Rightarrow$$
$$K=10^{1189.401532}=10^{0.401532}\times10^{1189}$$
$$\approx2.5208\times10^{1189}$$

53. $$K=234^{567}$$
$$\log K=\log\left(234^{567}\right)=567\log234$$
$$=1343.345$$
$$M=567^{234}$$
$$\log M=\log\left(567^{234}\right)=234\log567$$
$$=664.338$$
Since $\log K>\log M$, $K>M$. Thus, $234^{567}>567^{234}$.

54. $$K=4321^{8765}$$
$$\log K=\log\left(4321^{8765}\right)=8765\log4321$$
$$=31865.90$$
$$M=8765^{4321}$$
$$\log M=\log\left(8765^{4321}\right)=4321\log8765$$
$$=17036.63$$
Since $\log K>\log M$, $K>M$. Thus, $4321^{8765}>8765^{4321}$.

55. $K = 17^{200} \cdot 53^{67}$

$\log K = \log\left(17^{200} \cdot 53^{67}\right)$
$= \log\left(17^{200}\right) + \log\left(53^{67}\right)$
$= 200\log 17 + 67\log 53$
$= 361.6163$

There are 362 digits in the given product.

56. $K = 67^{200} \div 23^{150}$

$\log K = \log\left(67^{200} \div 23^{150}\right)$
$= \log\left(67^{200}\right) - \log\left(23^{150}\right)$
$= 200\log 67 - 150\log 23$
$= 160.9558$

There are 161 digits in the given quotient.

57. $\log_2 5 = \dfrac{\log 5}{\log 2} \approx 2.322$

58. $\log_4 11 = \dfrac{\log 11}{\log 4} \approx 1.730$

59. $\log_{1/2} 3 \approx -1.585$

60. $\log_{\sqrt{3}} 12.5 = \dfrac{\log 12.5}{\log\sqrt{3}} \approx 4.598$

61. $\log_{\sqrt{5}} \sqrt{17} = \dfrac{\log\sqrt{17}}{\log\sqrt{5}} \approx 1.760$

62. $\log_{15} 123 = \dfrac{\log 123}{\log 15} \approx 1.777$

63. $\log_2 7 + \log_4 3 = \dfrac{\log 7}{\log 2} + \dfrac{\log 3}{\log 4} \approx 3.6$

64. $\log_2 9 - \log_{\sqrt{2}} 5 = \dfrac{\log 9}{\log 2} - \dfrac{\log 5}{\log\sqrt{2}} \approx -1.474$

65. $\log_3 \sqrt{3} = \log_3 3^{1/2} = \dfrac{1}{2}$

66. $\log_{1/4} 4 = \log_{1/4}\left(\dfrac{1}{4}\right)^{-1} = -1$

67. $\log_3(\log_2 8) = \log_3(\log_2 2^3) = \log_3 3 = 1$

68. $2^{\log_2 2} = 2^1 = 2$

69. $5^{2\log_5 3 + \log_5 2} = 5^{\log_5 3^2 + \log_5 2} = 5^{\log_5(9\cdot 2)}$
$= 5^{\log_5 18} = 18$

70. $e^{3\ln 2 - 2\ln 3} e^{\ln(2^3 \div 3^2)} = \dfrac{8}{9}$

71. $\log 4 + 2\log 5 = \log(4\cdot 5^2) = \log 100$
$= \log 10^2 = 2$

72. $\log_2 160 - \log_2 5 = \log_2\left(\dfrac{160}{5}\right) = \log_2 32$
$= \log_2 2^5 = 5$

73. Substitute (10, 1) and (1, 2) in the equation $y = c + b\log x$ to obtain

$1 = c + b\log 10$ (1)
$2 = c + b\log 1$ (2)

Subtract equation (1) from equation (2) and solve the resulting equation for *b*.

$1 = b\log 1 - b\log 10 = b\cdot 0 - b\cdot 1 = -b \Rightarrow$
$b = -1$

Substitute the value for *b* into equation (1) and solve for *c*.

$1 = c - \log 10 \Rightarrow 1 = c - 1 \Rightarrow c = 2$

Substituting the values for *b* and *c* into $y = c + b\log x$ gives $y = 2 - \log x$.

74. Substitute (4, 10) and (2, 12) in the equation $y = c + b\log x$ to obtain

$10 = c + b\log 4$ (1)
$12 = c + b\log 2$ (2)

Subtract equation (2) from equation (1) and solve the resulting equation for *b*.

$-2 = b\log 4 - b\log 2 = b\left(\log 4 - \log 2\right)$
$= b\log 2 \Rightarrow$
$b = \dfrac{-2}{\log 2}$

Substitute the value for *b* into equation (2) and solve for *c*.

$12 = c + \left(\dfrac{-2}{\log 2}\right)\log 2 \Rightarrow 12 = c - 2 \Rightarrow c = 14$

Substituting the values for *b* and *c* into $y = c + b\log x$ gives

$y = 14 - \left(\dfrac{2}{\log 2}\right)\log x = 14 - 2\left(\dfrac{\log x}{\log 2}\right)$
$= 14 - 2\log_2 x.$

75. Substitute (*e*, 1) and (1, 2) in the equation $y = c + b\log x$ to obtain

$1 = c + b\log e$ (1)
$2 = c + b\log 1$ (2)

Since log 1 = 0, equation (2) becomes *c* = 2.

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(*continued*)

Substitute the value for c into equation (1) and solve for b.

$$1 = 2 + b\log e \Rightarrow -1 = b\log e \Rightarrow b = -\frac{1}{\log e}$$

Substituting the values for b and c into $y = c + b\log x$ gives

$$y = 2 - \left(\frac{1}{\log e}\right)\log x = 2 - \left(\frac{\log x}{\log e}\right) = 2 - \ln x.$$

76. Substitute (3, 1) and (9, 2) in the equation $y = c + b\log x$ to obtain

$$1 = c + b\log 3 \quad (1)$$
$$2 = c + b\log 9 \quad (2)$$

Subtract equation (1) from equation (2) and solve the resulting equation for b.

$$1 = b\log 9 - b\log 3 = b(\log 9 - \log 3)$$
$$= b\log\left(\frac{9}{3}\right) = b\log 3 \Rightarrow$$
$$b = \frac{1}{\log 3}$$

Substitute the value for b into equation (1) and solve for c.

$$1 = c + \left(\frac{1}{\log 3}\right)\log 3 \Rightarrow 1 = c + 1 \Rightarrow c = 0$$

Substituting the values for b and c into $y = c + b\log x$ gives

$$y = \left(\frac{1}{\log 3}\right)\log x = \frac{\log x}{\log 3} = \log_3 x.$$

77. Substitute (5, 4) and (25, 7) in the equation $y = c + b\log x$ to obtain

$$4 = c + b\log 5 \quad (1)$$
$$7 = c + b\log 25 \quad (2)$$

Subtract equation (1) from equation (2) and solve the resulting equation for b.

$$3 = b\log 25 - b\log 5 = b(\log 25 - \log 5)$$
$$= b\log\left(\frac{25}{5}\right) = b\log 5 \Rightarrow$$
$$b = \frac{3}{\log 5}$$

Substitute the value for b into equation (1) and solve for c.

$$4 = c + \left(\frac{3}{\log 5}\right)\log 5 \Rightarrow 4 = c + 3 \Rightarrow c = 1$$

Substituting the values for b and c into $y = c + b\log x$ gives

$$y = 1 + \left(\frac{3}{\log 5}\right)\log x = 1 + 3\left(\frac{\log x}{\log 5}\right)$$
$$= 1 + 3\log_5 x.$$

78. Substitute (4, 3) and (8, 5) in the equation $y = c + b\log x$ to obtain

$$3 = c + b\log 4 \quad (1)$$
$$5 = c + b\log 8 \quad (2)$$

Subtract equation (1) from equation (2) and solve the resulting equation for b.

$$2 = b\log 8 - b\log 4 = b(\log 8 - \log 4)$$
$$= b\log\left(\frac{8}{4}\right) = b\log 2 \Rightarrow$$
$$b = \frac{2}{\log 2}$$

Substitute the value for b into equation (1) and solve for c.

$$3 = c + \left(\frac{2}{\log 2}\right)\log 4 = c + \left(\frac{2}{\log 2}\right)\log\left(2^2\right)$$
$$= c + 2\left(\frac{2}{\log 2}\right)\log 2 = c + 4 \Rightarrow$$
$$3 = c + 4 \Rightarrow -1 = c$$

Substituting the values for b and c into $y = c + b\log x$ gives

$$y = -1 + \left(\frac{2}{\log 2}\right)\log x = -1 + 2\left(\frac{\log x}{\log 2}\right)$$
$$= -1 + 2\log_2 x.$$

79. Substitute (2, 4) and (4, 9) in the equation $y = c + b\log x$ to obtain

$$4 = c + b\log 2 \quad (1)$$
$$9 = c + b\log 4 \quad (2)$$

Subtract equation (1) from equation (2) and solve the resulting equation for b.

$$5 = b\log 4 - b\log 2 = b(\log 4 - \log 2)$$
$$= b\log\left(\frac{4}{2}\right) \Rightarrow 5 = b\log 2 \Rightarrow b = \frac{5}{\log 2}$$

Substitute the value for b into equation (1) and solve for c.

$$4 = c + \left(\frac{5}{\log 2}\right)\log 2 = c + 5 \Rightarrow -1 = c$$

Substituting the values for b and c into $y = c + b\log x$ gives

$$y = -1 + \left(\frac{5}{\log 2}\right)\log x = -1 + 5\left(\frac{\log x}{\log 2}\right)$$
$$= -1 + 5\log_2 x.$$

80. Substitute (1, 1) and (5, 7) in the equation $y = c + b\log x$ to obtain

$1 = c + b\log 1 \quad (1)$
$7 = c + b\log 5 \quad (2)$

Subtract equation (1) from equation (2) and solve the resulting equation for *b*.

$$6 = b\log 5 - b\log 1 = b(\log 5 - \log 1)$$
$$= b\log\left(\frac{5}{1}\right) \Rightarrow 6 = b\log 5 \Rightarrow b = \frac{6}{\log 5}$$

Substitute the value for *b* into equation (2) and solve for *c*.

$$7 = c + \left(\frac{6}{\log 5}\right)\log 5 = c + 6 \Rightarrow 1 = c$$

Substituting the values for *b* and *c* into $y = c + b\log x$ gives

$$y = 1 + \left(\frac{6}{\log 5}\right)\log x = 1 + 6\left(\frac{\log x}{\log 5}\right)$$
$$= 1 + 6\log_5 x.$$

81. $A(t) = A_0e^{kt} \Rightarrow 23 = 50e^{12k} \Rightarrow$
$0.46 = e^{12k} \Rightarrow \ln(0.46) = 12k \Rightarrow$
$k = \frac{\ln(0.46)}{12}$

To find the half-life, we use the formula

$$h = -\frac{\ln 2}{k} = -\frac{\ln 2}{\ln(0.46)/12} \approx 10.7.$$

The half-life is about 10.7 years.

82. $A(t) = A_0e^{kt} \Rightarrow 65 = 200e^{10k} \Rightarrow$
$0.325 = e^{10k} \Rightarrow \ln(0.325) = 10k \Rightarrow$
$k = \frac{\ln(0.325)}{10}$

To find the half-life, we use the formula

$$h = -\frac{\ln 2}{k} = -\frac{\ln 2}{\ln(0.325)/10} \approx 6.2.$$

The half-life is about 6.2 years.

83. $A(t) = A_0e^{kt} \Rightarrow 3.8 = 10.3e^{15k} \Rightarrow$
$\frac{3.8}{10.3} = e^{15k} \Rightarrow \ln\left(\frac{3.8}{10.3}\right) = 15k \Rightarrow$
$k = \frac{\ln\left(\frac{3.8}{10.3}\right)}{15}$

To find the half-life, we use the formula

$$h = -\frac{\ln 2}{k} = -\frac{\ln 2}{\ln(3.8/10.3)/15} \approx 10.4.$$

The half-life is about 10.4 hours.

84. $A(t) = A_0e^{kt} \Rightarrow 12.3 = 20.8e^{40k} \Rightarrow$
$\frac{12.3}{20.8} = e^{40k} \Rightarrow \ln\left(\frac{12.3}{20.8}\right) = 40k \Rightarrow$
$k = \frac{\ln\left(\frac{12.3}{20.8}\right)}{40}$

To find the half-life, we use the formula

$$h = -\frac{\ln 2}{k} = -\frac{\ln 2}{\ln(12.3/20.8)/40} \approx 52.8.$$

The half-life is about 52.8 minutes.

4.3 Applying the Concepts

85. $7.09 = 7e^{1k} \Rightarrow$
$\ln\frac{7.09}{7} = k \approx 0.012775 = 1.2775\%$

86. a. $12 = 7e^{0.012775t} \Rightarrow \ln\left(\frac{12}{7}\right) = 0.012775t \Rightarrow$

$t \approx 42.2$ years after 2011.

The world population will be 12 billion sometime during 2054.

b. $20 = 7e^{0.0127755t} \Rightarrow \ln\left(\frac{20}{7}\right) = 0.012775t \Rightarrow$

$t \approx 82.2$ years after 2011.

The world population will be 20 billion sometime during 2094.

87. $20 < 7e^{100k} \Rightarrow \ln\left(\frac{20}{7}\right) < 100k \Rightarrow$
$0.010498 = 1.0498\% < k$

The maximum rate of growth is 1.0498%.

88. $5 > 7e^{25k} \Rightarrow \ln\left(\frac{5}{7}\right) > 25k \Rightarrow$

$-0.013459 = -1.3459\% > k \Rightarrow$ the rate of growth must be less than 1.3459%.

89. $3500 = 1000e^{0.1t} \Rightarrow \ln 3.5 = 0.1t \Rightarrow$
$t \approx 12.53$ years

90. Assume the original investment is \$1.

$2 = e^{6k} \Rightarrow \ln 2 = 6k \Rightarrow k \approx 0.1155 = 11.55\%$

91. Because the half-life is 8 days, there will be 10 grams left after 8 days. Use this to find *k*:

$10 = 20e^{8k} \Rightarrow \frac{1}{2} = e^{8k} \Rightarrow \ln\left(\frac{1}{2}\right) = 8k \Rightarrow$
$k \approx -0.08664$
$A = 20e^{-0.08664(5)} \approx 12.969$ grams

92. Because the half-life is 13 years, there will be 5 grams left after 13 years. Use this to find k:

$$5 = 10e^{13k} \Rightarrow \frac{1}{2} = e^{13k} \Rightarrow \ln\left(\frac{1}{2}\right) = 13k \Rightarrow$$
$$k \approx -0.05332$$
$$A = 10e^{-0.05332(2)} \approx 8.9885 \text{ grams}$$

93. $\frac{1}{2} = e^{-0.055t} \Rightarrow \ln\left(\frac{1}{2}\right) = -0.055t \Rightarrow t \approx 12.6$ yr.

94. Find k using $A = 7.5, A_0 = 60,$ and $t = 63$:

$$7.5 = 60e^{63k} \Rightarrow \ln\left(\frac{7.5}{60}\right) = 63k \Rightarrow k \approx -0.033$$
$$\frac{1}{2} = e^{-0.033t} \Rightarrow \ln\left(\frac{1}{2}\right) = -0.033t \Rightarrow t \approx 21 \text{ hr.}$$

95. $1.5 = 5e^{-10k} \Rightarrow 0.3 = e^{-10k} \Rightarrow \ln 0.3 = -10k \Rightarrow$
$k \approx 0.1204$

96. $3 = 12e^{-0.1204t} \Rightarrow \frac{1}{4} = e^{-0.1204t} \Rightarrow$
$\ln(1/4) = -0.1204t \Rightarrow t \approx 11.5$ hours

97. Find k using $A = 8, A_0 = 16,$ and $t = 36$:

$$8 = 16e^{-36k} \Rightarrow \frac{1}{2} = e^{-36k} \Rightarrow$$
$$\ln\left(\frac{1}{2}\right) = 36k \Rightarrow k \approx 0.0193$$
$$A = 16e^{-0.0193(8)} \Rightarrow A \approx 13.7 \text{ grams.}$$

98. Find k: $\frac{1}{2} = e^{-12k} \Rightarrow \ln\left(\frac{1}{2}\right) = -12k \Rightarrow$

$$k \approx 0.0578$$
$$0.9 = e^{-0.0578t} \Rightarrow \ln 0.9 = -0.0578t \Rightarrow$$
$$t \approx 1.82 \text{ hours}$$

In exercises 99–102, we use the formula

$$P = \frac{r \cdot M}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} \div n,$$

where P = the payment, r = the annual interest rate, M = the mortgage amount, t = the number of years, and n = the number of payments per year.

99. $P = \dfrac{0.06 \cdot 120,000}{1 - \left(1 + \frac{0.06}{12}\right)^{-12 \cdot 20}} \div 12 = 859.72$

The monthly payment is \$859.72.
There are 240 payments so the total amount paid is $240 \cdot \$859.72 = \$206,332.80$.
The amount of interest paid is
206,332.80 − 120,000 = \$86,332.80.

100.
$$1200 = \frac{0.08 \cdot 150,000}{1 - \left(1 + \frac{0.08}{12}\right)^{-12t}} \div 12$$
$$14,400 = \frac{12,000}{1 - \left(1 + \frac{0.08}{12}\right)^{-12t}}$$
$$1 - \left(1 + \frac{0.08}{12}\right)^{-12t} = \frac{12,000}{14,400}$$
$$1 - \frac{5}{6} = \left(1 + \frac{1}{150}\right)^{-12t}$$
$$\frac{1}{6} = \left(\frac{151}{150}\right)^{-12t}$$
$$\log\left(\frac{1}{6}\right) = -12t \log\left(\frac{151}{150}\right)$$
$$t = -\frac{\log\left(\frac{1}{6}\right)}{12 \log\left(\frac{151}{150}\right)} \approx 22.5$$

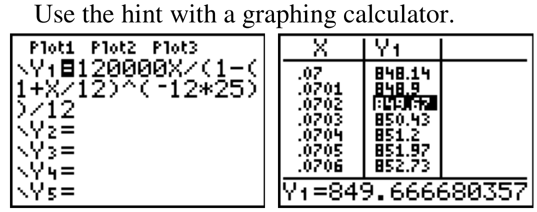
They will have to make payments for 22.5 years. They will make 270 payments of \$1200 each for a total of \$324,000. They will pay 324,000 − 150,000 = \$174,000 in interest.

101.
$$850 = \frac{0.085 \cdot M}{1 - \left(1 + \frac{0.085}{12}\right)^{-12 \cdot 30}} \div 12$$
$$10,200 = \frac{0.085 \cdot M}{1 - \left(1 + \frac{0.085}{12}\right)^{-12 \cdot 30}}$$
$$M = \frac{10,200\left[1 - \left(1 + \frac{0.085}{12}\right)^{-12 \cdot 30}\right]}{0.085}$$
$$\approx 110,545.60$$

Andy can afford a mortgage of about \$110,545.60.

102. $850 = \dfrac{r \cdot 120,000}{1 - \left(1 + \frac{r}{12}\right)^{-12 \cdot 25}} \div 12$

Use the hint with a graphing calculator.



At 7.02%, her monthly payment will be \$849.67.

4.3 Beyond the Basics

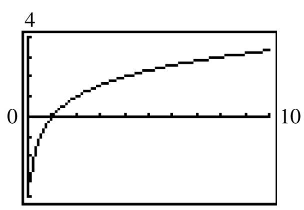
103. $\log_b\left(\sqrt{x^2+1}-x\right)+\log_b\left(\sqrt{x^2+1}+x\right)$
$=\log_b\left(\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)\right)$
$=\log_b\left(x^2+1-x^2\right)=\log_b 1=0$

104. Use reasoning similar to that in exercise 103:
$0=\log_b(x+1-x)$
$=\log_b\left(\left(\sqrt{x+1}+x\right)\left(\sqrt{x+1}-x\right)\right)$
$=\log_b\left(\sqrt{x+1}+x\right)+\log_b\left(\sqrt{x+1}-x\right)\Rightarrow$
$\log_b\left(\sqrt{x+1}+x\right)=-\log_b\left(\sqrt{x+1}-x\right)$

105. $\left(\log_b a\right)\left(\log_a b\right)=\dfrac{\log a}{\log b}\cdot\dfrac{\log b}{\log a}=1$

106. $\log\left(\dfrac{a}{b}\right)+\log\left(\dfrac{b}{a}\right)$
$=\log a-\log b+\log b-\log a=0$

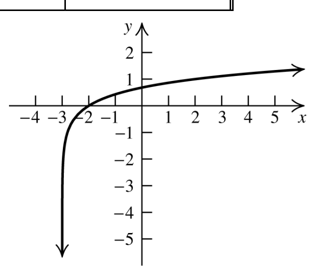
107.



[0, 10, 1] by [–4, 4,1]

108.

x	$\log_5(x+3)$	x	$\log_5(x+3)$
–3	Undefined	2	1
–2	0	3	1.11
–1	0.43	4	1.21
0	0.68	5	1.29
1	0.86		



109. a. $\log\left(\dfrac{a}{b}\right)+\log\left(\dfrac{b}{a}\right)+\log\left(\dfrac{c}{a}\right)+\log\left(\dfrac{a}{c}\right)=0$
(See exercise 106.)

b. $\log\left(\dfrac{a^2}{bc}\right)+\log\left(\dfrac{b^2}{ca}\right)+\log\left(\dfrac{c^2}{ab}\right)$
$=2\log a-(\log b+\log c)$
$+2\log b-(\log c+\log a)$
$+2\log c-(\log a+\log b)=0$

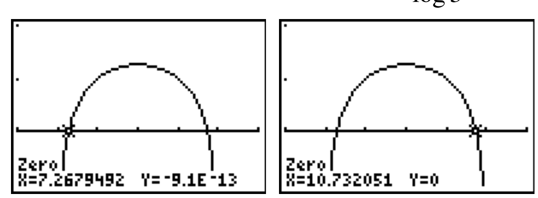
c. $\log_2 3\cdot\log_3 4=\dfrac{\log 3}{\log 2}\cdot\dfrac{\log 2^2}{\log 3}$
$=\dfrac{\log 3}{\log 2}\cdot\dfrac{2\log 2}{\log 3}=2$

d. $\log_a b\cdot\log_b c\cdot\log_c a$
$=\dfrac{\log b}{\log a}\cdot\dfrac{\log c}{\log b}\cdot\dfrac{\log a}{\log c}=1$

110. $\log_2(\log_2 N)=4\Rightarrow 2^4=16=\log_2 N\Rightarrow$
$2^{16}=N\Rightarrow N=65{,}536.$
There are 5 digits.

111. $f(x)=\log_4\left(\log_5\left(\log_3\left(18x-x^2-77\right)\right)\right)$
$18x-x^2-77=-\left(x^2-18x+77\right)$
$=-(x-11)(x-7)$
$\log_3\left(18x-x^2-77\right)$ is defined only for those values of x which make $18x-x^2-77>0.$ Thus, the domain of $\log_3\left(18x-x^2-77\right)$ is (7, 11).
$\log_5\left[\log_3\left(18x-x^2-77\right)\right]$ is defined only for those values of x in the interval (7, 11) which make $\log_3\left(18x-x^2-77\right)>0.$ We use a graphing calculator to solve this. Note that we use the change of base formula to define the function.

$$Y_1=\log_3\left(18x-x^2-77\right)=\frac{\log\left(18x-x^2-77\right)}{\log 3}.$$



[6, 12] by [−1 ,2]

Thus, the domain of $\log_5\left[\log_3\left(18x-x^2-77\right)\right]$ is approximately (7.27, 10.73).

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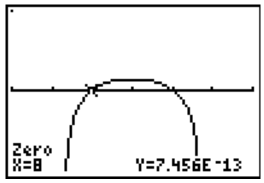
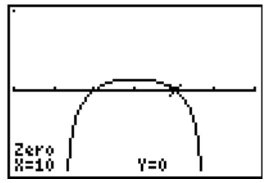
$\log_4\left(\log_5\left(\log_3\left(18x - x^2 - 77\right)\right)\right)$ is defined only for those values of x in the interval (7.27, 10.73) which make $\log_5\left[\log_3\left(18x - x^2 - 77\right)\right] > 0$.

We use a graphing calculator to solve this. Note that we use the change of base formula to define the function. $Y_2 = \log_5\left(\log_3\left(18x - x^2 - 77\right)\right) = \log_5\left(Y_1\right) = \dfrac{Y_1}{\log 5}$





[6, 12] by [−1 ,1]

Thus, the domain of $\log_4\left(\log_5\left(\log_3\left(18x - x^2 - 77\right)\right)\right)$ is (8, 10).

112. a. $f\left(\dfrac{1}{x}\right) = \log_a\left(\dfrac{1}{x}\right) = \log_a(x^{-1})$
$= -\log_a x = -(-\log_{a^{-1}} x) = \log_{1/a} x$

b. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\log_a(x+h) - \log_a x}{h} = \dfrac{1}{h}\cdot\log_a\left(\dfrac{x+h}{x}\right) = \dfrac{1}{h}\cdot\log_a\left(1 + \dfrac{h}{x}\right) = \log_a\left(1 + \dfrac{h}{x}\right)^{1/h}, h \neq 0$

113. a. False **b.** True **c.** False **d.** False **e.** True
f. True **g.** False **h.** True **i.** False **j.** True

114. a. Let $M = a^v$ and let $N = a^w$ for some real numbers v and w. Then
$\log_a \dfrac{M}{N} = \log_a \dfrac{a^v}{a^w} = \log_a a^{v-w} = v - w.$
$v = \log_a M$ and $w = \log_a N$, so $v - w = \log_a M - \log_a N$. Therefore, $\log_a \dfrac{M}{N} = \log_a M - \log_a N$.

b. Let $M = a^v$ for some real number v. Then $\log_a M^r = \log_a\left(a^v\right)^r = \log_a\left(a^{vr}\right) = vr$.
$v = \log_a M$, so $vr = r\log_a M$. Therefore $\log_a M^r = r\log_a M$.

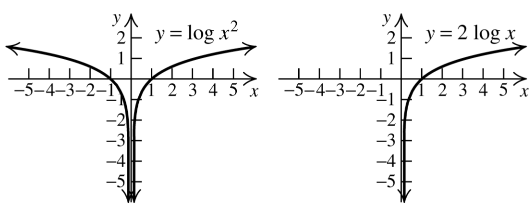
115. $\log\left(\dfrac{a+b}{3}\right) = \dfrac{1}{2}(\log a + \log b) \Rightarrow 2\log\left(\dfrac{a+b}{3}\right) = \log(ab) \Rightarrow 2\log(a+b) - 2\log 3 = \log(ab) \Rightarrow$
$\log(a+b)^2 - \log 9 = \log(ab) \Rightarrow \log(a+b)^2 = \log(ab) + \log 9 \Rightarrow \log(a+b)^2 = \log(9ab) \Rightarrow$
$(a+b)^2 = 9ab \Rightarrow a^2 + 2ab + b^2 = 9ab \Rightarrow a^2 + b^2 - 7ab = 0$

116. $$\frac{1}{1+\log_c ab}+\frac{1}{1+\log_a bc}+\frac{1}{1+\log_b ca}=\frac{1}{1+\frac{\log ab}{\log c}}+\frac{1}{1+\frac{\log bc}{\log a}}+\frac{1}{1+\frac{\log ca}{\log b}}$$
$$=\frac{\log c}{\log c+\log ab}+\frac{\log a}{\log a+\log bc}+\frac{\log b}{\log b+\log ca}$$
$$=\frac{\log c}{\log c+\log a+\log b}+\frac{\log a}{\log a+\log b+\log c}+\frac{\log b}{\log b+\log c+\log a}$$
$$=\frac{\log a+\log b+\log c}{\log a+\log b+\log c}=1$$

4.3 Critical Thinking/Discussion/Writing

117. In step 2, $\log\left(\frac{1}{2}\right)$ is negative, so $3<4\Rightarrow$ $3\log\left(\frac{1}{2}\right)>4\log\left(\frac{1}{2}\right)$.

118. The domain of $2\log x$ is $(0,\infty)$, while the domain of $\log(x^2)$ is $(-\infty,0)\cup(0,\infty)$.



119. If $p=2^m-1$ is a prime number, then the only way *p* and 2^m have a different number of digits is if $2^m=10^k$ or $2^{n-k}=5^k$. However, this is impossible because 2^{n-k} is even and 5^k is odd. Thus, *p* and 2^m have the same number of digits.
$$K=2^{43112609}$$
$$\log K=\log\left(2^{43112609}\right)=43112609\log 2$$
$$\approx 12978188.5$$
There are 12,978,189 digits in this prime number.

120. $\frac{1}{\log(1-x)}$ is defined only for those values of $\log(1-x)\neq 0$ and for those values of *x* such that $\log(1-x)$ is defined.
$\log(1-x)=0\Rightarrow 1-x=1\Rightarrow x=0$
$\log(1-x)$ is defined for
$1-x>0\Rightarrow 1>x$ or $x<1$.
Thus, the domain of $\frac{1}{\log(1-x)}$ is $(-\infty,\ 0)\cup(0,\ 1)$.
$\frac{1}{\ln(x+2)}$ is defined only for those values of $\ln(x+2)\neq 0$ and for those values of *x* such that $\ln(x+2)$ is defined.
$\ln(x+2)=0\Rightarrow x+2=1\Rightarrow x=-1$
$\ln(x+2)$ is defined for
$x+2>0\Rightarrow x>-2$ or $-2<x$.
Thus, the domain of $\frac{1}{\ln(x+2)}$ is $(-2,\ -1)\cup(-1,\ \infty)$.
The intersection of the two domains is the domain of $\frac{1}{\log(1-x)}+\frac{1}{\ln(x+2)}$. Thus, the domain is $(-2,\ -1)\cup(-1,\ 0)\cup(0,\ 1)$.

4.3 Maintaining Skills

121. $11\cdot 3^0=11\cdot 1=11$

122. $-4\cdot 5^x\cdot 5^{-x}=-4\cdot 5^{x-x}=-4\cdot 5^0=-4$

123. $4^x\cdot 2^{-2x+1}=\left(2^2\right)^x\cdot 2^{-2x+1}=2^{2x}\cdot 2^{-2x+1}$
$=2^{2x+(-2x+1)}=2^1=2$

124. $\left(7^x\right)^2\cdot\left(7^2\right)^{-x}=7^{2x}\cdot 7^{-2x}=7^{2x-2x}=7^0=1$

For exercises 125–128, let $t=5^x$. Then $5^{2x}=t^2$.

125. $5^{2x}-5^x=-1\Rightarrow t^2-t=-1\Rightarrow t^2-t+1=0$

126. $3\cdot 5^{2x}-2\cdot 5^x=7\Rightarrow 3t^2-2t=7\Rightarrow$
$3t^2-2t-7=0$

127. $\dfrac{5^x + 3\cdot 5^{-x}}{5^x} = \dfrac{1}{4} \Rightarrow \dfrac{t+\frac{3}{t}}{t} = \dfrac{1}{4} \Rightarrow \dfrac{\frac{t^2+3}{t}}{t} = \dfrac{1}{4} \Rightarrow$

$\dfrac{t^2+3}{t^2} = \dfrac{1}{4} \Rightarrow 4t^2 + 12 = t^2 \Rightarrow 3t^2 + 12 = 0 \Rightarrow$

$t^2 + 4 = 0$

128. $\dfrac{5^{-x} + 5^x}{5^{-x} - 5^x} = \dfrac{1}{2} \Rightarrow \dfrac{\frac{1}{t}+t}{\frac{1}{t}-t} = \dfrac{1}{2} \Rightarrow \dfrac{1+t^2}{1-t^2} = \dfrac{1}{2} \Rightarrow$

$2 + 2t^2 = 1 - t^2 = 3t^2 + 1 = 0$

129. $2x - (11 + x) = 8x + (7 + 2x)$

$x - 11 = 10x + 7$

$-18 = 9x \Rightarrow x = -2$

Solution set: $\{-2\}$

130. $4x - 1 + (6x - 2) = 3 - (5x + 1)$

$10x - 3 = 2 - 5x$

$15x = 5 \Rightarrow x = \dfrac{1}{3}$

Solution set: $\left\{\dfrac{1}{3}\right\}$

131. $x^2 + 3x - 1 = 3$

$x^2 + 3x - 4 = 0$

$(x-1)(x+4) = 0$

$x - 1 = 0 \mid x + 4 = 0$

$x = 1 \mid x = -4$

Solution set: $\{-4, 1\}$

132. $2x^2 - 7x = 3x + 48$

$2x^2 - 10x - 48 = 0$

$x^2 - 5x - 24 = 0$

$(x-8)(x+3) = 0$

$x - 8 = 0 \mid x + 3 = 0$

$x = 8 \mid x = -3$

Solution set: $\{-3, 8\}$

133. $2x < 7 + x \Rightarrow x < 7$

Solution set: $(-\infty, 7)$

134. $5x - 2 \le 19 - (1 - x)$

$5x - 2 \le 18 + x$

$4x \le 20 \Rightarrow x \le 5$

Solution set: $(-\infty, 5]$

135. $12x > 30 - 3x$

$15x > 30 \Rightarrow x > 2$

Solution set: $(2, \infty)$

136. $4x - 17 \ge 6 - (5 + 2x)$

$4x - 17 \ge 1 - 2x$

$6x \ge 18 \Rightarrow x \ge 3$

Solution set: $[3, \infty)$

4.4 Exponential and Logarithmic Equations and Inequalities

4.4 Practice Problems

1. a. $3^x = 243 \Rightarrow 3^x = 3^5 \Rightarrow x = 5$

b. $8^x = 4 \Rightarrow \left(2^3\right)^x = 2^2 \Rightarrow 2^{3x} = 2^2 \Rightarrow$

$3x = 2 \Rightarrow x = \dfrac{2}{3}$

2. $7 \cdot 3^{x+1} = 11 \Rightarrow 3^{x+1} = \dfrac{11}{7} \Rightarrow$

$\ln\left(3^{x+1}\right) = \ln\left(\dfrac{11}{7}\right) \Rightarrow (x+1)\ln 3 = \ln\left(\dfrac{11}{7}\right) \Rightarrow$

$x + 1 = \dfrac{\ln(11/7)}{\ln 3} \Rightarrow x = \dfrac{\ln(11/7)}{\ln 3} - 1 \approx -0.589$

3. $3^{x+1} = 2^{2x}$

$\ln\left(3^{x+1}\right) = \ln\left(2^{2x}\right)$

$(x+1)\ln 3 = 2x\ln 2$

$x\ln 3 + \ln 3 = 2x\ln 2$

$x\ln 3 - 2x\ln 2 = -\ln 3$

$x(\ln 3 - 2\ln 2) = -\ln 3$

$x = -\dfrac{\ln 3}{\ln 3 - 2\ln 2} \approx 3.819$

4. $e^{2x} - 4e^x - 5 = 0$

$\left(e^x - 5\right)\left(e^x + 1\right) = 0$

$e^x - 5 = 0$ or $e^x + 1 = 0$

$e^x - 5 = 0 \Rightarrow e^x = 5 \Rightarrow \ln\left(e^x\right) = \ln 5 \Rightarrow$

$x\ln e = \ln 5 \Rightarrow x = \ln 5 \approx 1.609$

$e^x + 1 = 0 \Rightarrow e^x = -1$, which is not possible.

5. The given model is $P(t) = P_0(1+r)^t$.

a. In 2020, ten years after the base year, the population of the United States will be $P(10) = 308(1 + .011)^{10} \approx 343.61$ million. The population of Pakistan will be $P(10) = 185(1 + .033)^{10} \approx 255.96$ million.

b. To find when the population of the U.S. will be 350 million, solve for *t*:

$$350 = 308(1+.011)^t \Rightarrow \frac{350}{308} = 1.011^t \Rightarrow$$

$$\ln\left(\frac{350}{308}\right) = t\ln 1.011 \Rightarrow$$

$$t = \frac{\ln(350/308)}{\ln 1.011} \approx 11.69$$

The population of the U.S. will be 350 million approximately 11.69 years after 2010, sometime in the year 2022.

c. To find when the population of the two countries will be the same, solve for *t*:

$$308(1+.011)^t = 185(1+.033)^t$$

$$308(1.011)^t = 185(1.033)^t$$

$$\frac{308}{185} = \frac{1.033^t}{1.011^t} = \left(\frac{1.033}{1.011}\right)^t$$

$$\ln\left(\frac{308}{185}\right) = t\ln\left(\frac{1.033}{1.011}\right)$$

$$\frac{\ln(308/185)}{\ln(1.033/1.011)} \approx 23.68 = t$$

The populations of the two countries will be the same about 23.68 years after 2010, sometime in the year 2033.

6. $1 + 2\ln x = 4 \Rightarrow 2\ln x = 3 \Rightarrow \ln x = \frac{3}{2} \Rightarrow x = e^{3/2}$

7. $\log_3(x-8) + \log_3 x = 2$

$$\log_3[x(x-8)] = 2$$

$$x^2 - 8x = 3^2$$

$$x^2 - 8x - 9 = 0$$

$$(x-9)(x+1) = 0$$

$x - 9 = 0$ or $x + 1 = 0$

$x = 9$ $\quad x = -1$

Now check each possible solution in the original equation.

$$\log_3(9-8) + \log_3 9 \stackrel{?}{=} 2$$

$$\log_3 1 + \log_3 9 \stackrel{?}{=} 2$$

$$0 + 2 = 2 \checkmark$$

$$\log_3(-1-8) + \log_3(-1) \stackrel{?}{=} 2$$

This is not possible because logarithms are not defined for negative values.
The solution set is {9}.

8. $\ln(x+5) + \ln(x+1) = \ln(x-1)$

$$\ln[(x+5)(x+1)] = \ln(x-1)$$

$$\ln(x^2 + 6x + 5) = \ln(x-1)$$

$$x^2 + 6x + 5 = x - 1$$

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

$x + 2 = 0 \Rightarrow x = -2$ or $x + 3 = 0 \Rightarrow x = -3$

Now check each possible solution in the original equation.

$$\ln(-2+5) + \ln(-2+1) \stackrel{?}{=} \ln(-2-1)$$

$$\ln 3 + \ln(-1) = \ln(-3)$$

This is not possible because logarithms are not defined for negative values.

$$\ln(-3+5) + \ln(-3+1) \stackrel{?}{=} \ln(-3-1)$$

$$\ln 2 + \ln(-2) = \ln(-4)$$

This is not possible because logarithms are not defined for negative values.
The solution set is $\varnothing$.

9. The year 1987 represents $t = 0$, so the year 2005 represents $t = 18$. Using equation (19) in example 9 in the text, we have

$$6.5 = \frac{35}{1 + 6e^{-18k}} \Rightarrow 6.5 + 39e^{-18k} = 35 \Rightarrow$$

$$e^{-18k} = \frac{28.5}{39} \Rightarrow -18k = \ln\left(\frac{28.5}{39}\right) \Rightarrow$$

$$k = -\frac{\ln(28.5/39)}{18} \approx .0174 = 1.74\%$$

The growth rate was about 1.74%.

10. $3(0.5)^x + 7 > 19 \Rightarrow 3(0.5)^x > 12 \Rightarrow$

$$(0.5)^x > 4 \Rightarrow x\ln 0.5 > \ln 4 \Rightarrow x < \frac{\ln 4}{\ln 0.5} \Rightarrow$$

$$x < -2$$

11. Since $1 - 3x$ must be positive, $x < \frac{1}{3}$.

$$\ln(1-3x) > 2 \Rightarrow 1 - 3x > e^2 \Rightarrow$$

$$-3x > e^2 - 1 \Rightarrow x < \frac{e^2 - 1}{-3} \Rightarrow$$

$$x < \frac{1 - e^2}{3} \text{ or } x < -2.13$$

4.4 Basic Concepts and Skills

1. An equation that contains terms of the form a^x is called a(n) <u>exponential</u> equation.

2. An equation that contains terms of the form $\log_a x$ is called a(n) <u>logarithmic</u> equation.

3. The equation $y = \dfrac{m}{1 + ae^{-bx}}$ represents a(n) <u>logistic</u> model.

4. True

5. False. A logarithmic equation can have a negative solution as long as the solution does not make the parameter of the logarithm negative.

6. True.
$8^{2x} = 4^{3x} \Rightarrow \left(2^3\right)^{2x} = \left(2^2\right)^{3x} \Rightarrow 2^{6x} = 2^{6x}$

7. $2^x = 16 \Rightarrow 2^x = 2^4 \Rightarrow x = 4$

8. $3^x = 243 \Rightarrow 3^x = 3^5 \Rightarrow x = 5$

9. $8^x = 32 \Rightarrow 2^{3x} = 2^5 \Rightarrow 3x = 5 \Rightarrow x = \dfrac{5}{3}$

10. $5^{x-1} = 1 \Rightarrow 5^{x-1} = 5^0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$

11. $4^{|x|} = 128 \Rightarrow 2^{2|x|} = 2^7 \Rightarrow 2|x| = 7 \Rightarrow x = \pm\dfrac{7}{2}$

12. $9^{|x|} = 243 \Rightarrow 3^{2|x|} = 3^5 \Rightarrow 2|x| = 5 \Rightarrow x = \pm\dfrac{5}{2}$

13. $5^{-|x|} = 625 \Rightarrow 5^{-|x|} = 5^4 \Rightarrow -|x| = 4 \Rightarrow$ there is no solution.

14. $3^{-|x|} = 81 \Rightarrow 3^{-|x|} = 3^4 \Rightarrow -|x| = 4 \Rightarrow$ there is no solution.

15. $\ln x = 0 \Rightarrow x = 1$

16. $\ln(x-1) = 1 \Rightarrow e^1 = x - 1 \Rightarrow x = 1 + e$

17. $\log_2 x = -1 \Rightarrow 2^{-1} = x \Rightarrow x = \dfrac{1}{2}$

18. $\log_2(x+1) = 3 \Rightarrow 2^3 = x + 1 \Rightarrow x = 7$

19. $\log_3 |x| = 2 \Rightarrow 3^2 = |x| \Rightarrow x = \pm 9$

20. $\log_2 |x+1| = 3 \Rightarrow 2^3 = |x+1| \Rightarrow x + 1 = 8 \Rightarrow$
$x = 7$ or $x + 1 = -8 \Rightarrow x = -9$

21. $\dfrac{1}{2}\log x - 2 = 0 \Rightarrow \dfrac{1}{2}\log x = 2 \Rightarrow \log x = 4 \Rightarrow$
$10^4 = 10{,}000 = x$

22. $\dfrac{1}{3}\log(x+1) - 1 = 0 \Rightarrow \log(x+1) = 3 \Rightarrow$
$10^3 = x + 1 \Rightarrow x = 999$

In exercises 23−52, the equations can be solved using either the common logarithm or the natural logarithm.

23. $2^x = 3 \Rightarrow x\ln 2 = \ln 3 \Rightarrow x = \dfrac{\ln 3}{\ln 2} \approx 1.585$

24. $3^x = 5 \Rightarrow x\ln 3 = \ln 5 \Rightarrow x = \dfrac{\ln 5}{\ln 3} \approx 1.465$

25. $2^{2x+3} = 15 \Rightarrow (2x+3)\ln 2 = \ln 15 \Rightarrow$
$2x + 3 = \dfrac{\ln 15}{\ln 2} \Rightarrow$
$x = \dfrac{\frac{\ln 15}{\ln 2} - 3}{2} = \dfrac{\ln 15 - 3\ln 2}{2\ln 2} \approx 0.453$

26. $3^{2x+5} = 17 \Rightarrow (2x+5)\ln 3 = \ln 17 \Rightarrow$
$2x + 5 = \dfrac{\ln 17}{\ln 3} \Rightarrow$
$x = \dfrac{\frac{\ln 17}{\ln 3} - 5}{2} = \dfrac{\ln 17 - 5\ln 3}{2\ln 3} \approx -1.211$

27. $5 \cdot 2^x - 7 = 10 \Rightarrow 2^x = \dfrac{17}{5} \Rightarrow$
$x\ln 2 = \ln\left(\dfrac{17}{5}\right) \Rightarrow x = \dfrac{\ln 17 - \ln 5}{\ln 2} \approx 1.766$

28. $3 \cdot 5^x + 4 = 11 \Rightarrow 5^x = \dfrac{7}{3} \Rightarrow x\ln 5 = \ln\left(\dfrac{7}{3}\right) \Rightarrow$
$x = \dfrac{\ln 7 - \ln 3}{\ln 5} \approx 0.526$

29. $3 \cdot 4^{2x-1} + 4 = 14 \Rightarrow 4^{2x-1} = \dfrac{10}{3} \Rightarrow$
$(2x-1)\ln 4 = \ln\left(\dfrac{10}{3}\right) \Rightarrow$
$x = \dfrac{\frac{\ln 10 - \ln 3}{\ln 4} + 1}{2} = \dfrac{\ln 10 - \ln 3 + \ln 4}{2\ln 4} \approx 0.934$

30. $2 \cdot 3^{4x-5} - 7 = 10 \Rightarrow 3^{4x-5} = \dfrac{17}{2} \Rightarrow$
$(4x-5)\ln 3 = \ln\left(\dfrac{17}{2}\right) \Rightarrow$
$x = \dfrac{\frac{\ln 17 - \ln 2}{\ln 3} + 5}{4} = \dfrac{\ln 17 - \ln 2 + 5\ln 3}{4\ln 3} \approx 1.737$

31. $5^{1-x} = 2^x \Rightarrow (1-x)\ln 5 = x\ln 2 \Rightarrow$
$\ln 5 - x\ln 5 = x\ln 2 \Rightarrow \ln 5 = x(\ln 5 + \ln 2) \Rightarrow$
$x = \dfrac{\ln 5}{\ln 5 + \ln 2} \approx 0.699$

32. $3^{2x-1} = 2^{x+1} \Rightarrow (2x-1)\ln 3 = (x+1)\ln 2 \Rightarrow$
$2x\ln 3 - \ln 3 = x\ln 2 + \ln 2 \Rightarrow$
$x(2\ln 3 - \ln 2) = \ln 2 + \ln 3 \Rightarrow$
$x = \dfrac{\ln 2 + \ln 3}{2\ln 3 - \ln 2} \approx 1.191$

33. $2^{1-x} = 3^{4x+6} \Rightarrow (1-x)\ln 2 = (4x+6)\ln 3 \Rightarrow$
$\ln 2 - x\ln 2 = 4x\ln 3 + 6\ln 3 \Rightarrow$
$\ln 2 - 6\ln 3 = 4x\ln 3 + x\ln 2 \Rightarrow$
$\ln 2 - 6\ln 3 = x(4\ln 3 + \ln 2) \Rightarrow$
$\dfrac{\ln 2 - 6\ln 3}{4\ln 3 + \ln 2} = x \Rightarrow x \approx -1.159$

34. $5^{2x+1} = 3^{x-1} \Rightarrow (2x+1)\ln 5 = (x-1)\ln 3 \Rightarrow$
$2x\ln 5 + \ln 5 = x\ln 3 - \ln 3 \Rightarrow$
$2x\ln 5 - x\ln 3 = -\ln 3 - \ln 5 \Rightarrow$
$x(2\ln 5 - \ln 3) = -\ln 3 - \ln 5 \Rightarrow$
$x = \dfrac{-\ln 3 - \ln 5}{2\ln 5 - \ln 3} \approx -1.277$

35.
$$\begin{aligned} 2\cdot 3^{x-1} &= 5^{x+1} \\ \ln\left(2\cdot 3^{x-1}\right) &= \ln\left(5^{x+1}\right) \\ \ln 2 + (x-1)\ln 3 &= (x+1)\ln 5 \\ \ln 2 + x\ln 3 - \ln 3 &= x\ln 5 + \ln 5 \\ x\ln 3 - x\ln 5 &= \ln 5 - \ln 2 + \ln 3 \\ x(\ln 3 - \ln 5) &= \ln 5 - \ln 2 + \ln 3 \\ x &= \frac{\ln 5 - \ln 2 + \ln 3}{\ln 3 - \ln 5} \\ &= \frac{\ln 2 - \ln 3 - \ln 5}{\ln 5 - \ln 3} \approx -3.944 \end{aligned}$$

36.
$$\begin{aligned} 5\cdot 2^{2x+1} &= 7\cdot 3^{x-1} \\ \ln\left(5\cdot 2^{2x+1}\right) &= \ln\left(7\cdot 3^{x-1}\right) \\ \ln 5 + (2x+1)\ln 2 &= \ln 7 + (x-1)\ln 3 \\ \ln 5 + 2x\ln 2 + \ln 2 &= \ln 7 + x\ln 3 - \ln 3 \\ 2x\ln 2 - x\ln 3 &= \ln 7 - \ln 3 - \ln 5 - \ln 2 \\ x(2\ln 2 - \ln 3) &= \ln 7 - \ln 3 - \ln 5 - \ln 2 \\ x &= \frac{\ln 7 - \ln 3 - \ln 5 - \ln 2}{2\ln 2 - \ln 3} \\ &\approx -5.059 \end{aligned}$$

37. $1.065^t = 2 \Rightarrow t\ln 1.065 = \ln 2 \Rightarrow$
$t = \dfrac{\ln 2}{\ln 1.065} \approx 11.007$

38. $1.0725^t = 2 \Rightarrow t\ln 1.0725 = \ln 2 \Rightarrow$
$t = \dfrac{\ln 2}{\ln 1.0725} \approx 9.903$

39. Let $y = 2^x$. Then $2^{2x} - 4\cdot 2^x = 21 \Rightarrow$
$y^2 - 4y - 21 = 0 \Rightarrow (y-7)(y+3) = 0 \Rightarrow$
$y = 7$ or $y = -3$. Reject the negative solution.
$2^x = 7 \Rightarrow x\ln 2 = \ln 7 \Rightarrow x = \dfrac{\ln 7}{\ln 2} \approx 2.807$

40. $4^x - 4^{-x} = 2 \Rightarrow 4^x\left(4^x - 4^{-x} = 2\right) \Rightarrow$
$4^{2x} - 4^0 = 2\cdot 4^x \Rightarrow 4^{2x} - 2\cdot 4^x - 1 = 0.$
Let $y = 4^x$. Then $4^{2x} - 2\cdot 4^x - 1 = 0 \Rightarrow$
$y^2 - 2y - 1 = 0 \Rightarrow y = \dfrac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$
Reject the negative solution.
$4^x = 1 + \sqrt{2} \Rightarrow x\ln 4 = \ln\left(1+\sqrt{2}\right) \Rightarrow$
$x = \dfrac{\ln\left(1+\sqrt{2}\right)}{\ln 4} \approx 0.636$

41. $9^x - 6\cdot 3^x + 8 = 0 \Rightarrow 3^{2x} - 6\cdot 3^x + 8 = 0.$
Let $y = 3^x$. Then $3^{2x} - 6\cdot 3^x + 8 = 0 \Rightarrow$
$y^2 - 6y + 8 = 0 \Rightarrow (y-4)(y-2) = 0 \Rightarrow y = 2$
or $y = 4$. Substituting, we have $3^x = 2 \Rightarrow$
$x\ln 3 = \ln 2 \Rightarrow x = \dfrac{\ln 2}{\ln 3} \approx 0.631$ or $3^x = 4 \Rightarrow$
$x\ln 3 = \ln 4 \Rightarrow x = \dfrac{\ln 4}{\ln 3} \approx 1.262.$
The solution set is $\{0.631, 1.262\}$.

42. $\dfrac{3^x + 5\cdot 3^{-x}}{3} = 2 \Rightarrow 3^x + 5\cdot 3^{-x} = 6 \Rightarrow$
$3^x\left(3^x + 5\cdot 3^{-x} = 6\right) \Rightarrow 3^{2x} + 5\cdot 3^0 = 6\cdot 3^x \Rightarrow$
$3^{2x} - 6\cdot 3^x + 5 = 0.$ Let $y = 3^x$.Then
$3^{2x} - 6\cdot 3^x + 5 = 0 \Rightarrow y^2 - 6y + 5 = 0 \Rightarrow$
$(y-5)(y-1) = 0 \Rightarrow y = 1$ or $y = 5$.
Substituting, we have $3^x = 1 \Rightarrow x = 0$ or
$3^x = 5 \Rightarrow x\ln 3 = \ln 5 \Rightarrow x = \dfrac{\ln 5}{\ln 3} \approx 1.465.$
The solution set is $\left\{0, \dfrac{\ln 5}{\ln 3} \approx 1.465\right\}$.

43.
$$\begin{aligned} 3^{3x} - 4\cdot 3^{2x} + 2\cdot 3^x &= 8 \\ 3^{3x} - 4\cdot 3^{2x} + 2\cdot 3^x - 8 &= 0 \\ 3^{2x}\left(3^x - 4\right) + 2\left(3^x - 4\right) &= 0 \\ \left(3^{2x} + 2\right)\left(3^x - 4\right) &= 0 \end{aligned}$$
$3^{2x} + 2 = 0 \Rightarrow 3^{2x} = -2 \Rightarrow$ there is no solution.
$3^x - 4 = 0 \Rightarrow 3^x = 4 \Rightarrow x\ln 3 = \ln 4 \Rightarrow$
$x = \dfrac{\ln 4}{\ln 3} \approx 1.262$
Solution set: $\left\{\dfrac{\ln 4}{\ln 3} \approx 1.262\right\}$

44. $2^{3x}+3\cdot 2^{2x}-2^x=3$
$2^{3x}+3\cdot 2^{2x}-2^x-3=0$
$2^{2x}\left(2^x+3\right)-1\left(2^x+3\right)=0$
$\left(2^{2x}-1\right)\left(2^x+3\right)=0$
$2^{2x}-1=0\Rightarrow 2^{2x}=1\Rightarrow 2x\ln 2=\ln 1\Rightarrow$
$2x\ln 2=0\Rightarrow x=0$
$2^x+3=0\Rightarrow 2^x=-3\Rightarrow$ there is no solution.
Solution set: $\{0\}$

45. $\dfrac{3^x-3^{-x}}{3^x+3^{-x}}=\dfrac{1}{4}\Rightarrow 4\cdot 3^x-4\cdot 3^{-x}=3^x+3^{-x}\Rightarrow$
$3\cdot 3^x-5\cdot 3^{-x}=0\Rightarrow 3^x\left(3\cdot 3^x-5\cdot 3^{-x}=0\right)\Rightarrow$
$3\cdot 3^{2x}-5\cdot 3^0=0\Rightarrow\ 3\cdot 3^{2x}=5\Rightarrow 3^{2x}=\dfrac{5}{3}\Rightarrow$
$2x\ln 3=\ln\left(\dfrac{5}{3}\right)\Rightarrow$
$x=\dfrac{\ln(5/3)}{2\ln 3}=\dfrac{\ln 5-\ln 3}{2\ln 3}\approx 0.232$

46. $\dfrac{e^x-e^{-x}}{e^x+e^{-x}}=\dfrac{1}{3}\Rightarrow 3\cdot e^x-3\cdot e^{-x}=e^x+e^{-x}\Rightarrow$
$2\cdot e^x-4\cdot e^{-x}=0\Rightarrow e^x\left(2\cdot e^x-4\cdot e^{-x}=0\right)\Rightarrow$
$2\cdot e^{2x}-4\cdot e^0=0\Rightarrow\ 2\cdot e^{2x}=4\Rightarrow e^{2x}=2\Rightarrow$
$2x=\ln 2\Rightarrow x=\dfrac{\ln 2}{2}\approx 0.347$

47. $\dfrac{4}{2+3^x}=1\Rightarrow 4=2+3^x\Rightarrow 2=3^x\Rightarrow$
$\ln 2=x\ln 3\Rightarrow x=\dfrac{\ln 2}{\ln 3}\approx 0.631$

48. $\dfrac{7}{2^x-1}=3\Rightarrow 7=3\cdot 2^x-3\Rightarrow 10=3\cdot 2^x\Rightarrow$
$\ln 10=\ln 3+x\ln 2\Rightarrow x=\dfrac{\ln 10-\ln 3}{\ln 2}\approx 1.737$

49. $\dfrac{17}{5-3^x}=7\Rightarrow 17=35-7\cdot 3^x\Rightarrow$
$7\cdot 3^x=18\Rightarrow \ln 7+x\ln 3=\ln 18\Rightarrow$
$x=\dfrac{\ln 18-\ln 7}{\ln 3}\approx 0.860$

50. $\dfrac{15}{3+2\cdot 5^x}=4\Rightarrow 15=12+8\cdot 5^x\Rightarrow 3=8\cdot 5^x\Rightarrow$
$\ln 3=\ln 8+x\ln 5\Rightarrow x=\dfrac{\ln 3-\ln 8}{\ln 5}\approx -0.609$

51. $\dfrac{5}{2+3^x}=4\Rightarrow 5=4(2+3^x)\Rightarrow$
$5=8+4\cdot 3^x\Rightarrow -\dfrac{3}{4}=3^x\Rightarrow$ there is no solution.

52. $\dfrac{7}{3+5\cdot 2^x}=4\Rightarrow 7=12+20\cdot 2^x\Rightarrow -\dfrac{1}{4}=2^x\Rightarrow$
there is no solution.

53. $3+\log(2x+5)=2\Rightarrow \log(2x+5)=-1\Rightarrow$
$10^{-1}=2x+5\Rightarrow -\dfrac{49}{10}=2x\Rightarrow x=-\dfrac{49}{20}$

54. $1+\log(3x-4)=0\Rightarrow \log(3x-4)=-1\Rightarrow$
$10^{-1}=3x-4\Rightarrow \dfrac{41}{10}=3x\Rightarrow x=\dfrac{41}{30}$

55. $\log\left(x^2-x-5\right)=0\Rightarrow x^2-x-5=10^0\Rightarrow$
$x^2-x-6=0\Rightarrow (x-3)(x+2)=0\Rightarrow x=-2$
or $x=3$

56. $\log\left(x^2-6x+9\right)=0\Rightarrow x^2-6x+9=10^0\Rightarrow$
$x^2-6x+8=0\Rightarrow (x-4)(x-2)=0\Rightarrow x=2$
or $x=4$

57. $\log_4\left(x^2-7x+14\right)=1\Rightarrow x^2-7x+14=4^1\Rightarrow$
$x^2-7x+10=(x-2)(x-5)\Rightarrow x=2$ or $x=5$

58. $\log_4\left(x^2+5x+10\right)=1\Rightarrow x^2+5x+10=4^1\Rightarrow$
$x^2+5x+6=0\Rightarrow (x+2)(x+3)=0\Rightarrow x=-2$
or $x=-3$

59. $\ln(2x-3)-\ln(x+5)=0\Rightarrow$
$\ln(2x-3)=\ln(x+5)\Rightarrow 2x-3=x+5\Rightarrow x=8$

60. $\log(x+8)+\log(x-1)=1\Rightarrow$
$\log\left((x+8)(x-1)\right)=1\Rightarrow (x+8)(x-1)=10\Rightarrow$
$x^2+7x-18=0\Rightarrow (x+9)(x-2)=0\Rightarrow x=2$
or $x=-9$.
Reject -9 because the logarithm of a negative number is undefined. The solution is $\{2\}$.

61. $\log x+\log(x+9)=1\Rightarrow \log\left(x(x+9)\right)=1\Rightarrow$
$x^2+9x=10\Rightarrow x^2+9x-10=0\Rightarrow$
$(x+10)(x-1)=0\Rightarrow x=-10$ or $x=1$.
Reject the negative solution because the logarithm of a negative number is undefined. The solution is $\{1\}$.

62. $\log_5(3x-1)-\log_5(2x+7)=0\Rightarrow$
$\log_5\dfrac{3x-1}{2x+7}=0\Rightarrow \dfrac{3x-1}{2x+7}=5^0=1\Rightarrow$
$3x-1=2x+7\Rightarrow x=8$

63. $\log_a(5x-2)-\log_a(3x+4)=0 \Rightarrow$
$\log_a \frac{5x-2}{3x+4}=0 \Rightarrow \frac{5x-2}{3x+4}=a^0=1 \Rightarrow$
$5x-2=3x+4 \Rightarrow x=3$

64. $\log(x-1)+\log(x+2)=1 \Rightarrow$
$\log\left((x-1)(x+2)\right)=1 \Rightarrow x^2+x-2=10 \Rightarrow$
$x^2+x-12=0 \Rightarrow (x+4)(x-3)=0 \Rightarrow$
$x=-4$ or $x=3$.
Reject the negative solution because the logarithm of a negative number is undefined. The solution is $\{3\}$.

65. $\log_6(x+2)+\log_6(x-3)=1 \Rightarrow$
$\log_6\left((x+2)(x-3)\right)=1 \Rightarrow x^2-x-6=6 \Rightarrow$
$x^2-x-12=0 \Rightarrow (x-4)(x+3)=0 \Rightarrow$
$x=4$ or $x=-3$.
Reject the negative solution because the logarithm of a negative number is undefined. The solution is $\{4\}$.

66. $\log_2(3x-2)-\log_2(5x+1)=3 \Rightarrow$
$\log_2 \frac{3x-2}{5x+1}=3 \Rightarrow \frac{3x-2}{5x+1}=2^3=8 \Rightarrow$
$3x-2=40x+8 \Rightarrow -10=37x \Rightarrow x=-\frac{10}{37}.$
$3\left(-\frac{10}{37}-2\right)=-\frac{252}{37}$, so there is no solution.

67. $\log_3(2x-7)-\log_3(4x-1)=2 \Rightarrow$
$\log_3 \frac{2x-7}{4x-1}=2 \Rightarrow \frac{2x-7}{4x-1}=3^2=9 \Rightarrow$
$2x-7=36x-9 \Rightarrow 2=34x \Rightarrow x=\frac{1}{17}.$
$2\left(\frac{1}{17}\right)-7=-\frac{117}{17}$, so there is no solution.

68. $\log_4\sqrt{x+3}-\log_4\sqrt{2x-1}=\frac{1}{4} \Rightarrow$
$\log_4 \frac{\sqrt{x+3}}{\sqrt{2x-1}}=\frac{1}{4} \Rightarrow \frac{\sqrt{x+3}}{\sqrt{2x-1}}=4^{1/4}=\sqrt{2} \Rightarrow$
$\sqrt{\frac{x+3}{2x-1}}=\sqrt{2} \Rightarrow \frac{x+3}{2x-1}=2 \Rightarrow$
$x+3=4x-2 \Rightarrow 3x=5 \Rightarrow x=\frac{5}{3}$

69. $\log_7 3x+\log_7(2x-1)=\log_7(16x-10) \Rightarrow$
$\log_7\left(3x(2x-1)\right)=\log_7(16x-10) \Rightarrow$
$6x^2-3x=16x-10 \Rightarrow 6x^2-19x+10=0 \Rightarrow$
$(2x-5)(3x-2)=0 \Rightarrow x=\frac{5}{2}$ or $x=\frac{2}{3}$

70. $\log_3(x+1)+\log_3 2x=\log_3(3x+1) \Rightarrow$
$\log_3\left(2x(x+1)\right)=\log_3(3x+1) \Rightarrow$
$2x^2+2x=3x+1 \Rightarrow 2x^2-x-1=0 \Rightarrow$
$(2x+1)(x-1)=0 \Rightarrow x=-\frac{1}{2}$ or $x=1.$
Reject $x=-\frac{1}{2}$ because $2\left(-\frac{1}{2}\right)=-1$ and there is no logarithm of a negative number.
The solution is $\{1\}$.

71. $f(x)=20+a\cdot 2^{kx};\ f(0)=50;\ f(1)=140$
$50=20+a\cdot 2^{k\cdot 0} \Rightarrow 50=20+a \Rightarrow a=30$
$140=20+30\cdot 2^{k\cdot 1} \Rightarrow 4=2^k \Rightarrow k=2$
$f(2)=20+30\cdot 2^{2\cdot 2}=20+480=500$

72. $f(x)=40+a\cdot 4^{kx};\ f(0)=-216;\ f(2)=39$
$-216=40+a\cdot 4^{k\cdot 0} \Rightarrow -216=40+a \Rightarrow$
$a=-256$
$39=40-256\cdot 4^{k\cdot 2} \Rightarrow \frac{1}{256}=4^{2k} \Rightarrow$
$4^{-4}=4^{2k} \Rightarrow -4=2k \Rightarrow k=-2$
$f(1)=40-256\cdot 4^{-2\cdot 1}=40-16=24$

73. $f(x)=16+a\cdot 3^{kx};\ f(0)=21;\ f(4)=61$
$21=16+a\cdot 3^{k\cdot 0} \Rightarrow a=5$
$61=16+5\cdot 3^{k\cdot 4} \Rightarrow 9=3^{4k} \Rightarrow 3^2=3^{4k} \Rightarrow$
$2=4k \Rightarrow k=\frac{1}{2}$
$f(2)=16+5\cdot 3^{\frac{1}{2}\cdot 2}=16+15=31$

74. $f(x)=50+a\cdot 2^{kx};\ f(0)=34;\ f(4)=46$
$34=50+a\cdot 2^{k\cdot 0} \Rightarrow 34=50+a \Rightarrow a=-16$
$46=50-16\cdot 2^{k\cdot 4} \Rightarrow \frac{1}{4}=2^{4k} \Rightarrow$
$2^{-2}=2^{4k} \Rightarrow -2=4k \Rightarrow k=-\frac{1}{2}$
$f(2)=50-16\cdot 2^{2(-1/2)}=50-16\left(\frac{1}{2}\right)=42$

75. $f(x)=\frac{10}{3+ae^{kx}};\ f(0)=2;\ f(1)=\frac{1}{2}$
$2=\frac{10}{3+ae^{k\cdot 0}} \Rightarrow 2=\frac{10}{3+a} \Rightarrow 6+2a=10 \Rightarrow$
$2a=4 \Rightarrow a=2$
$\frac{1}{2}=\frac{10}{3+2e^{k\cdot 1}} \Rightarrow \frac{1}{2}=\frac{10}{3+2e^k} \Rightarrow$
$3+2e^k=20 \Rightarrow e^k=\frac{17}{2} \Rightarrow k=\ln\frac{17}{2}$
$f(2)=\frac{10}{3+2e^{2\left(\ln\frac{17}{2}\right)}}\approx 0.068=\frac{4}{59}$

76. $f(x) = \frac{6}{a + 2e^{kx}}$; $f(0) = 1$; $f(1) = 0.8$

$1 = \frac{6}{a + 2e^{k \cdot 0}} = \frac{6}{a + 2} \Rightarrow a + 2 = 6 \Rightarrow a = 4$

$0.8 = \frac{6}{4 + 2e^{k \cdot 1}} = \frac{6}{4 + 2e^{k}} \Rightarrow$

$3.2 + 1.6e^{k} = 6 \Rightarrow e^{k} = 1.75 \Rightarrow k = \ln 1.75$

$f(2) = \frac{6}{4 + 2e^{2 \ln 1.75}} \approx 0.593 = \frac{16}{27}$

77. $f(x) = \frac{4}{a + 4e^{kx}}$; $f(0) = 2$; $f(1) = 9$

$2 = \frac{4}{a + 4e^{k \cdot 0}} = \frac{4}{a + 4} \Rightarrow 2a + 8 = 4 \Rightarrow a = -2$

$9 = \frac{4}{-2 + 4e^{k \cdot 1}} = \frac{4}{-2 + 4e^{k}} \Rightarrow$

$-18 + 36e^{k} = 4 \Rightarrow e^{k} = \frac{22}{36} = \frac{11}{18} \Rightarrow k = \ln \frac{11}{18}$

$f(2) = \frac{4}{-2 + 4e^{2 \ln(11/18)}} \approx -7.902$

78. $f(x) = \frac{a}{1 + 3e^{kx}}$; $f(0) = -1$; $f(2) = -2$

$-1 = \frac{a}{1 + 3e^{k \cdot 0}} = \frac{a}{1 + 3} \Rightarrow a = -4$

$-2 = \frac{-4}{1 + 3e^{k \cdot 2}} \Rightarrow -2 - 6e^{2k} = -4 \Rightarrow$

$e^{2k} = \frac{1}{3} \Rightarrow 2k = \ln \frac{1}{3} \Rightarrow k = \frac{1}{2} \ln \frac{1}{3}$

$f(1) = \frac{-4}{1 + 3e^{\left(\frac{1}{2} \ln \frac{1}{3}\right)(1)}} \approx -1.464$

79. $5(0.3)^{x} + 1 \le 11 \Rightarrow (0.3)^{x} \le 2 \Rightarrow$

$x \ln 0.3 \le \ln 2 \Rightarrow x \ge \frac{\ln 2}{\ln 0.3}$ (Note that $\ln 0.3 < 0$)

80. $0.1^{x} - 4 > 15 \Rightarrow 0.1^{x} > 19 \Rightarrow x \ln 0.1 > \ln 19 \Rightarrow$

$x < \frac{\ln 19}{\ln 0.1}$ (Note that $\ln 0.1 < 0$)

81. $-3(1.2)^{x} + 11 \ge 8 \Rightarrow -3(1.2)^{x} \ge -3 \Rightarrow$

$1.2^{x} \le 1 \Rightarrow x \ln 1.2 \le \ln 1 \Rightarrow x \le 0$

82. $-7(0.4)^{x} + 19 < 5 \Rightarrow -7(0.4)^{x} < -14 \Rightarrow$

$0.4^{x} > 2 \Rightarrow x \ln 0.4 > \ln 2 \Rightarrow x < \frac{\ln 2}{\ln 0.4}$

83. Note that the domain of $\log(5x + 15)$ is $(-3, \infty)$ since $5x + 15$ must be greater than 0.

$\log(5x + 15) < 2 \Rightarrow 5x + 15 < 10^{2} \Rightarrow$

$5x < 85 \Rightarrow x < 17$

Solution set: $(-3, 17)$

84. Note that the domain of $\log(2x + 0.9)$ is $(-0.45, \infty)$ since $2x + 0.9$ must be greater than 0.

$\log(2x + 0.9) > -1 \Rightarrow 2x + 0.9 > 10^{-1} \Rightarrow$

$2x > -0.8 \Rightarrow x > -0.4$

Solution set: $(-0.4, \infty)$

85. Note that the domain of $\ln(x - 5)$ is $(5, \infty)$ since $x - 5$ must be greater than 0.

$\ln(x - 5) \ge 1 \Rightarrow x - 5 \ge e \Rightarrow x \ge e + 5$

Solution set: $[e + 5, \infty)$

86. Note that the domain of $\ln(4x + 10)$ is $\left(-\frac{5}{2}, \infty\right)$ since $4x + 10$ must be greater than 0.

$\ln(4x + 10) \le 2 \Rightarrow 4x + 10 \le e^{2} \Rightarrow$

$4x \le e^{2} - 10 \Rightarrow x \le \frac{e^{2} - 10}{4}$

Solution set: $\left(-\frac{5}{2}, \frac{e^{2} - 10}{4}\right]$

87. Note that the domain of $\log_2(3x - 7)$ is $\left(\frac{7}{3}, \infty\right)$ since $3x - 7$ must be greater than 0.

$\log_2(3x - 7) < 3 \Rightarrow 3x - 7 < 2^{3} \Rightarrow 3x < 15 \Rightarrow$

$x < 5$

Solution set: $\left(\frac{7}{3}, 5\right)$

88. Note that the domain of $\log_2(5x - 4)$ is $\left(\frac{4}{5}, \infty\right)$ since $5x - 4$ must be greater than 0.

$\log_2(5x - 4) > 4 \Rightarrow 5x - 4 > 2^{4} \Rightarrow$

$5x > 20 \Rightarrow x > 4$

Solution set: $(4, \infty)$

4.4 Applying the Concepts

89. a. $18,000 = 10,000\left(1 + \frac{0.06}{1}\right)^{(1)t} \Rightarrow$

$1.8 = 1.06^{t} \Rightarrow t = \frac{\ln 1.8}{\ln 1.06} \approx 10.087$ years

b. $18,000 = 10,000\left(1 + \frac{0.06}{4}\right)^{4t} \Rightarrow$

$1.8 = 1.015^{4t} \Rightarrow 4t = \frac{\ln 1.8}{\ln 1.015} \Rightarrow t \approx 9.870$ yr

c. $18,000 = 10,000\left(1+\frac{0.06}{12}\right)^{12t} \Rightarrow$
$1.8 = 1.005^{12t} \Rightarrow 12t = \frac{\ln 1.8}{\ln 1.005} \Rightarrow$
$t \approx 9.821$ years

d. $18,000 = 10,000\left(1+\frac{0.06}{365}\right)^{365t} \Rightarrow$
$1.8 = \left(1+\frac{0.06}{365}\right)^{365t} \Rightarrow$
$365t = \frac{\ln 1.8}{\ln\left(1+\frac{0.06}{365}\right)} \Rightarrow t \approx 9.797$ years

e. $18,000 = 10,000e^{0.06t} \Rightarrow 1.8 = e^{0.06t} \Rightarrow$
$\ln 1.8 = 0.06t \Rightarrow t \approx 9.796$ years

90. a. $200 = 100\left(1+\frac{0.072}{1}\right)^{(1)t} \Rightarrow 2 = 1.072^t \Rightarrow$
$t = \frac{\ln 2}{\ln 1.072} \approx 9.970$ years

b. $200 = 100\left(1+\frac{0.072}{4}\right)^{4t} \Rightarrow 2 = 1.018^{4t} \Rightarrow$
$4t = \frac{\ln 2}{\ln 1.018} \Rightarrow t \approx 9.713$ years

c. $200 = 100\left(1+\frac{0.072}{12}\right)^{12t} \Rightarrow 2 = 1.006^{12t} \Rightarrow$
$12t = \frac{\ln 2}{\ln 1.006} \Rightarrow t \approx 9.656$ years

d. $200 = 100e^{0.072t} \Rightarrow 2 = e^{0.072t} \Rightarrow$
$\ln 2 = 0.072t \Rightarrow t \approx 9.627$ years

91. $40,000 = 20,000e^{8r} \Rightarrow 2 = e^{8r} \Rightarrow$
$\ln 2 = 8r \Rightarrow r = \frac{\ln 2}{8} \approx 0.0866 = 8.66\%$
$c_5 = 20,000e^{5(0.0866)} \approx \$30,837.52$

92. a. $2 = 1(1+r)^t \Rightarrow \ln 2 = t\ln(1+r) \Rightarrow$
$t = \frac{\ln 2}{\ln(1+r)}$ years

b. $2 = (1)e^{rt} \Rightarrow \ln 2 = rt \Rightarrow t = \frac{\ln 2}{r}$ years

93. a. $\log\left(\frac{I}{12}\right) = -0.025(30) \Rightarrow$
$\frac{I}{12} = 10^{-0.025(30)} \Rightarrow I \approx 2.134$ lumens

b. $\log\left(\frac{4}{12}\right) = -0.025x \Rightarrow x \approx 19.08$ feet

94. Using the continuous compounding formula, we have $2 = 1e^{rt} \Rightarrow \ln 2 = rt \Rightarrow \frac{\ln 2}{r} = t.$ So,
$\ln 2 \approx 0.69 \Rightarrow t = \frac{0.69}{r} \approx \frac{69}{100r} \approx \frac{70}{100r}$.

95. a. First, find *a* by substituting $t = 0$ and $f(0) = 1000$: $1000 = \frac{20,000}{1+ae^{-k(0)}} \Rightarrow$
$1000(1+a) = 20,000 \Rightarrow 1+a = 20 \Rightarrow a = 19.$
Now find *k* using $t = 4$ and $f(t) = 8999.$
$8999 = \frac{20,000}{1+19e^{-k(4)}} \Rightarrow$
$8999 + 170,981e^{-4k} = 20,000 \Rightarrow$
$e^{-4k} = \frac{11,001}{170,981} \Rightarrow -4k = \ln\left(\frac{11,001}{170,981}\right) \Rightarrow$
$k \approx 0.68589$
Use this value of *k* to find the number of people infected:
$f(8) = \frac{20,000}{1+19e^{-0.68589(8)}} \approx 18,542$

b. $12,400 = \frac{20,000}{1+19e^{-0.68589t}} \Rightarrow$
$12,400 + 12,400\cdot 19e^{-0.68589t} = 20,000 \Rightarrow$
$12,400\cdot 19e^{-0.68589t} = 7600 \Rightarrow$
$e^{-0.68589t} = \frac{7600}{12,400\cdot 19} \Rightarrow$
$-0.68589t = \ln\left(\frac{7600}{12,400\cdot 19}\right) \Rightarrow$
$t = \frac{\ln\left(\frac{7600}{12,400\cdot 19}\right)}{-0.68589} \approx 5$
12,400 people will be infected after 5 weeks.

96. a. First find *a* by substituting $t = 0$ and $f(0) = 1$:
$1 = \frac{5000}{1+ae^{-k(0)}} \Rightarrow 1+a = 5000 \Rightarrow$
$a = 4999.$ Now find *k* using $t = 10$ and $f(0) = 2500$

$$2500 = \frac{5000}{1+4999e^{-k(10)}}$$

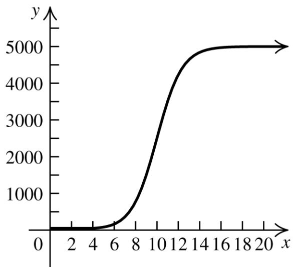
$2500(1+4999e^{-10k}) = 5000$

(*continued on next page*)

(*continued*)

$$1+4999e^{-10k}=2 \Rightarrow e^{-10k}=\frac{1}{4999} \Rightarrow$$
$$-10k=\ln\left(\frac{1}{4999}\right) \Rightarrow k \approx 0.852$$

So, the equation is $f(t)=\dfrac{5000}{1+4999e^{-0.852t}}$



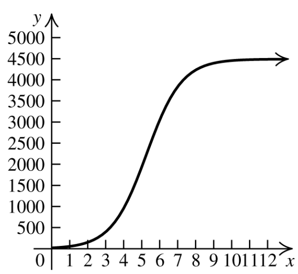
b. $R(15)=\dfrac{5000}{1+4999e^{-0.852(15)}} \approx 4930$

c. As $t \to \infty$, $R(t) \to \dfrac{5000}{1}=5000$.

97. a. $\dfrac{4490}{1+e^{5.4094-1.0255(0)}} \approx 20$

b. The carrying capacity is the numerator, 4490.

c.



98. a. First find a by substituting $t=0$ and $f(0)=1$:

$$1=\frac{20000}{20+ae^{-0\cdot k}} \Rightarrow 20+a=20000 \Rightarrow$$
$$a=19980$$

Now find k using $t = 8$ and $f(8)=845$.

$$845=\frac{20000}{20+19980e^{-8k}}$$
$$20+19980e^{-8k}=\frac{20000}{845}$$
$$19980e^{-8k}=\frac{20,000}{845}-20$$
$$e^{-8k}=\frac{\frac{20000}{845}-20}{19980}$$
$$-8k=\ln\left(\frac{\frac{20000}{845}-20}{19980}\right)$$
$$k=\frac{\ln\left(\frac{\frac{20000}{845}-20}{19980}\right)}{-8} \approx 1.075333$$

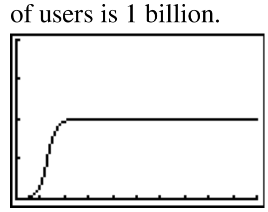
b. The year 2015 is 11 years after 2004.

$$f(11)=\frac{20000}{20+19980e^{-1.075333\cdot 11}} \approx 992.7$$

In 2015, there will be about 993 million Facebook users.

c.
$$2000=\frac{20000}{20+19980e^{-1.075333t}} \Rightarrow$$
$$2000\left(20+19980e^{-1.075333t}\right)=20000 \Rightarrow$$
$$20+19980e^{-1.075333t}=10 \Rightarrow$$
$$19980e^{-1.075333t}=-10 \Rightarrow$$
$$e^{-1.075333t}=-\frac{1}{1998} \Rightarrow$$
$$-1.075333t=\ln\left(-\frac{1}{1998}\right)$$

This is not possible because the domain of a logarithmic function is $(0, \infty)$. Thus, there will never be 2 billion users, according to the model. By graphing the function, we see that the maximum number of users is 1 billion.



[0, 50] by [0, 2000]

4.4 Beyond the Basics

99.
$$P=\frac{M}{1+e^{-kt}} \Rightarrow P\left(1+e^{-kt}\right)=M \Rightarrow$$
$$P+Pe^{-kt}=M \Rightarrow \frac{M-P}{P}=e^{-kt} \Rightarrow$$
$$\frac{M-P}{P}=\frac{1}{e^{kt}} \Rightarrow \frac{P}{M-P}=e^{kt} \Rightarrow$$
$$\ln\left(\frac{P}{M-P}\right)=kt \Rightarrow t=\frac{1}{k}\ln\left(\frac{P}{M-P}\right)$$

100. a. $2^x=e^{kx} \Rightarrow x\ln 2=kx \Rightarrow k=\ln 2 \approx 0.693$

b. $e^x=2^{kx} \Rightarrow x=kx\ln 2 \Rightarrow k=\dfrac{1}{\ln 2} \approx 1.443$

101. a. $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5} = k \Rightarrow$
$\log x = 2k \Rightarrow x = 10^{2k}.$
$\log y = 3k \Rightarrow y = 10^{3k}.$
$\log z = 5k \Rightarrow z = 10^{5k}.$
So, $xy = 10^{2k} \cdot 10^{3k} = 10^{5k} = z.$

b. $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5} \Rightarrow$
$3\log x = 2\log y \Rightarrow \log x^3 = \log y^2 \Rightarrow$
$x^3 = y^2.$
$5\log x = 2\log z \Rightarrow \log x^5 = \log z^2 \Rightarrow$
$x^5 = z^2.$
So, $y^2 z^2 = x^3 x^5 = x^8$

102. $P(1+i)^n = R\frac{(1+i)^n - 1}{i} \Rightarrow$
$iP(1+i)^n = R\left((1+i)^n - 1\right) \Rightarrow$
$\frac{iP}{R} = \frac{(1+i)^n - 1}{(1+i)^n} = 1 - \frac{1}{(1+i)^n} \Rightarrow$
$1 - \frac{iP}{R} = \frac{R - iP}{R} = \frac{1}{(1+i)^n} \Rightarrow \frac{R}{R - iP} = (1+i)^n \Rightarrow$
$\ln\left(\frac{R}{R - iP}\right) = n\ln(1+i) \Rightarrow n = \frac{\ln\left(\frac{R}{R - iP}\right)}{\ln(1+i)}$

103. $(\log x)^2 = \log x \Rightarrow (\log x)^2 - \log x = 0 \Rightarrow$
$\log x(\log x - 1) = 0 \Rightarrow \log x = 0 \Rightarrow x = 1$ or
$\log x - 1 = 0 \Rightarrow \log x = 1 \Rightarrow x = 10$

104. $(\log_2 x)(\log_2 8x) = 10 \Rightarrow$
$(\log_2 x)(\log_2 8 + \log_2 x) = 10 \Rightarrow$
$(\log_2 x)(3 + \log_2 x) = 10$
Let $u = \log_2 x.$ Then we have
$u(3+u) = 10 \Rightarrow u^2 + 3u - 10 = 0 \Rightarrow$
$(u+5)(u-2) = 0 \Rightarrow u = -5, 2 \Rightarrow$
$\log_2 x = -5 \Rightarrow x = 2^{-5} = \frac{1}{32}$ or
$\log_2 x = 2 \Rightarrow x = 2^2 = 4$

105. $(\log_3 x)(\log_3 3x) = 2 \Rightarrow$
$(\log_3 x)(\log_3 3 + \log_3 x) = 2 \Rightarrow$
$(\log_3 x)(1 + \log_3 x) = 2$
Let $u = \log_3 x.$ Then we have
$u(1+u) = 2 \Rightarrow u^2 + u - 2 = 0 \Rightarrow$
$(u+2)(u-1) = 0 \Rightarrow u = -2, 1 \Rightarrow$
$\log_3 x = -2 \Rightarrow x = 3^{-2} = \frac{1}{9}$ or
$\log_3 x = 1 \Rightarrow x = 3^1 = 3$

106. $\log_2 x + \log_4 x = 6 \Rightarrow \log_2 x + \frac{\log_2 x}{\log_2 4} = 6 \Rightarrow$
$\log_2 x + \frac{\log_2 x}{2} = 6 \Rightarrow \frac{3}{2}\log_2 x = 6 \Rightarrow$
$\log_2 x = 4 \Rightarrow x = 2^4 = 16$

107. $\log_4 x^2(x-1)^2 - \log_2(x-1) = 1 \Rightarrow$
$\frac{\log_2 x^2(x-1)^2}{\log_2 4} - \log_2(x-1) = 1 \Rightarrow$
$\frac{\log_2 [x(x-1)]^2}{2} - \log_2(x-1) = 1 \Rightarrow$
$\frac{2\log_2 [x(x-1)]}{2} - \log_2(x-1) = 1 \Rightarrow$
$\log_2 [x(x-1)] - \log_2(x-1) = 1 \Rightarrow$
$\log_2 \frac{x(x-1)}{x-1} = 1 \Rightarrow \log_2 x = 1 \Rightarrow x = 2.$

108. $\frac{\log(7x-12)}{\log x} = 2 \Rightarrow \log(7x-12) = 2\log x \Rightarrow$
$\log(7x-12) = \log x^2 \Rightarrow 7x - 12 = x^2 \Rightarrow$
$x^2 - 7x + 12 = 0 \Rightarrow (x-3)(x-4) = 0 \Rightarrow$
$x = 3, 4$
Be sure to check both solutions to verify that they are valid solutions.

109. $\frac{\log(3x-5)}{2} = \log x \Rightarrow \log(3x-5) = 2\log x \Rightarrow$
$\log(3x-5) = \log x^2 \Rightarrow 3x - 5 = x^2 \Rightarrow$
$x^2 - 3x + 5 = 0$
$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(5)}}{2(1)} = \frac{3 \pm \sqrt{9-20}}{2}$
$= \frac{3 \pm \sqrt{-11}}{2}$
Since logarithms are not defined for complex numbers, there is no solution. Solution set: $\varnothing$

110. $\log(x-4)-\log x=\log\left(\frac{1}{10-x}\right)\Rightarrow$
$\log\left(\frac{x-4}{x}\right)=\log\left(\frac{1}{10-x}\right)\Rightarrow$
$\frac{x-4}{x}=\frac{1}{10-x}\Rightarrow(x-4)(10-x)=x\Rightarrow$
$-x^2+14x-40=x\Rightarrow -x^2+13x-40=0\Rightarrow$
$x^2-13x+40=0\Rightarrow(x-5)(x-8)=0\Rightarrow$
$x=5,\ x=8$

111. $f(x)=y=3^x+5$
Interchange x and y, then solve for y.
$x=3^y+5\Rightarrow x-5=3^y\Rightarrow$
$\log_3(x-5)=y=f^{-1}(x)$

112. $f(x)=y=2^{-x}+4$
Interchange x and y, then solve for y.
$x=2^{-y}+4\Rightarrow x-4=2^{-y}\Rightarrow$
$\log_2(x-4)=-y\Rightarrow -\log_2(x-4)=f^{-1}(x)$

113. $f(x)=y=3\cdot 4^x+7$
Interchange x and y, then solve for y.
$x=3\cdot 4^y+7\Rightarrow\frac{x-7}{3}=4^y\Rightarrow$
$\log_4\left(\frac{x-7}{3}\right)=y=f^{-1}(x)$

114. $f(x)=y=2\cdot 3^{x-1}-5$
Interchange x and y, then solve for y.
$x=2\cdot 3^{y-1}-5\Rightarrow\frac{x+5}{2}=3^{y-1}\Rightarrow$
$\log_3\left(\frac{x+5}{2}\right)=y-1\Rightarrow$
$\log_3\left(\frac{x+5}{2}\right)+1=f^{-1}(x)$

115. $f(x)=y=1+\log_2(x-1)$
Interchange x and y, then solve for y.
$x=1+\log_2(y-1)\Rightarrow x-1=\log_2(y-1)\Rightarrow$
$2^{x-1}=y-1\Rightarrow 2^{x-1}+1=y=f^{-1}(x)$

116. $f(x)=y=7+2\log_6(3x-1)$
Interchange x and y, then solve for y.
$x=7+2\log_6(3y-1)\Rightarrow$
$\frac{x-7}{2}=\log_6(3y-1)\Rightarrow 6^{(x-7)/2}=3y-1\Rightarrow$
$\frac{6^{(x-7)/2}+1}{3}=y=f^{-1}(x)$

117. $f(x)=y=\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)$
Interchange x and y, then solve for y.
$x=\frac{1}{2}\ln\left(\frac{y-1}{y+1}\right)\Rightarrow 2x=\ln\left(\frac{y-1}{y+1}\right)\Rightarrow$
$e^{2x}=\frac{y-1}{y+1}\Rightarrow(y+1)e^{2x}=y-1\Rightarrow$
$ye^{2x}+e^{2x}=y-1\Rightarrow e^{2x}+1=y-ye^{2x}\Rightarrow$
$e^{2x}+1=y\left(1-e^{2x}\right)\Rightarrow\frac{1+e^{2x}}{1-e^{2x}}=y=f^{-1}(x)$

118. $f(x)=y=\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$
Interchange x and y, then solve for y.
$x=\frac{1}{2}\log\left(\frac{1+y}{1-y}\right)\Rightarrow 2x=\log\left(\frac{1+y}{1-y}\right)\Rightarrow$
$10^{2x}=\frac{1+y}{1-y}\Rightarrow 10^{2x}-10^{2x}y=1+y\Rightarrow$
$10^{2x}-1=10^{2x}y+y\Rightarrow 10^{2x}-1=y\left(10^{2x}+1\right)\Rightarrow$
$y=f^{-1}(x)=\frac{10^{2x}-1}{10^{2x}+1}$

119. $7^n>43^{67}\Rightarrow n\ln 7>67\ln 43\Rightarrow$
$n>\frac{67\ln 43}{\ln 7}\Rightarrow n>129.5$
Thus, the smallest integer for which $7^n>43^{67}$ is 130.

120. $9^n>e^{321}\Rightarrow n\ln 9>321\Rightarrow$
$n>\frac{321}{\ln 9}\Rightarrow n>146.09$
Thus, the smallest integer for which $9^n>e^{321}$ is 147.

121. $8^{1/n}<1.01\Rightarrow\frac{1}{n}\ln 8<\ln 1.01\Rightarrow$
$\frac{\ln 8}{\ln 1.01}<n\Rightarrow 208.98<n$
Thus, the smallest integer for which $8^{1/n}<1.01$ is 209.

122. $12^{1/n}<1.001\Rightarrow\frac{1}{n}\ln 12<\ln 1.001\Rightarrow$
$\frac{\ln 12}{\ln 1.001}<n\Rightarrow 2486.14<n$
Thus, the smallest integer for which $12^{1/n}<1.001$ is 2487.

123. If 31^n has 567 digits, then $566 \le \log 31^n < 567$. (See Section 4.3, Example 4.)

$$566 \le \log 31^n < 567 \Rightarrow 566 \le n \log 31 < 567 \Rightarrow \frac{566}{\log 31} \le n < \frac{567}{\log 31} \Rightarrow 379.5 \le n < 380.2$$

Thus, $n = 380$.

124. If n^{123} has 456 digits, then $455 \le \log n^{123} < 456$. (See Section 4.3, Example 4.)

$$455 \le \log n^{123} < 456 \Rightarrow 455 \le 123 \log n < 456 \Rightarrow \frac{455}{123} \le \log n < \frac{456}{123} \Rightarrow$$
$$10^{455/123} \le n < 10^{456/123} \Rightarrow 5002.5 \le n < 5097.03$$

Thus, n is any integer between 5003 and 5097.

125. $$\frac{x^{\log y}}{x^{\log z}} \cdot \frac{y^{\log z}}{y^{\log x}} \cdot \frac{z^{\log x}}{z^{\log y}} = \frac{\left(10^{\log x}\right)^{\log y}}{\left(10^{\log x}\right)^{\log z}} \cdot \frac{\left(10^{\log y}\right)^{\log z}}{\left(10^{\log y}\right)^{\log x}} \cdot \frac{\left(10^{\log z}\right)^{\log x}}{\left(10^{\log z}\right)^{\log y}} = \frac{10^{\log x \log y}}{10^{\log x \log z}} \cdot \frac{10^{\log y \log z}}{10^{\log x \log y}} \cdot \frac{10^{\log x \log z}}{10^{\log y \log z}}$$
$$= \frac{10^{\log x \log y + \log y \log z + \log x \log z}}{10^{\log x \log z + \log x \log y + \log y \log z}} = 1$$

126. $$\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{xz} xyz} = \frac{1}{\frac{\log xyz}{\log xy}} + \frac{1}{\frac{\log xyz}{\log yz}} + \frac{1}{\frac{\log xyz}{\log xz}} = \frac{\log xy}{\log xyz} + \frac{\log yz}{\log xyz} + \frac{\log xz}{\log xyz}$$
$$= \frac{\log x + \log y + \log y + \log z + \log x + \log z}{\log x + \log y + \log z} = \frac{2(\log x + \log y + \log z)}{\log x + \log y + \log z} = 2$$

4.4 Critical Thinking/Discussion/Writing

127. $$\frac{P}{2} = \frac{P}{1 + ae^{-kt}} \Rightarrow \frac{1}{2} = \frac{1}{1 + ae^{-kt}} \Rightarrow$$
$$1 + ae^{-kt} = 2 \Rightarrow ae^{-kt} = 1 \Rightarrow e^{-kt} = \frac{1}{a} = a^{-1} \Rightarrow$$
$$-kt = \ln\left(a^{-1}\right) = -\ln a \Rightarrow t = \frac{\ln a}{k}$$

128. a. $$\log_4 (x-1)^2 = 3 \Rightarrow (x-1)^2 = 4^3 = 64 \Rightarrow$$
$$x^2 - 2x - 63 = 0 \Rightarrow (x+7)(x-9) = 0 \Rightarrow$$
$$x = -7, 9$$

b. $$2\log_4 (x-1) = 3 \Rightarrow \log_4 (x-1) = \frac{3}{2} \Rightarrow$$
$$x - 1 = 4^{3/2} = 8 \Rightarrow x = 9$$

c. $$2\log_4 |x-1| = 3 \Rightarrow \log_4 |x-1| = \frac{3}{2} \Rightarrow$$
$$|x-1| = 4^{3/2} = 8 \Rightarrow x - 1 = 8 \Rightarrow x = 9 \text{ or}$$
$$x - 1 = -8 \Rightarrow x = -7$$

The three equations do not have identical solutions because the equation in (a) has a quadratic term, the equation in (b) has a linear term, and the equation in (c) has an absolute value.

4.4 Maintaining Skills

129. $9^2 = 81 \Rightarrow \log_9 81 = 2$

130. $3^{-2} = \frac{1}{9} = \log_3\left(\frac{1}{9}\right) = -2$

131. $2 \cdot 10^x + 1 = 7 \Rightarrow 2 \cdot 10^x = 6 \Rightarrow 10^x = 3 \Rightarrow$
$\log 3 = x$

132. $3e^{2x} + 5 = 17 \Rightarrow 3e^{2x} = 12 \Rightarrow e^{2x} = 4 \Rightarrow$
$\ln 4 = 2x$

133. $\log_2 64 = 6 \Rightarrow 2^6 = 64$

134. $\log_{1/2} 16 = -4 \Rightarrow \left(\frac{1}{2}\right)^{-4} = 16$

135. $\log\left(\frac{A}{2}\right) = 3 \Rightarrow 10^3 = \frac{A}{2} \Rightarrow A = 2 \cdot 10^3$

136. $\ln\left(\frac{A}{P}\right) = kt \Rightarrow e^{kt} = \frac{A}{P} \Rightarrow A = Pe^{kt}$

137. $\log_5 x = -3 \Rightarrow x = 5^{-3} = \frac{1}{125}$

138. $\log_3 \frac{1}{27} = x-1 \Rightarrow 3^{x-1} = \frac{1}{27} \Rightarrow 3^{x-1} = \frac{1}{3^3} \Rightarrow$
$3^{x-1} = 3^{-3} \Rightarrow x-1=-3 \Rightarrow x=-2$

139. $\log_x 1000 = 3 \Rightarrow 1000 = x^3 \Rightarrow 10^3 = x^3 \Rightarrow$
$x = 10$

140. $\log_2\left(x^2-6x+10\right)=1 \Rightarrow x^2-6x+10=2 \Rightarrow$
$x^2-6x+8=0 \Rightarrow (x-2)(x-4)=0 \Rightarrow$
$x=2,\ x=4$
Be sure to verify that each solution is valid.

141. $2^{x+1}=32 \Rightarrow 2^{x+1}=2^5 \Rightarrow x+1=5 \Rightarrow x=4$

142. $3^{2x-1}=7 \Rightarrow (2x-1)\ln 3 = \ln 7 \Rightarrow$
$2x-1=\frac{\ln 7}{\ln 3} \Rightarrow 2x = \frac{\ln 7}{\ln 3}+1 \Rightarrow x = \frac{1}{2}\left(\frac{\ln 7}{\ln 3}+1\right)$

143.
$$\begin{aligned} 3^{x+1} &= 5^{2x-3} \\ (x+1)\ln 3 &= (2x-3)\ln 5 \\ x\ln 3+\ln 3 &= 2x\ln 5 - 3\ln 5 \\ \ln 3 + 3\ln 5 &= 2x\ln 5 - x\ln 3 \\ \ln 3 + 3\ln 5 &= x(2\ln 5 - \ln 3) \\ x &= \frac{\ln 3+3\ln 5}{2\ln 5-\ln 3} \end{aligned}$$

144. $\log\left(x^2+x\right)=\log(3x+3) \Rightarrow$
$x^2+x=3x+3 \Rightarrow x^2-2x-3=0 \Rightarrow$
$(x-3)(x+1)=0 \Rightarrow x=3,\ x=-1$
If $x=-1$, then we have
$\log\left((-1)^2+(-1)\right)=\log(3(-1)+3) \Rightarrow$
$\log 0 = \log 0$, which is not valid. So the only solution is $x = 3$.

4.5 Logarithmic Scales

4.5 Practice Problems

1. a.
$$\begin{aligned} \text{pH} &= -\log\left[\text{H}^+\right] = -\log\left(2.68\times10^{-6}\right) \\ &= -\left(\log 2.68 + \log 10^{-6}\right) \\ &= -\log 2.68 + 6 \approx 5.57 \end{aligned}$$

b.
$$\begin{aligned} 8.47 &= -\log\left[\text{H}^+\right] \\ -8.47 &= \log\left[\text{H}^+\right] \\ \left[\text{H}^+\right] &= 10^{-8.47} = 10^{0.53}\times10^{-9} \approx 3.39\times10^{-9} \end{aligned}$$

2. $2.8 = \text{pH}_{\text{acid rain}} = -\log\left[\text{H}^+\right]_{\text{acid rain}} \Rightarrow$
$\left[\text{H}^+\right]_{\text{acid rain}} = 10^{-2.8}$
$6.2 = \text{pH}_{\text{ordinary rain}} = -\log\left[\text{H}^+\right]_{\text{ordinary rain}} \Rightarrow$
$\left[\text{H}^+\right]_{\text{ordinary rain}} = 10^{-6.2}$
$$\frac{\left[\text{H}^+\right]_{\text{acid rain}}}{\left[\text{H}^+\right]_{\text{ordinary rain}}} = \frac{10^{-2.8}}{10^{-6.2}} = 10^{-2.8-(-6.2)} = 10^{3.4} \approx 2512$$
This acid rain is about 2512 times more acidic than the ordinary rain.

3. $M = \log\left(\frac{I}{I_0}\right)$
$6.5 = \log\left(\frac{I}{I_0}\right) \Rightarrow \left(\frac{I}{I_0}\right) = 10^{6.5} \Rightarrow$
$I = 10^{6.5} I_0 \approx 3{,}162{,}278 I_0$

4. Let I_M denote the intensity of the Mozambique earthquake and let I_C denote the intensity of the southern California earthquake. Then we have
$7.0 = \log\left(\frac{I_M}{I_0}\right) \Rightarrow \frac{I_M}{I_0} = 10^{7.0} \Rightarrow I_M = 10^{7.0} I_0$
$5.2 = \log\left(\frac{I_C}{I_0}\right) \Rightarrow \frac{I_C}{I_0} = 10^{5.2} \Rightarrow I_C = 10^{5.2} I_0$
$\frac{I_M}{I_C} = \frac{10^{7.0} I_0}{10^{5.2} I_0} = \frac{10^{7.0}}{10^{5.2}} = 10^{7.0-5.2} = 10^{1.8} \approx 63.1$
The intensity of the Mozambique earthquake was about 63 times that of the southern California earthquake.

5. Let M_{2011} and E_{2011} represent the magnitude and energy of the 2011 Japan earthquake. Let M_{1997} and E_{1997} represent the magnitude and energy of the 1977 Iran earthquake. From Table 4.7, we have $M_{2011} = 9.0$ and $M_{1997} = 7.5$.
$$\frac{E_{2011}}{E_{1997}} = 10^{1.5(M_{2011}-M_{1997})} = 10^{1.5(9.0-7.5)} = 10^{2.25} \approx 177.8$$
The energy released by the 2011 Japan earthquake was about 178 times that released by the 1997 Iran earthquake.

6. $I = 200\times10^{-7}\ \text{W/m}^2$ and
$I_0 = 10^{-12}\ \text{W/m}^2$.

$$L = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{200\times10^{-7}}{10^{-12}}\right)$$
$$= 10\log\left(200\times10^{5}\right) = 10\left[\log 200 + \log 10^{5}\right]$$
$$= 10\left[\log 200 + 5\log 10\right] = 10\left[\log 200 + 5\right]$$
$$= 10\log 200 + 50 \approx 73.01$$

The decibel level is approximately 73 dB.

7. $I = I_0 \times 10^{L/10}$

$I_{75} = I_0 \times 10^{75/10}$

$I_{55} = I_0 \times 10^{55/10}$

$$\frac{I_{75}}{I_{55}} = \frac{I_0\times10^{75/10}}{I_0\times10^{55/10}} = 10^{20/10} = 10^2 = 100$$

A 75 dB sound is 100 times more intense than a 55 dB sound.

8. $I = I_0 \times 10^{L/10}$

$$I_{48} = 10^{-12}\times10^{48/10} = 10^{-12}\times10^{4.8} = 10^{-7.2}$$
$$= 10^{0.8}\times10^{-8} \approx 6.3\times10^{-8}\ \text{W/m}^2$$

The intensity of a 48 dB sound is about $6.3\times10^{-8}\ \text{W/m}^2$.

9. B is two semitones above A, so $P(f) = 200$.

$f_0 = 440$ Hz

$$P(f) = 1200\log_2\frac{f}{f_0}$$
$$200 = 1200\log_2\frac{f}{440} \Rightarrow \frac{1}{6} = \log_2\frac{f}{440} \Rightarrow$$
$$2^{1/6} = \frac{f}{440} \Rightarrow f = 2^{1/6}\cdot 440 \approx 493.9$$

The frequency of B is about 494 Hz.

10. If the reference frequency is f_0 and it increases by 25%, then $f = f_0 + 0.25f_0 = 1.25f_0$. Then,

$$P(f) = 1200\log_2\frac{1.25f_0}{f_0}$$
$$= 1200\log_2 1.25 = 1200\frac{\log 1.25}{\log 2}$$
$$\approx 386.3$$

Since an equal tempered major third is 400 cents, the difference is about 400 − 386.3 = 13.7 cents, or a little less than 14 cents.

11. a. $m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$

$$2 - 0 = \frac{5}{2}\log\left(\frac{b_1}{b_2}\right) \Rightarrow \frac{4}{5} = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{b_1}{b_2} = 10^{4/5} \approx 6.3$$

A magnitude 0 star is approximately 6.3 times brighter than a magnitude 2 star.

b. Let $m_1 = 4.6$ and $b_2 = 150b_1$.

$$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$$
$$m_2 - 4.6 = 2.5\log\left(\frac{b_1}{1.50b_1}\right) = 2.5\log\left(\frac{1}{1.50}\right)$$
$$= 2.5\log 1.50^{-1} = -2.5\log 1.50$$
$$m_2 = -2.5\log 1.50 + 4.6 \approx 4.1598$$

The magnitude of the star that is 50% brighter than a star of magnitude 4.6 is about 4.1598.

4.5 Basic Concepts and Skills

1. If the pH value of a solution is less than 7, the solution is <u>acidic</u>.

2. As $[\text{H}^+]$ increases, the pH value of a substance <u>decreases</u>.

3. On the Richter scale, the magnitude of an earthquake $M = \underline{\log\frac{I}{I_0}}$.

4. The energy E released by an earthquake of magnitude M is given by $\log E = \underline{4.4 + 1.5M}$.

5. The loudness L of a sound of intensity I is given by $\underline{L = 10\log\left(\frac{I}{I_0}\right)}$.

6. If f_0 is a reference frequency and f is the frequency of a note, then the change in pitch $\underline{P(f) = 1200\log_2\frac{f}{f_0}}$.

7. If two stars have magnitudes m_1 and m_2 with apparent brightness b_1 and b_2, respectively, then $m_2 - m_1 = \underline{2.5\log\frac{b_1}{b_2}}$.

8. False. The acidity of a solution decreases as its pH value increases.

9. True

10. True

11. $\text{pH} = -\log\left[\text{H}^+\right] = -\log\left(10^{-8}\right) = 8\log 10 = 8$
The substance is a base.

12. $\text{pH} = -\log\left[\text{H}^+\right] = -\log\left(10^{-4}\right) = 4\log 10 = 4$
The substance is an acid.

13. $\text{pH} = -\log\left[\text{H}^+\right] = -\log\left(2.3\times 10^{-5}\right)$
$= -\left(\log 2.3 + \log 10^{-5}\right)$
$= -\left(\log 2.3 - 5\log 10\right) = -\left(\log 2.3 - 5\right)$
$= -\log 2.3 + 5 \approx 4.64$
The substance is an acid.

14. $\text{pH} = -\log\left[\text{H}^+\right] = -\log\left(4.7\times 10^{-9}\right)$
$= -\left(\log 4.7 + \log 10^{-9}\right)$
$= -\left(\log 4.7 - 9\log 10\right) = -\left(\log 4.7 - 9\right)$
$= -\log 4.7 + 9 \approx 8.33$
The substance is a base.

15. $\text{pH} = 6 = -\log\left[\text{H}^+\right] \Rightarrow \left[\text{H}^+\right] = 10^{-6}$
$\left[\text{H}^+\right]\left[\text{OH}^-\right] = 10^{-14}$
$10^{-6}\left[\text{OH}^-\right] = 10^{-14}$
$\left[\text{OH}^-\right] = \dfrac{10^{-14}}{10^{-6}} = 10^{-8}$

16. $\text{pH} = 8 = -\log\left[\text{H}^+\right] \Rightarrow \left[\text{H}^+\right] = 10^{-8}$
$\left[\text{H}^+\right]\left[\text{OH}^-\right] = 10^{-14}$
$10^{-8}\left[\text{OH}^-\right] = 10^{-14}$
$\left[\text{OH}^-\right] = \dfrac{10^{-14}}{10^{-8}} = 10^{-6}$

17. $\text{pH} = 9.5 = -\log\left[\text{H}^+\right] \Rightarrow \left[\text{H}^+\right] = 10^{-9.5} \Rightarrow$
$\left[\text{H}^+\right] = 10^{0.5}\times 10^{-10} \approx 3.16\times 10^{-10}$
$\left[\text{H}^+\right]\left[\text{OH}^-\right] = 10^{-14}$
$\left(3.16\times 10^{-10}\right)\left[\text{OH}^-\right] = 10^{-14}$
$\left[\text{OH}^-\right] = \dfrac{10^{-14}}{3.16\times 10^{-10}} = \dfrac{1}{3.16}\times 10^{-4}$
$\approx 0.316\times 10^{-4} = 3.16\times 10^{-5}$

18. $\text{pH} = 3.7 = -\log\left[\text{H}^+\right] \Rightarrow \left[\text{H}^+\right] = 10^{-3.7} \Rightarrow$
$\left[\text{H}^+\right] = 10^{0.3}\times 10^{-4} \approx 2.0\times 10^{-4}$
$\left[\text{H}^+\right]\left[\text{OH}^-\right] = 10^{-14}$
$\left(2.0\times 10^{-4}\right)\left[\text{OH}^-\right] = 10^{-14}$
$\left[\text{OH}^-\right] = \dfrac{10^{-14}}{2.0\times 10^{-4}}$
$= \dfrac{1}{2.0}\times 10^{-10}$
$\approx 0.5\times 10^{-10}$
$= 5.0\times 10^{-11}$

19. a. $M = \log\left(\dfrac{I}{I_0}\right)$
$5 = \log\left(\dfrac{I}{I_0}\right) \Rightarrow \left(\dfrac{I}{I_0}\right) = 10^5 \Rightarrow I = 10^5 I_0$

b. $\log E = 4.4 + 1.5M$
$\log E = 4.4 + 1.5(5) = 11.9 \Rightarrow$
$E = 10^{11.9} = 10^{0.9}\times 10^{11}$
$\approx 7.94\times 10^{11}$ joules

20. a. $M = \log\left(\dfrac{I}{I_0}\right)$
$2 = \log\left(\dfrac{I}{I_0}\right) \Rightarrow \left(\dfrac{I}{I_0}\right) = 10^2 \Rightarrow I = 100 I_0$

b. $\log E = 4.4 + 1.5M$
$\log E = 4.4 + 1.5(2) = 7.4 \Rightarrow$
$E = 10^{7.4} = 10^{0.4}\times 10^{7}$
$\approx 2.51\times 10^{7}$ joules

21. a. $M = \log\left(\dfrac{I}{I_0}\right)$
$7.8 = \log\left(\dfrac{I}{I_0}\right) \Rightarrow \left(\dfrac{I}{I_0}\right) = 10^{7.8} \Rightarrow$
$I = 10^{0.8}\times 10^7 I_0 \approx 6.3\times 10^7 I_0$

b. $\log E = 4.4 + 1.5M$
$\log E = 4.4 + 1.5(7.8) = 16.1 \Rightarrow$
$E = 10^{16.1} = 10^{0.1}\times 10^{16}$
$\approx 1.26\times 10^{16}$ joules

22. a. $M = \log\left(\frac{I}{I_0}\right)$

$3.7 = \log\left(\frac{I}{I_0}\right) \Rightarrow \left(\frac{I}{I_0}\right) = 10^{3.7} \Rightarrow$

$I = 10^{0.7} \times 10^3 I_0 \approx 5.012 \times 10^3 I_0$

$= 5012 I_0$

b. $\log E = 4.4 + 1.5M$

$\log E = 4.4 + 1.5(3.7) = 9.95 \Rightarrow$

$E = 10^{9.95} = 10^{0.95} \times 10^9$

$\approx 8.91 \times 10^9$ joules

23. a. $\log E = 4.4 + 1.5M$

$\log 10^{13.4} = 4.4 + 1.5M \Rightarrow 13.4 = 4.4 + 1.5M$

$M = \frac{13.4 - 4.4}{1.5} = \frac{9}{1.5} = 6$

b. $M = \log\left(\frac{I}{I_0}\right)$

$6 = \log\left(\frac{I}{I_0}\right) \Rightarrow \left(\frac{I}{I_0}\right) = 10^6 \Rightarrow I = 10^6 I_0$

24. a. $\log E = 4.4 + 1.5M$

$\log 10^{10.4} = 4.4 + 1.5M \Rightarrow 10.4 = 4.4 + 1.5M$

$M = \frac{10.4 - 4.4}{1.5} = \frac{6}{1.5} = 4$

b. $M = \log\left(\frac{I}{I_0}\right)$

$4 = \log\left(\frac{I}{I_0}\right) \Rightarrow \left(\frac{I}{I_0}\right) = 10^4 \Rightarrow I = 10^4 I_0$

25. a. $\log E = 4.4 + 1.5M$

$\log 10^{12} = 4.4 + 1.5M \Rightarrow 12 = 4.4 + 1.5M$

$M = \frac{12 - 4.4}{1.5} = \frac{7.6}{1.5} \approx 5.1$

b. $M = \log\left(\frac{I}{I_0}\right)$

$5.1 = \log\left(\frac{I}{I_0}\right) \Rightarrow \left(\frac{I}{I_0}\right) = 10^{5.1}$

$I = 10^{5.1} I_0 = 10^{0.1} \times 10^5 I_0 \approx 1.26 \times 10^5 I_0$

26. a. $\log E = 4.4 + 1.5M$

$\log 10^9 = 4.4 + 1.5M \Rightarrow 9 = 4.4 + 1.5M$

$M = \frac{9 - 4.4}{1.5} = \frac{4.6}{1.5} \approx 3.1$

b. $M = \log\left(\frac{I}{I_0}\right)$

$3.1 = \log\left(\frac{I}{I_0}\right) \Rightarrow \left(\frac{I}{I_0}\right) = 10^{3.1}$

$I = 10^{3.1} I_0 = 10^{0.1} \times 10^3 I_0 \approx 1.26 \times 10^3 I_0$

27. $L = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{10^{-8}}{10^{-12}}\right) = 10\log 10^4$

$= 40$

28. $L = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{10^{-10}}{10^{-12}}\right)$

$= 10\log 10^2 = 20$

29. $L = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{3.5 \times 10^{-7}}{10^{-12}}\right)$

$= 10\log\left(3.5 \times 10^5\right) = 10\left(\log 3.5 + \log 10^5\right)$

$= 10\log 3.5 + 50 \approx 55.4$

30. $L = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{2.37 \times 10^{-5}}{10^{-12}}\right)$

$= 10\log\left(2.37 \times 10^7\right) = 10\left(\log 2.37 + \log 10^7\right)$

$= 10\log 2.37 + 70 \approx 73.7$

31. $I = I_0 \times 10^{L/10} = 10^{-12} \times 10^{80/10}$

$= 10^{-12} \times 10^8 = 10^{-4}$

32. $I = I_0 \times 10^{L/10} = 10^{-12} \times 10^{90/10}$

$= 10^{-12} \times 10^9 = 10^{-3}$

33. $I = I_0 \times 10^{L/10} = 10^{-12} \times 10^{64.7/10}$

$= 10^{-12} \times 10^{6.47} = 10^{-12} \times 10^6 \times 10^{0.47}$

$\approx 2.95 \times 10^{-6}$

34. $I = I_0 \times 10^{L/10} = 10^{-12} \times 10^{37.4/10}$

$= 10^{-12} \times 10^{3.74} = 10^{-12} \times 10^3 \times 10^{0.74}$

$\approx 5.50 \times 10^{-9}$

35. A# is one semitone above A, so $P(f) = 100$.

$f_0 = 440$ Hz

$P(f) = 1200\log_2 \frac{f}{f_0}$

$100 = 1200\log_2 \frac{f}{440} \Rightarrow \frac{1}{12} = \log_2 \frac{f}{440} \Rightarrow$

$2^{1/12} = \frac{f}{440} \Rightarrow f = 2^{1/12} \cdot 440 \approx 466$ Hz

The frequency of A# is about 466 Hz.

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(*continued*)

C is three semitones above A, so $P(f) = 300.$

$f_0 = 440$ Hz

$P(f) = 1200\log_2 \frac{f}{f_0}$

$300 = 1200\log_2 \frac{f}{440} \Rightarrow \frac{1}{4} = \log_2 \frac{f}{440} \Rightarrow$

$2^{1/4} = \frac{f}{440} \Rightarrow f = 2^{1/4} \cdot 440 \approx 523$ Hz

The frequency of C is about 523 Hz.

36. D is five semitones above A, so $P(f) = 500.$

$f_0 = 440$ Hz

$P(f) = 1200\log_2 \frac{f}{f_0}$

$500 = 1200\log_2 \frac{f}{440} \Rightarrow \frac{5}{12} = \log_2 \frac{f}{440} \Rightarrow$

$2^{5/12} = \frac{f}{440} \Rightarrow f = 2^{5/12} \cdot 440 \approx 587$ Hz

The frequency of D is about 587 Hz.
E is seven semitones above A, so $P(f) = 700.$ $f_0 = 440$ Hz

$P(f) = 1200\log_2 \frac{f}{f_0}$

$700 = 1200\log_2 \frac{f}{440} \Rightarrow \frac{7}{12} = \log_2 \frac{f}{440} \Rightarrow$

$2^{7/12} = \frac{f}{440} \Rightarrow f = 2^{7/12} \cdot 440 \approx 659$ Hz

The frequency of E is about 659 Hz.

37. $P(f) = 1200\log_2 \frac{f}{f_0} = 1200\log_2 \left(\frac{10}{9}\right)$

$= 1200\left(\frac{\log\left(\frac{10}{9}\right)}{\log 2}\right) \approx 182$

The difference, 200 − 182 = 18 cents, is noticeable.

38. $P(f) = 1200\log_2 \frac{f}{f_0} = 1200\log_2 \left(\frac{9}{8}\right)$

$= 1200\left(\frac{\log\left(\frac{9}{8}\right)}{\log 2}\right) \approx 204$

The difference, 204 − 200 = 4 cents, is barely noticeable.

39. Let the magnitude and brightness of star *A* be m_1 and b_1, and let the magnitude of star *B* be m_2 and b_2.

$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$

$20 - 4 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow 16 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$6.4 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow \frac{b_1}{b_2} = 10^{6.4} \Rightarrow$

$b_1 = 10^{0.4} \times 10^6 b_2 \approx 2.5 \times 10^6 b_2$

Star *A* is 2.5×10^6 times as bright as star *B*.

40. Let the magnitude and brightness of star *A* be m_1 and b_1, and let the magnitude of star *B* be m_2 and b_2.

$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$

$5 - (-2) = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow 7 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$2.8 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow \frac{b_1}{b_2} = 10^{2.8} \Rightarrow$

$b_1 = 10^{0.8} \times 10^2 b_2 \approx 6.3 \times 10^2 b_2$

Star *A* is 6.3×10^2 times as bright as star *B*.

41. Let the magnitude and brightness of star *A* be m_1 and b_1, and let the magnitude of star *B* be $m_2 = m_1 - 2$ and b_2.

$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$

$(m_1 - 2) - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$-2 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow -0.8 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$\frac{b_1}{b_2} = 10^{-0.8} \approx \frac{1}{6.3} \Rightarrow 6.3b_1 = b_2$

Star *B* is about 6.3 times brighter than star *A*.

42. Let the magnitude and brightness of star *A* be m_1 and b_1, and let the magnitude of star *B* be $m_2 = m_1 - 1$ and b_2.

$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$

$(m_1 - 1) - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$-1 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow -0.4 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$\frac{b_1}{b_2} = 10^{-0.4} \approx \frac{1}{2.5} \Rightarrow 2.5b_1 = b_2$

Star *B* is about 2.5 times brighter than star *A*.

4.5 Applying the Concepts

43. $\text{pH} = -\log\left[H^+\right] = -\log\left(3.98\times10^{-8}\right)$
$= -\left(\log 3.98 + \log 10^{-8}\right) = -\left(\log 3.98 - 8\right)$
$= -\log 3.98 + 8 \approx 7.4$
The pH of human blood is about 7.4; it is basic.

44. $\text{pH} = -\log\left[H^+\right] = -\log\left(3.16\times10^{-7}\right)$
$= -\left(\log 3.16 + \log 10^{-7}\right) = -\left(\log 3.16 - 7\right)$
$= -\log 3.16 + 7 \approx 6.5$
The pH of milk is about 6.5; it is acidic.

45. a. $\text{pH} = 3.15 = -\log\left[H^+\right] \Rightarrow \left[H^+\right] = 10^{-3.15} \Rightarrow$
$\left[H^+\right] = 10^{0.85}\times10^{-4} \approx 7.1\times10^{-4}$

b. $\text{pH} = 7.2 = -\log\left[H^+\right] \Rightarrow \left[H^+\right] = 10^{-7.2} \Rightarrow$
$\left[H^+\right] = 10^{0.8}\times10^{-8} \approx 6.3\times10^{-8}$

c. $\text{pH} = 7.78 = -\log\left[H^+\right] \Rightarrow \left[H^+\right] = 10^{-7.78} \Rightarrow$
$\left[H^+\right] = 10^{0.22}\times10^{-8} \approx 1.7\times10^{-8}$

d. $\text{pH} = 3 = -\log\left[H^+\right] \Rightarrow \left[H^+\right] = 10^{-3} \Rightarrow$
$\left[H^+\right] = 10^{-3}$

46. a. $\text{pH} = 1.0 = -\log\left[H^+\right] \Rightarrow \left[H^+\right] = 10^{-1.0} = 0.1$

b. $\text{pH} = 8.7 = -\log\left[H^+\right] \Rightarrow \left[H^+\right] = 10^{-8.7} \Rightarrow$
$\left[H^+\right] = 10^{0.3}\times10^{-9} \approx 2.0\times10^{-9}$

c. $\text{pH} = 10.6 = -\log\left[H^+\right] \Rightarrow \left[H^+\right] = 10^{-10.6} \Rightarrow$
$\left[H^+\right] = 10^{0.4}\times10^{-11} \approx 2.5\times10^{-11}$

d. $\text{pH} = 2.3 = -\log\left[H^+\right] \Rightarrow \left[H^+\right] = 10^{-2.3} \Rightarrow$
$\left[H^+\right] = 10^{0.7}\times10^{-3} \approx 5.0\times10^{-3} = 0.005$

47. $\text{pH}_{\text{acid rain}} = 3.8 = -\log\left[H^+\right] \Rightarrow$
$\left[H^+\right] = 10^{-3.8} \Rightarrow$
$\left[H^+\right] = 10^{0.2}\times10^{-4} \approx 1.6\times10^{-4}$
The average concentration of hydrogen ions in the acid rain is 1.6×10^{-4} moles per liter.
$\text{pH}_{\text{ordinary rain}} = 6 = -\log\left[H^+\right] \Rightarrow$
$\left[H^+\right] = 10^{-6}$

$$\frac{\left[H^+\right]_{\text{acid rain}}}{\left[H^+\right]_{\text{ordinary rain}}} = \frac{1.6\times10^{-4}}{10^{-6}} = 1.6\times10^{2} = 160$$

The acid rain is 160 times more acidic than ordinary rain.

48. $\text{pH}_A = \text{pH}_B + 1.0 = -\log\left[H^+{}_A\right] \Rightarrow$
$\left[H^+{}_A\right] = 10^{-(\text{pH}_B + 1.0)} = 10^{(-\text{pH}_B - 1.0)}$
$= 10^{(-\text{pH}_B)}10^{-1.0} = \dfrac{10^{(-\text{pH}_B)}}{10}$
$\text{pH}_B = -\log\left[H^+{}_B\right] \Rightarrow \left[H^+{}_B\right] = 10^{(-\text{pH}_B)}$

$$\frac{\left[H^+{}_A\right]}{\left[H^+{}_B\right]} = \frac{\frac{10^{(-\text{pH}_B)}}{10}}{10^{(-\text{pH}_B)}} = \frac{10^{(-\text{pH}_B)}}{10\times10^{(-\text{pH}_B)}} \Rightarrow$$

$10\left[H^+{}_A\right] = \left[H^+{}_B\right]$

49. $\left[H^+{}_A\right] = 100\left[H^+{}_B\right]$
$\text{pH}_A = -\log\left[H^+{}_A\right] = -\log\left(100\left[H^+{}_B\right]\right)$
$= -\left(\log 100 + \log\left[H^+{}_B\right]\right)$
$= -\left(2 + \log\left[H^+{}_B\right]\right) = -\log\left[H^+{}_B\right] - 2$
$= \text{pH}_B - 2$

50. Let x = the original pH of the solution. Then $x + 1.5$ = the new pH of the solution.
$x = -\log\left[H^+{}_{\text{original}}\right] \Rightarrow 10^{-x} = \left[H^+{}_{\text{original}}\right]$
$x + 1.5 = -\log\left[H^+{}_{\text{new}}\right] \Rightarrow$
$10^{-(x+1.5)} = \left[H^+{}_{\text{new}}\right] \Rightarrow$
$10^{(-x-1.5)} = \left[H^+{}_{\text{new}}\right] \Rightarrow$
$10^{-x}\times10^{-1.5} = \left[H^+{}_{\text{new}}\right] \Rightarrow$
$\left[H^+{}_{\text{original}}\right]\times10^{0.5}\times10^{-2} = \left[H^+{}_{\text{new}}\right] \Rightarrow$
$\left[H^+{}_{\text{original}}\right]\times3.2\times10^{-2} = \left[H^+{}_{\text{new}}\right]$
$\left[H^+\right]$ decreases by a factor of 0.032. The increase in pH makes the solution more basic.

51. $\left[H^+{}_{\text{new}}\right] = 50\left[H^+{}_{\text{original}}\right]$
$\text{pH}_{\text{new}} = -\log\left(50\left[H^+{}_{\text{original}}\right]\right)$
$= -\log 50 - \log\left[H^+{}_{\text{original}}\right]$
$= -\log 50 - \text{pH}_{\text{original}}$
The pH decreases by $\log 50 \approx 1.7$.
The solution becomes more acidic.

52. a. $M = \log\left(\frac{I}{I_0}\right)$

$8.6 = \log\left(\frac{I}{I_0}\right) \Rightarrow \left(\frac{I}{I_0}\right) = 10^{8.6} \Rightarrow$

$I = 10^{8.6} I_0 = 10^{0.6} \times 10^8 I_0 \approx 3.98 \times 10^8 I_0$

b. $\log E = 4.4 + 1.5(8.6) = 17.3 \Rightarrow$

$E = 10^{17.3} = 10^{0.3} \times 10^{17}$

$\approx 1.995 \times 10^{17}$ joules

53. a. $M = \log\left(\frac{I}{I_0}\right)$

$7.8 = \log\left(\frac{I}{I_0}\right) \Rightarrow \left(\frac{I}{I_0}\right) = 10^{7.8} \Rightarrow$

$I = 10^{7.8} I_0 = 10^{0.8} \times 10^7 I_0 \approx 6.31 \times 10^7 I_0$

b. $\log E = 4.4 + 1.5(7.8) = 16.1 \Rightarrow$

$E = 10^{16.1} = 10^{0.1} \times 10^{16}$

$\approx 1.259 \times 10^{16}$ joules

54. a. Let I_{1923} denote the intensity of the 1923 earthquake and let I_{2011} denote the intensity of the 2011 earthquake. Then we have

$9.0 = \log\left(\frac{I_{2011}}{I_0}\right) \Rightarrow \frac{I_{2011}}{I_0} = 10^{9.0} \Rightarrow$

$I_{2011} = 10^{9.0} I_0$

$8.3 = \log\left(\frac{I_{1923}}{I_0}\right) \Rightarrow \frac{I_{1923}}{I_0} = 10^{8.3} \Rightarrow$

$I_{1923} = 10^{8.3} I_0$

$\frac{I_{2011}}{I_{1923}} = \frac{10^{9.0} I_0}{10^{8.3} I_0} = 10^{0.7} \approx 5.0 \Rightarrow$

$I_{2011} \approx 5.0 I_{1923}$

The intensity of the 2011 earthquake was about 5 times that of the 1923 earthquake.

b. Let M_{2011} and E_{2011} represent the magnitude and energy of the 2011 earthquake. Let M_{1923} and E_{1923} represent the magnitude and energy of the 1977 Iran earthquake. $M_{2011} = 9.0$ and $M_{1923} = 8.3$.

$\frac{E_{2011}}{E_{1923}} = 10^{1.5(M_{2011} - M_{1923})} = 10^{1.5(9.0-8.3)}$

$= 10^{1.05} \approx 11.2 \Rightarrow E_{2011} \approx 11.2 E_{1923}$

The energy released by the 2011 earthquake was about 11.2 times that released by the 1923 earthquake.

55. a. $M_A = M_B + 1$. Then we have

$M_A = M_B + 1 = \log\left(\frac{I_A}{I_0}\right) \Rightarrow$

$10^{(M_B+1)} = \frac{I_A}{I_0} \Rightarrow 10^{M_B} \times 10 = \frac{I_A}{I_0} \Rightarrow$

$10^{M_B} \times 10 I_0 = I_A \quad (1)$

$M_B = \log\left(\frac{I_B}{I_0}\right) \Rightarrow 10^{M_B} = \frac{I_B}{I_0} \Rightarrow$

$10^{M_B} \times I_0 = I_B$

Using equation (1), we have

$10^{M_B} \times 10 I_0 = I_A \Rightarrow 10^{M_B} \times I_0 \times 10 = I_A \Rightarrow$

$I_B \times 10 = I_A$

The intensity of the earthquake A was 10 times that of earthquake B.

b. $\frac{E_A}{E_B} = 10^{1.5(M_A - M_B)} = 10^{1.5(M_B + 1 - M_B)}$

$= 10^{1.5} \approx 31.6 \Rightarrow E_A \approx 31.6 E_B$

The energy released by the earthquake A was about 31.6 times that released by earthquake B.

56. a. $M_A = M_B + 1.5$. Then we have

$M_A = M_B + 1.5 = \log\left(\frac{I_A}{I_0}\right) \Rightarrow$

$10^{(M_B+1.5)} = \frac{I_A}{I_0} \Rightarrow 10^{M_B} \times 10^{1.5} = \frac{I_A}{I_0} \Rightarrow$

$10^{M_B} \times 31.6 I_0 = I_A \quad (1)$

$M_B = \log\left(\frac{I_B}{I_0}\right) \Rightarrow$

$10^{M_B} = \frac{I_B}{I_0} \Rightarrow 10^{M_B} \times I_0 = I_B$

Using equation (1), we have

$10^{M_B} \times 31.6 I_0 = I_A \Rightarrow$

$10^{M_B} \times I_0 \times 31.6 = I_A \Rightarrow I_B \times 31.6 = I_A$

The intensity of the earthquake A was about 31.6 times that of earthquake B.

b. $\frac{E_A}{E_B} = 10^{1.5(M_A - M_B)} = 10^{1.5(M_B + 1.5 - M_B)}$

$= 10^{2.25} \approx 177.8 \Rightarrow E_A \approx 177.8 E_B$

The energy released by the earthquake A was about 177.8 times that released by earthquake B.

57. $M = \log\left(\frac{I}{I_0}\right)$

$$I_A = 150 I_B \Rightarrow$$
$$\log\left(\frac{I_A}{I_0}\right) = \log\left(\frac{150 I_B}{I_0}\right) = \log\left(150 \cdot \frac{I_B}{I_0}\right)$$
$$= \log 150 + \log\left(\frac{I_B}{I_0}\right) = \log 150 + M_B$$

The difference in Richter scale readings is $\log 150 \approx 2.18$.

58. $E_A = 150 E_B \Rightarrow \frac{E_A}{E_B} = 150 = 10^{1.5(M_A - M_B)} \Rightarrow$

$$\log 150 = 1.5(M_A - M_B) \Rightarrow$$
$$M_A - M_B = \frac{\log 150}{1.5} \approx 1.45$$

The difference in Richter scale readings is $\frac{\log 150}{1.5} \approx 1.45$.

59. $L = 10 \log\left(\frac{I}{I_0}\right) = 10 \log\left(\frac{5.2 \times 10^{-5}}{10^{-12}}\right)$

$$= 10 \log\left(5.2 \times 10^{7}\right) = 10\left[\log 5.2 + \log 10^{7}\right]$$
$$= 10\left[\log 5.2 + 7\right] = 10 \log 5.2 + 70 \approx 77.2 \text{ dB}$$

60. $L = 10 \log\left(\frac{I}{I_0}\right) = 10 \log\left(\frac{2.5 \times 10^{2}}{10^{-12}}\right)$

$$= 10 \log\left(2.5 \times 10^{14}\right) = 10\left[\log 2.5 + \log 10^{14}\right]$$
$$= 10\left[\log 5.2 + 14\right] = 10 \log 2.5 + 140 \approx 144.0 \text{ dB}$$

61. $I = I_0 \times 10^{L/10}$

$$I_{130} = I_0 \times 10^{130/10}$$
$$I_{65} = I_0 \times 10^{65/10}$$
$$\frac{I_{130}}{I_{65}} = \frac{I_0 \times 10^{130/10}}{I_0 \times 10^{65/10}} = 10^{65/10} = 10^{6.5}$$
$$= 10^{0.5} \times 10^{6} \approx 3.16 \times 10^{6}$$

The 130 dB sound is about 3.16×10^{6} times as intense as the 65 dB sound.

62. $I = I_0 \times 10^{L/10}$

$$I_{75} = I_0 \times 10^{75/10}$$
$$I_{62} = I_0 \times 10^{62/10}$$
$$\frac{I_{75}}{I_{62}} = \frac{I_0 \times 10^{75/10}}{I_0 \times 10^{62/10}} = 10^{13/10} = 10^{1.3} \approx 20.0$$

The 75 dB sound is about 20 times as intense as the 62 dB sound.

63. A sound at the threshold of pain has intensity 10 W/m^2. A sound that is 1000 times as intense has intensity 10^4 W/m^2.

$$L = 10 \log\left(\frac{I}{I_0}\right) = 10 \log\left(\frac{10^4}{10^{-12}}\right) = 10 \log 10^{16}$$
$$= 160 \text{ dB}$$

64. $I = I_0 \times 10^{L/10}$

$$I_{L_2} = I_0 \times 10^{L_2/10} = I_0 \times 10^{0.1L_2}$$
$$I_{L_1} = I_0 \times 10^{L_1/10} = I_0 \times 10^{0.1L_1}$$
$$\frac{I_{L_2}}{I_{L_1}} = \frac{I_0 \times 10^{0.1L_2}}{I_0 \times 10^{0.1L_1}} = 10^{0.1L_2 - 0.1L_1} = 10^{0.1(L_2 - L_1)}$$

65. $L_2 = L_1 + 1$

$$I_{L_2} = I_0 \times 10^{L_2/10} = I_0 \times 10^{0.1(L_1+1)}$$
$$I_{L_1} = I_0 \times 10^{L_1/10} = I_0 \times 10^{0.1L_1}$$
$$\frac{I_{L_2}}{I_{L_1}} = \frac{I_0 \times 10^{0.1(L_1+1)}}{I_0 \times 10^{0.1L_1}} = 10^{0.1(L_1+1-L_1)}$$
$$= 10^{0.1} \approx 1.26 \Rightarrow I_{L_2} \approx 1.26 \times I_{L_1}$$

The intensity of the louder sound is about 1.26 times the intensity of the softer sound.

66. $I = I_0 \times 10^{L/10}$

$$I_{70} = I_0 \times 10^{70/10} = I_0 \times 10^{7}$$
$$I_{29} = I_0 \times 10^{29/10} = I_0 \times 10^{2.9}$$
$$\frac{I_{70}}{I_{29}} = \frac{I_0 \times 10^{7}}{I_0 \times 10^{2.9}} = 10^{4.1} \Rightarrow$$
$$I_{70} = 10^{0.1} \times 10^{4} I_{29} \approx 1.26 \times 10^{4} I_{29}$$

The intensity of the outside sound is about 1.26×10^{4} times the intensity of the inside sound.

67. $P(f) = 1200 \log_2 \frac{f}{f_0}$

$$P(441) = 1200 \log_2\left(\frac{441}{440}\right) = 1200\left(\frac{\log\left(\frac{441}{440}\right)}{\log 2}\right)$$
$$\approx 3.93 \approx 4$$

The difference in pitch is about 4 cents higher.

68. $P(f) = 1200 \log_2 \frac{f}{f_0}$

$$P(441) = 1200 \log_2\left(\frac{438}{440}\right) = 1200\left(\frac{\log\left(\frac{438}{440}\right)}{\log 2}\right)$$
$$\approx -7.89 \approx -8$$

The difference in pitch is about 8 cents lower.

69. Let m_1 represent the magnitude of the sun and let m_2 represent the magnitude of the full Moon.

$$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$$
$$-13-(-27) = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{14}{2.5} = 5.6 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{b_1}{b_2} = 10^{5.6} = 10^{0.6} \times 10^5 \approx 3.98 \times 10^5 \Rightarrow$$
$$b_1 = 3.98 \times 10^5 b_2$$

The sun is about 1.63×10^5 times brighter than the full Moon.

70. Let m_1 represent the magnitude of the full Moon and let m_2 represent the magnitude of Vega.

$$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$$
$$0.03-(-13) = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{13.03}{2.5} = 5.212 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{b_1}{b_2} = 10^{5.212} = 10^{0.212} \times 10^5 \approx 1.63 \times 10^5 \Rightarrow$$
$$b_1 = 1.63 \times 10^5 b_2$$

The full Moon is about 1.63×10^5 times brighter than Vega.

71. Let m_1 represent the magnitude of the Sun and let m_2 represent the magnitude of Venus.

$$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$$
$$-4-(-27) = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{23}{2.5} = 9.2 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{b_1}{b_2} = 10^{9.2} = 10^{0.2} \times 10^9 \approx 1.58 \times 10^9 \Rightarrow$$
$$b_1 = 1.58 \times 10^9 b_2$$

The Sun is about 1.58×10^9 times brighter than Venus.

72. Let m_1 represent the magnitude of the Venus and let m_2 represent the magnitude of Neptune.

$$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$$
$$7.8-(-4) = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{11.8}{2.5} = 4.72 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{b_1}{b_2} = 10^{4.72} = 10^{0.72} \times 10^4 \approx 5.2481 \times 10^4 \Rightarrow$$
$$b_1 = 52,481 b_2$$

The Venus is about 52,481 times brighter than Neptune.

73. Let m_1 and b_1 represent the magnitude and brightness of the unknown star and let m_2 and b_2 represent the magnitude and brightness of Saturn.

$$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right);\ b_1 = 560 b_2$$
$$1.47 - m_1 = 2.5\log\left(\frac{560 b_2}{b_2}\right) \Rightarrow$$
$$1.47 - m_1 = 2.5\log 560 \Rightarrow$$
$$1.47 - 2.5\log 560 = m_1 \Rightarrow -5.4 \approx m_1$$

The magnitude of the unknown star is about −5.4.

74. Let m_1 and b_1 represent the magnitude and brightness of the unknown star and let m_2 and b_2 represent the magnitude and brightness of Neptune.

$$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right);\ b_1 = 10^9 b_2$$
$$7.8 - m_1 = 2.5\log\left(\frac{10^9 b_2}{b_2}\right) \Rightarrow$$
$$7.8 - m_1 = 2.5\log\left(10^9\right) \Rightarrow$$
$$7.8 - 22.5 = m_1 \Rightarrow -14.7 \approx m_1$$

The magnitude of the unknown star is about −14.7.

4.5 Beyond the Basics

75. $L = 10\log\left(\frac{4\times10^4}{r^2}\right)$

a. $r = 10$ ft

$L = 10\log\left(\frac{4\times10^4}{10^2}\right) \approx 26$ dB

$r = 25$ ft

$L = 10\log\left(\frac{4\times10^4}{25^2}\right) \approx 18$ dB

$r = 50$ ft

$L = 10\log\left(\frac{4\times10^4}{50^2}\right) \approx 12$ dB

$r = 100$ ft

$L = 10\log\left(\frac{4\times10^4}{100^2}\right) \approx 6$ dB

b. $0 = 10\log\left(\frac{4\times10^4}{r^2}\right) \Rightarrow 0 = \log\left(\frac{4\times10^4}{r^2}\right) \Rightarrow$

$10^0 = 1 = \frac{4\times10^4}{r^2} \Rightarrow r^2 = 40{,}000 \Rightarrow$

$r = 200$ ft

The decibel level drops to 0 when a listener is 200 ft away.

c. $L = 10\log\left(\frac{4\times10^4}{r^2}\right)$

$= 10\left(\log 4 + \log\left(10^4\right) - \log\left(r^2\right)\right)$

$= 10\left(\left(4 + \log 4\right) - 2\log r\right)$

$= 40 + 10\log 4 - 20\log r$

$a = 40 + 10\log 4 = 10\left(4 + \log 4\right)$

$b = -20$

d. $L = 10\log\left(\frac{4\times10^4}{r^2}\right) = 10\log\left(\frac{2\times10^2}{r}\right)^2 \Rightarrow$

$L = 20\log\left(\frac{200}{r}\right) \Rightarrow \frac{L}{20} = \log\left(\frac{200}{r}\right) \Rightarrow$

$10^{L/20} = \frac{200}{r} \Rightarrow r = \frac{200}{10^{L/20}}$

76. $I = \frac{K}{r^2}$

a. $I_1 = \frac{K}{r_1^2};\ I_2 = \frac{K}{r_2^2}$

$L_1 = 10\log\left(\frac{I_1}{I_0}\right) = 10\log\left(\frac{\frac{K}{r_1^2}}{I_0}\right)$

$L_2 = 10\log\left(\frac{I_2}{I_0}\right) = 10\log\left(\frac{\frac{K}{r_2^2}}{I_0}\right)$

$L_1 - L_2 = 10\log\left(\frac{\frac{K}{r_1^2}}{I_0}\right) - 10\log\left(\frac{\frac{K}{r_2^2}}{I_0}\right)$

$= 10\left(\log K - \log r_1^2 - \log I_0\right)$

$\quad - 10\left(\log K - \log r_2^2 - \log I_0\right)$

$= 10\left(\log K - 2\log r_1 - \log I_0\right)$

$\quad - 10\left(\log K - 2\log r_2 - \log I_0\right)$

$= 10\log K - 20\log r_1 - 10\log I_0$

$\quad - 10\log K + 20\log r_2 + 10\log I_0$

$= 20\log r_2 - 20\log r_1$

$= 20\left(\log r_2 - \log r_1\right) = 20\log\left(\frac{r_2}{r_1}\right)$

b. Using the result from part (a), we have

$140 - L_2 = 20\log\left(\frac{100}{30}\right) \Rightarrow$

$L_2 = 140 - 20\log\left(\frac{10}{3}\right) \approx 129.5$ dB

$140 - L_2 = 20\log\left(\frac{300}{30}\right) \Rightarrow$

$L_2 = 140 - 20\log\left(10\right) = 120$ dB

At 100 m, the jet sound will have a decibel level of about 129.5. At 300 m, the jet sound will have a decibel level of 120.

c. $L_1 - L_2 = 20\log\left(\frac{r_2}{r_1}\right) \Rightarrow$

$\frac{L_1 - L_2}{20} = \log\left(\frac{r_2}{r_1}\right) \Rightarrow \frac{r_2}{r_1} = 10^{(L_1 - L_2)/20}$

77. The energy released by the earthquake is given by

$E = \left(2.5\times10^4\right)\times10^{1.5(8)} = 2.5\times10^{16}$ joules.

$\frac{E_{\text{earthquake}}}{E_{\text{bomb}}} = \frac{2.5\times10^{16}}{5\times10^{15}} = .5\times10 = 5$

The earthquake released five times as much energy as the nuclear bomb did.

78. $40 = 1200\log_2 \frac{f}{f_0} \Rightarrow \frac{1}{30} = \log_2 \frac{f}{f_0} \Rightarrow$

$2^{1/30} = \frac{f}{f_0} \Rightarrow 2^{-1/30} = \frac{f_0}{f}$

By transposing f and f_0, Adrien computed the following:

$P(f) = 1200\log_2 \frac{f_0}{f} = 1200\log_2\left(2^{-1/30}\right)$

$= 1200\left(-\frac{1}{30}\right) = -40$ cents

79. $10 = 1200\log_2 \frac{f}{880} \Rightarrow \frac{1}{120} = \log_2 \frac{f}{880} \Rightarrow$

$2^{1/120} = \frac{f}{880} \Rightarrow f = 880 \times 2^{1/120} \approx 885$

$-10 = 1200\log_2 \frac{f}{880} \Rightarrow -\frac{1}{120} = \log_2 \frac{f}{880} \Rightarrow$

$2^{-1/120} = \frac{f}{880} \Rightarrow f = 880 \times 2^{-1/120} \approx 875$

The frequency range is [875, 885].

80. $m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$\frac{m_2 - m_1}{2.5} = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$0.4(m_2 - m_1) = \log\left(\frac{b_1}{b_2}\right) \Rightarrow \frac{b_1}{b_2} = 10^{0.4(m_2 - m_1)}$

81. Let m_1 and b_1 represent the magnitude and brightness of the unknown star and let m_2 and b_2 represent the magnitude and brightness of the given star.

$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right);\ b_2 = 176b_1$

$m_2 - 3.42 = 2.5\log\left(\frac{b_1}{176b_1}\right) \Rightarrow$

$m_2 - 3.42 = 2.5\log\left(\frac{1}{176}\right) \Rightarrow$

$m_2 = 2.5\log\left(\frac{1}{176}\right) + 3.42 \approx -2.2$

The magnitude of the unknown star is about −2.2.

82. a. $m - M = 5\log\frac{D}{10} \Rightarrow \frac{m-M}{5} = \log\frac{D}{10} \Rightarrow$

$\frac{D}{10} = 10^{(m-M)/5} \Rightarrow$

$D = 10 \times 10^{(m-M)/5} = 10^{1+(m-M)/5}$

b. Using the result from part (a), we have

$D = 10^{1+(0-4.39)/5} \approx 1.32$

Alpha Centauri is about 1.32 parsecs from Earth.

c. $0 - 4.39 = 2.5\log\frac{L}{b} \Rightarrow -\frac{4.39}{2.5} = \log\frac{L}{b} \Rightarrow$

$\frac{L}{b} = 10^{-4.39/2.5} \approx 0.0175 \Rightarrow L \approx 0.0175b$ or

$\frac{b}{L} = 10^{4.39/2.5} \approx 57.0 \Rightarrow b \approx 57.0L$

4.5 Maintaining Skills

83. $3x - 2y = 12 \Rightarrow -2y = -3x + 12 \Rightarrow$

$y = \frac{3}{2}x - 6$

The slope is $\frac{3}{2}$ and the y-intercept is −6.

84. $3x + 5y = 15 \Rightarrow 5y = -3x + 15 \Rightarrow y = -\frac{3}{5}x + 3$

The slope is $-\frac{3}{5}$ and the y-intercept is 3.

85. $e^{1.3}x + e^{2.5}y = e^{4.7} \Rightarrow e^{2.5}y = -e^{1.3}x + e^{4.7} \Rightarrow$

$y = -\frac{e^{1.3}}{e^{2.5}}x + \frac{e^{4.7}}{e^{2.5}} = -e^{-1.2}x + e^{2.2}$

The slope is $-e^{-1.2}$ and the y-intercept is $e^{2.2}$.

86. $x\log 2 - y\log 4 = \log 8 \Rightarrow$

$-y\log 4 = -x\log 2 + \log 8 \Rightarrow$

$y = \frac{\log 2}{\log 4}x - \frac{\log 8}{\log 4} = \frac{\log 2}{\log 2^2}x - \frac{\log 2^3}{\log 2^2} \Rightarrow$

$y = \frac{\log 2}{2\log 2}x - \frac{3\log 2}{2\log 2} = \frac{1}{2}x - \frac{3}{2}$

The slope is $\frac{1}{2}$ and the y-intercept is $-\frac{3}{2}$.

For exercises 87–94, recall that the standard form for the equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the center of the circle and r is the radius. You may need to complete the square in order to put the equation in standard form.

87. $x^2 + (y-2)^2 = 9$

center: (0, 2); radius: 3

88. $(x+1)^2 + y^2 = 5$

center: (−1, 0); radius: $\sqrt{5}$

89. $(x-3)^2+(y+1)^2=7$
center: (3, −1); radius: $\sqrt{7}$

90. $(x+2)^2+(y-3)^2=16$
center: (−2, 3); radius: 4

91. $x^2+y^2-2x-3=0 \Rightarrow x^2-2x+y^2=3 \Rightarrow$
$(x^2-2x+1)+y^2=3+1 \Rightarrow$
$(x-1)^2+y^2=4$
center: (1, 0); radius: 2

92. $x^2+y^2+4y=0 \Rightarrow$
$x^2+(y^2+4y+4)=0+4 \Rightarrow$
$x^2+(y+2)^2=4$
center: (0, −2); radius: 2

93.
$$\begin{aligned} x^2+y^2-2x+4y-4&=0\\ x^2-2x+y^2+4y&=4\\ (x^2-2x+1)+(y^2+4y+4)&=4+1+4\\ (x-1)^2+(y+2)^2&=9 \end{aligned}$$
center: (1, −2); radius: 3

94.
$$\begin{aligned} x^2+y^2+4x-6y-3&=0\\ x^2+4x+y^2-6y&=3\\ (x^2+4x+4)+(y^2-6y+9)&=3+4+9\\ (x+2)^2+(y-3)^2&=16 \end{aligned}$$
center: (−2, 3); radius: 4

Chapter 4 Review Exercises

Basic Concepts and Skills

1. False. $f(x)=a^x$ is an exponential function if $a>0$ and $a \neq 1$.

2. True

3. False. The domain of $f(x)=\log(2-x)$ is $(-\infty, 2)$.

4. True

5. True

6. True

7. False. $\ln M + \ln N = \ln(MN)$

8. True

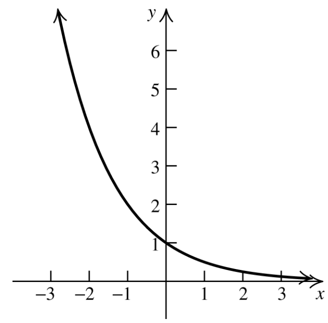
9. True

10. True

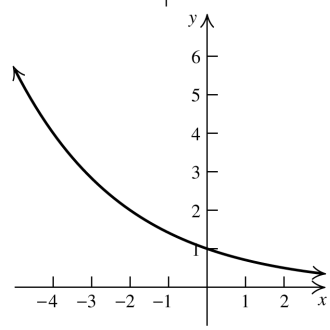
11. h **12.** c **13.** f **14.** e

15. d **16.** b **17.** a **18.** g

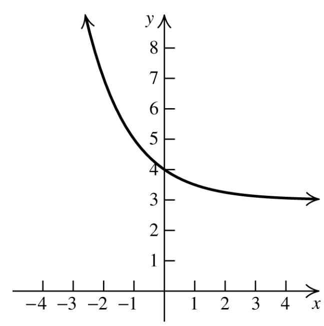
19. Domain: $(-\infty, \infty)$
range: $(0, \infty)$
asymptote: $y=0$



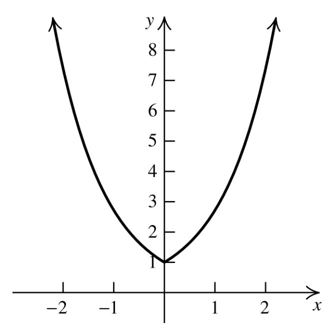
20. Domain: $(-\infty, \infty)$
range: $(0, \infty)$
asymptote: $y=0$



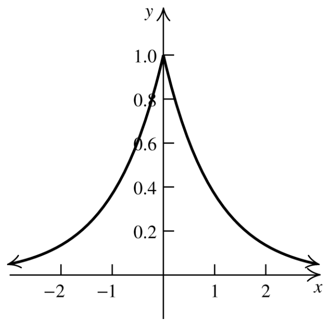
21. Domain: $(-\infty, \infty)$
range: $(3, \infty)$
asymptote: $y=3$



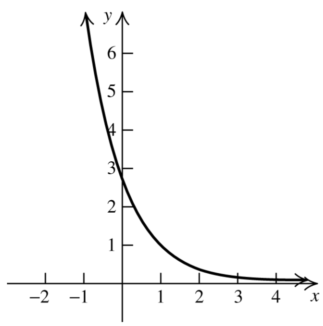
22. Domain: $(-\infty, \infty)$
range: $[1, \infty)$
no asymptote



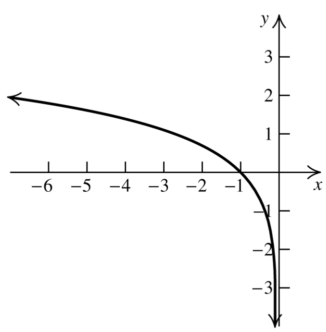
23. Domain: $(-\infty, \infty)$
range: $(0, 1]$
asymptote: $y = 0$



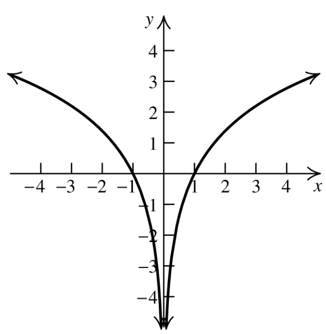
24. Domain: $(-\infty, \infty)$
range: $(0, \infty)$
asymptote: $y = 0$



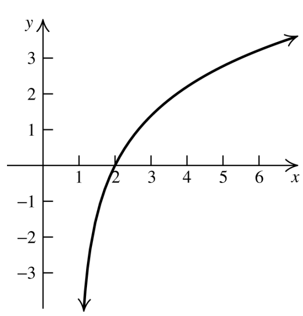
25. Domain: $(-\infty, 0)$
range: $(-\infty, \infty)$
asymptote: $x = 0$



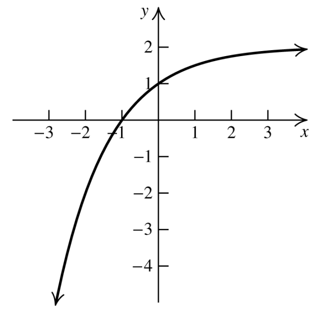
26. Domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$
asymptote: $x = 0$



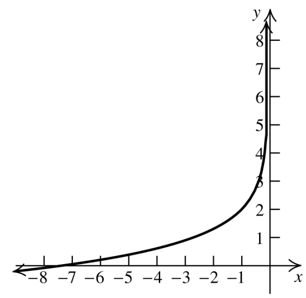
27. Domain: $(1, \infty)$
range: $(-\infty, \infty)$
asymptote: $x = 1$



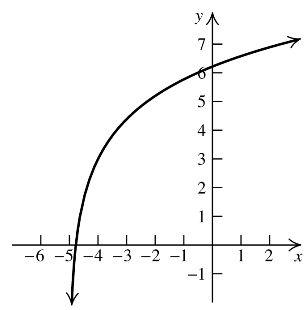
28. Domain: $(-\infty, \infty)$
range: $(-\infty, 2)$
asymptote: $y = 2$



29. Domain: $(-\infty, 0)$
range: $(-\infty, \infty)$
asymptote: $x = 0$



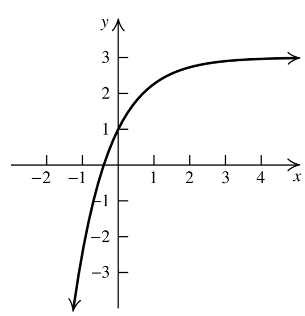
30. Domain: $(-5, \infty)$
range: $(-\infty, \infty)$
asymptote: $x = -5$



31. a. y-intercept: $y = 3 - 2e^0 = 1$

x-intercept: $0 = 3 - 2e^{-x} \Rightarrow 2e^{-x} = 3 \Rightarrow$
$e^{-x} = \dfrac{3}{2} \Rightarrow e^x = \dfrac{2}{3} \Rightarrow x = \ln\left(\dfrac{2}{3}\right)$

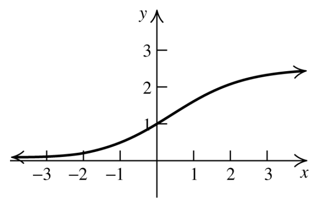
b. As $x \to \infty$, $f(x) \to 3$.
As $x \to -\infty$, $f(x) \to -\infty$.



32. a. y-intercept: $y = \dfrac{5}{2 + 3e^0} = 1$

x-intercept: $0 = \dfrac{5}{2 + 3e^x} \Rightarrow 0 = 5 \Rightarrow$ there is no x-intercept.

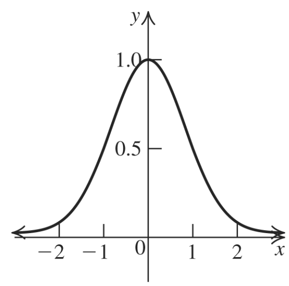
b. As $x \to \infty,\ f(x) \to \frac{5}{2}.$
As $x \to -\infty,\ f(x) \to 0.$



33. a. y-intercept: $y = e^{0^2} = 1$

x-intercept: $0 = e^{-x^2} \Rightarrow \ln 0 = -x^2 \Rightarrow$ there is no x-intercept.

b. As $x \to \infty,\ f(x) \to 0.$
As $x \to -\infty,\ f(x) \to 0.$



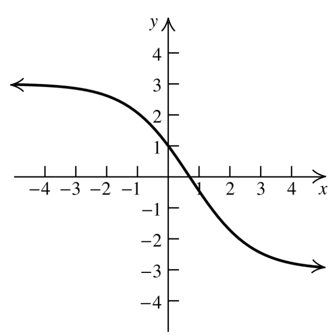
34. a. y-intercept: $y = 3 - \frac{6}{1+2e^0} = 1$

x-intercept: $0 = 3 - \frac{6}{1+2e^{-x}} \Rightarrow$

$3 = \frac{6}{1+2e^{-x}} \Rightarrow 3 + 6e^{-x} = 6 \Rightarrow$

$6e^{-x} = 3 \Rightarrow e^{-x} = \frac{1}{2} \Rightarrow e^x = 2 \Rightarrow x = \ln 2$

b. As $x \to \infty,\ f(x) \to -3.$
As $x \to -\infty,\ f(x) \to 3.$



35. $10 = a(2^{0k}) \Rightarrow a = 10$
$640 = 10(2^{3k}) \Rightarrow 64 = 8^k \Rightarrow k = 2$
$f(2) = 10(2^{2(2)}) = 160$

36. $10 = 50 - a(5^{0k}) \Rightarrow 10 = 50 - a \Rightarrow a = 40$
$0 = 50 - 40(5^{2k}) \Rightarrow \frac{5}{4} = 5^{2k} \Rightarrow$
$\ln\left(\frac{5}{4}\right) = 2k \ln 5 \Rightarrow k = \frac{\ln\left(\frac{5}{4}\right)}{2\ln 5} \approx 0.06932$
$f(1) = 50 - 40(5^{0.06932(1)}) \approx 5.28$

37. $1 = \frac{4}{1+ae^{-k(0)}} = \frac{4}{1+a} \Rightarrow 1 + a = 4 \Rightarrow a = 3$
$\frac{1}{2} = \frac{4}{1+3e^{-3k}} \Rightarrow 1 + 3e^{-3k} = 8 \Rightarrow e^{-3k} = \frac{7}{3} \Rightarrow$
$-3k = \ln\left(\frac{7}{3}\right) \Rightarrow k \approx -0.2824$
$f(4) = \frac{4}{1+3e^{-(-0.2824)(4)}} \approx 0.38898$

38. $5 = 6 - \frac{3}{1+ae^{-k(0)}} \Rightarrow -1 = -\frac{3}{1+a} \Rightarrow$
$-1 - a = -3 \Rightarrow a = 2$
$4 = 6 - \frac{3}{1+2e^{-4k}} \Rightarrow -2 = -\frac{3}{1+2e^{-4k}} \Rightarrow$
$2 + 4e^{-4k} = 3 \Rightarrow e^{-4k} = \frac{1}{4} \Rightarrow$
$-4k = \ln\left(\frac{1}{4}\right) \Rightarrow k \approx 0.3466$
$f(10) = 6 - \frac{3}{1+2e^{-0.3466(10)}} \approx 3.1764$

39. The graph passes through (0, 3), so we have $3 = ca^0 \Rightarrow c = 3.$ The graph passes through (2, 12), so we have $12 = 3a^2 \Rightarrow 4 = a^2 \Rightarrow$ $a = 2.$ The equation is $f(x) = 3 \cdot 2^x.$

40. The graph passes through (0, 4), so we have $4 = ca^0 \Rightarrow c = 4.$ The graph passes through (4, 1), so we have $1 = 4a^4 \Rightarrow \frac{1}{4} = a^4 \Rightarrow$
$a = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}.$

The equation is $f(x) = 4\left(\frac{\sqrt{2}}{2}\right)^x.$

41. The graph has been shifted right one unit, so the equation is of the form $y = \log_a(x-1).$ The graph passes through the point (6, 1), so we have
$1 = \log_a(6-1) \Rightarrow 1 = \log_a 5 \Rightarrow a = 5.$
Thus, the equation is $y = \log_5(x-1).$

42. The graph has been shifted right two units and reflected about the y-axis, so the equation is of the form $y = \log_a(2-x)$. The graph passes through the point (–3, 1), so we have $1 = \log_a(2-(-3)) \Rightarrow 1 = \log_a 5 \Rightarrow a = 5.$ Thus, the equation is $y = \log_5(2-x)$.

43. a. $y = -\left(2^{x-1}+3\right) = -2^{x-1} - 3$

b. $y = -2^{x-1} + 3$

44. a. $y = \frac{1}{3}\ln\left(\frac{x}{2}+1\right)$

b. $y = \ln\left[-(3x-1)\right] = \ln(1-3x)$

45. $\ln(xy^2z^3) = \ln x + 2\ln y + 3\ln z$

46. $\log\left(x^3\sqrt{y-1}\right) = 3\log x + \frac{1}{2}\log(y-1)$

47. $\ln\left[\frac{x\sqrt{x^2+1}}{(x^2+3)^2}\right] = \ln x + \frac{1}{2}\ln(x^2+1) - 2\ln(x^2+3)$

48. $\ln\sqrt{\frac{x^3+5}{x^3-7}} = \ln\left(\frac{\sqrt{x^3+5}}{\sqrt{x^3-7}}\right)$
$= \frac{1}{2}\left(\ln(x^3+5) - \ln(x^3-7)\right)$

49. $\ln y = \ln x + \ln 3 = \ln(3x) \Rightarrow y = 3x$

50. $\ln y = \ln C + kx \Rightarrow \ln y = \ln C + kx\ln e \Rightarrow$
$\ln y = \ln C + \ln e^{kx} \Rightarrow \ln y = \ln(Ce^{kx}) \Rightarrow$
$y = Ce^{kx}$

51. $\ln y = \ln(x-3) - \ln(y+2) \Rightarrow$
$\ln y + \ln(y+2) = \ln(x-3) \Rightarrow$
$\ln(y(y+2)) = \ln(x-3) \Rightarrow$
$\ln(y^2+2y) = \ln(x-3) \Rightarrow y^2+2y = x-3 \Rightarrow$
$y^2+2y+1 = x-3+1 \Rightarrow (y+1)^2 = x-2 \Rightarrow$
$y+1 = \pm\sqrt{x-2} \Rightarrow y = -1 \pm \sqrt{x-2}$
We disregard the negative solution, so $y = -1+\sqrt{x-2}$.

52. $\ln y = \ln x - \ln(x^2y) - 2\ln y = \ln\left(\frac{\frac{x}{x^2y}}{y^2}\right)$
$= \ln\left(\frac{x}{x^2y^3}\right) \Rightarrow y = \frac{1}{xy^3} \Rightarrow y^4 = \frac{1}{x} \Rightarrow$
$y = \left(\frac{1}{x}\right)^{1/4}$

53. $\ln y = \frac{1}{2}\ln(x-1) + \frac{1}{2}\ln(x+1) - \ln(x^2+1) \Rightarrow$
$\ln y = \ln\left(\frac{\sqrt{x-1}\sqrt{x+1}}{x^2+1}\right) = \ln\left(\frac{\sqrt{x^2-1}}{x^2+1}\right) \Rightarrow$
$y = \frac{\sqrt{x^2-1}}{x^2+1}$

54. $\ln(y-1) = \frac{1}{x} + \ln y = \frac{1}{x}\ln e + \ln y \Rightarrow$
$\ln(y-1) = \ln\left(e^{1/x}y\right) \Rightarrow y - 1 = e^{1/x}y \Rightarrow$
$y - e^{1/x}y = 1 \Rightarrow y\left(1-e^{1/x}\right) = 1 \Rightarrow y = \frac{1}{1-e^{1/x}}$

55. $3^x = 81 \Rightarrow 3^x = 3^4 \Rightarrow x = 4$

56. $5^{x-1} = 625 \Rightarrow 5^{x-1} = 5^4 \Rightarrow x-1 = 4 \Rightarrow x = 5$

57. $2^{x^2+2x} = 16 \Rightarrow 2^{x^2+2x} = 2^4 \Rightarrow x^2+2x = 4 \Rightarrow$
$x^2+2x-4 = 0 \Rightarrow x = \frac{-2\pm\sqrt{4+16}}{2} = -1\pm\sqrt{5}$

58. $3^{x^2-6x+8} = 1 = 3^0 \Rightarrow x^2-6x+8 = 0 \Rightarrow$
$(x-4)(x-2) = 0 \Rightarrow x = 2$ or $x = 4$

59. $3^x = 23 \Rightarrow x\ln 3 = \ln 23 \Rightarrow x = \frac{\ln 23}{\ln 3} \approx 2.854$

60. $2^{x-1} = 5.2 \Rightarrow (x-1)\ln 2 = \ln 5.2 \Rightarrow$
$x-1 = \frac{\ln 5.2}{\ln 2} \Rightarrow x = 1 + \frac{\ln 5.2}{\ln 2} \approx 3.379$

61. $273^x = 19 \Rightarrow x\ln 273 = \ln 19 \Rightarrow$
$x = \frac{\ln 19}{\ln 273} \approx 0.525$

62. $27 = 9^x \cdot 3^{x^2} \Rightarrow 3^3 = 3^{2x}\cdot 3^{x^2} = 3^{x^2+2x} \Rightarrow$
$x^2+2x = 3 \Rightarrow x^2+2x-3 = 0 \Rightarrow$
$(x+3)(x-1) = 0 \Rightarrow x = -3$ or $x = 1$

63. $3^{2x} = 7^x \Rightarrow 2x\ln 3 = x\ln 7 \Rightarrow$
$2x\ln 3 - x\ln 7 = 0 \Rightarrow x\left(2\ln 3 - \ln 7\right) = 0 \Rightarrow$
$x = 0$

64. $2^{x+4} = 3^{x+1} \Rightarrow (x+4)\ln 2 = (x+1)\ln 3 \Rightarrow$
$x\ln 2 + 4\ln 2 = x\ln 3 + \ln 3 \Rightarrow$
$x\ln 2 - x\ln 3 = \ln 3 - 4\ln 2 \Rightarrow$
$x(\ln 2 - \ln 3) = \ln 3 - 4\ln 2 \Rightarrow$
$x = \dfrac{\ln 3 - 4\ln 2}{\ln 2 - \ln 3} \approx 4.129$

65. $1.7^{3x} = 3^{2x-1} \Rightarrow 3x\ln 1.7 = (2x-1)\ln 3 \Rightarrow$
$3x\ln 1.7 = 2x\ln 3 - \ln 3 \Rightarrow$
$3x\ln 1.7 - 2x\ln 3 = -\ln 3 \Rightarrow$
$x(3\ln 1.7 - 2\ln 3) = -\ln 3 \Rightarrow$
$x = \dfrac{-\ln 3}{3\ln 1.7 - 2\ln 3} \approx 1.815$

66.
$$3\left(2^{x+5}\right) = 5\left(7^{2x-3}\right)$$
$$\ln\left[3\left(2^{x+5}\right)\right] = \ln\left[5\left(7^{2x-3}\right)\right]$$
$$\ln 3 + (x+5)\ln 2 = \ln 5 + (2x-3)\ln 7$$
$$\ln 3 + x\ln 2 + 5\ln 2 = \ln 5 + 2x\ln 7 - 3\ln 7$$
$$x\ln 2 - 2x\ln 7 = \ln 5 - 3\ln 7 - \ln 3 - 5\ln 2$$
$$x(\ln 2 - 2\ln 7) = \ln 5 - 3\ln 7 - \ln 3 - 5\ln 2$$
$$x = \frac{\ln 5 - 3\ln 7 - \ln 3 - 5\ln 2}{\ln 2 - 2\ln 7} \approx 2.749$$

67. $\log_3(x+2) - \log_3(x-1) = 1 \Rightarrow$
$\log_3\left(\dfrac{x+2}{x-1}\right) = 1 \Rightarrow \dfrac{x+2}{x-1} = 3 \Rightarrow$
$x + 2 = 3x - 3 \Rightarrow 2x = 5 \Rightarrow x = \dfrac{5}{2}$

68. $\log_3(x+12) - \log_3(x+4) = 2 \Rightarrow$
$\log_3\left(\dfrac{x+12}{x+4}\right) = 2 \Rightarrow \dfrac{x+12}{x+4} = 3^2 \Rightarrow$
$x + 12 = 9x + 36 \Rightarrow -8x = 24 \Rightarrow x = -3$

69. $\log_2(x+2) + \log_2(x+4) = 3 \Rightarrow$
$\log_2[(x+2)(x+4)] = 3 \Rightarrow$
$x^2 + 6x + 8 = 2^3 \Rightarrow x^2 + 6x = 0 \Rightarrow$
$x(x+6) = 0 \Rightarrow x = -6, 0$
Reject -6 since $\log_2(-6+2) = \log_2(-4)$ is not defined. Solution set: $\{0\}$

70. $\log_5(3x+7) + \log_5(x-5) = 2 \Rightarrow$
$\log_5[(3x+7)(x-5)] = 2 \Rightarrow$
$(3x+7)(x-5) = 5^2 = 25 \Rightarrow$
$3x^2 - 8x - 35 = 25 \Rightarrow 3x^2 - 8x - 60 = 0 \Rightarrow$
$(3x+10)(x-6) = 0 \Rightarrow$
$x = -\dfrac{10}{3}$ (reject this) or $x = 6$

71. $\log_5\left(x^2 - 5x + 6\right) - \log_5(x-2) = 1 \Rightarrow$
$\log_5\left(\dfrac{x^2 - 5x + 6}{x-2}\right) = 1 \Rightarrow \dfrac{x^2 - 5x + 6}{x-2} = 5 \Rightarrow$
$x^2 - 5x + 6 = 5x - 10 \Rightarrow x^2 - 10x + 16 = 0 \Rightarrow$
$(x-2)(x-8) = 0 \Rightarrow x = 2, 8$
If $x = 2$, then $\log_5(x-2) = \log_5 0$, which is undefined, so reject $x = 2$. Solution set: $\{8\}$

72. $\log_3\left(x^2 - x - 6\right) - \log_3(x-3) = 1 \Rightarrow$
$\log_3\left(\dfrac{x^2 - x - 6}{x-3}\right) = 1 \Rightarrow \dfrac{x^2 - x - 6}{x-3} = 3 \Rightarrow$
$x^2 - x - 6 = 3x - 9 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow$
$(x-1)(x-3) = 0 \Rightarrow x = 1, 3$
If $x = 1$, then $\log_3(x-3) = \log_3(-2)$, which is undefined, so reject $x = 1$. If $x = 3$, then $\log_3(x-3) = \log_3 0$, which is undefined, so reject $x = 3$. Solution set: $\varnothing$

73. $\log_6(x-2) + \log_6(x+1)$
$= \log_6(x+4) + \log_6(x-3) \Rightarrow$
$\log_6[(x-2)(x+1)] = \log_6[(x+4)(x-3)] \Rightarrow$
$x^2 - x - 2 = x^2 + x - 12 \Rightarrow -2x = -10 \Rightarrow x = 5$

74. $\ln(x-2) - \ln(x+2) = \ln(x-1) - \ln(2x+1) \Rightarrow$
$\ln\left(\dfrac{x-2}{x+2}\right) = \ln\left(\dfrac{x-1}{2x+1}\right) \Rightarrow \dfrac{x-2}{x+2} = \dfrac{x-1}{2x+1} \Rightarrow$
$(x-2)(2x+1) = (x-1)(x+2) \Rightarrow$
$2x^2 - 3x - 2 = x^2 + x - 2 \Rightarrow x^2 - 4x = 0 \Rightarrow$
$x(x-4) = 0 \Rightarrow x = 0$ (reject this), 4
Solution set: $\{4\}$

75. $2\ln 3x = 3\ln x \Rightarrow \ln(3x)^2 = \ln x^3 \Rightarrow$
$9x^2 = x^3 \Rightarrow x = 9$

76. $2\log x = \ln e \Rightarrow 2\log x = 1 \Rightarrow \log x = 1/2 \Rightarrow$
$x = 10^{1/2} = \sqrt{10}$

77. Multipling both sides of $2^x - 8\cdot 2^{-x} - 7 = 0$ by 2^x we have $2^{2x} - 8\cdot 2^{-x+x} - 7\cdot 2^x = 0 \Rightarrow$ $2^{2x} - 7\cdot 2^x - 8 = 0$. Letting $u = 2^x$, we have $u^2 - 7u - 8 = 0 \Rightarrow (u-8)(u+1) = 0 \Rightarrow u = 8$ or $u = -1$. (Reject this solution) $2^x = 8 \Rightarrow x = 3$

78. Multipling both sides of $3^x - 24\cdot 3^{-x} = 10$ by 3^x we have $3^{2x} - 24\cdot 3^{-x+x} - 10\cdot 3^x = 0 \Rightarrow$ $3^{2x} - 10\cdot 3^x - 24 = 0$. Letting $u = 3^x$, we have $u^2 - 10u - 24 = 0 \Rightarrow (u-12)(u+2) = 0 \Rightarrow$ $u = 12$ or $u = -2$. (Reject this solution) $3^x = 12 \Rightarrow x\ln 3 = \ln 12 \Rightarrow x = \frac{\ln 12}{\ln 3} \approx 2.262$

79. $3\left(0.2^x\right) + 5 \le 20 \Rightarrow 3\left(0.2^x\right) \le 15 \Rightarrow$ $\left(0.2^x\right) \le 5 \Rightarrow x\ln 0.2 \le \ln 5 \Rightarrow x \ge \frac{\ln 5}{\ln 0.2} \Rightarrow$ $x \ge -1$

80. $-4(0.2)^x + 15 < 6 \Rightarrow -4(0.2)^x < -9 \Rightarrow$ $0.2^x > \frac{9}{4} \Rightarrow x\ln 0.2 > \ln\left(\frac{9}{4}\right) \Rightarrow$ $x < \frac{\ln(9/4)}{\ln 0.2} \approx -0.504$

81. Note that the domain of $\log(2x+7)$ is $\left(-\frac{7}{2}, \infty\right)$ since $2x + 7$ must be greater than 0. $\log(2x+7) < 2 \Rightarrow 2x+7 < 10^2 \Rightarrow 2x < 93 \Rightarrow$ $x < \frac{93}{2}$

Solution set: $\left(-\frac{7}{2}, \frac{93}{2}\right)$

82. Note that the domain of $\ln(3x+5)$ is $\left(-\frac{5}{3}, \infty\right)$ since $3x + 5$ must be greater than 0. $\ln(3x+5) \le 1 \Rightarrow 3x+5 \le e \Rightarrow 3x \le e-5 \Rightarrow$ $x \le \frac{e-5}{3}$

Solution set: $\left\{-\frac{5}{3}, \frac{e-5}{3}\right\}$

Applying the Concepts

83. $2P = P(1+0.0625)^t \Rightarrow 2 = 1.0625^t \Rightarrow$ $\ln 2 = t\ln 1.0625 \Rightarrow t = \frac{\ln 2}{\ln 1.0625} \approx 11.4$

It will take about 11.4 years to double the investment.

84. $3P = Pe^t \Rightarrow 3 = e^t \Rightarrow t = \ln 3 \approx 1.10$

It will take about 1.10 years or about 1 year 1 month to triple the investment.

85. Find k: $\frac{1}{2} = e^{20k} \Rightarrow \ln\left(\frac{1}{2}\right) = 20k \Rightarrow$

$k = \frac{\ln(1/2)}{20}$

$0.25 = e^{\frac{\ln(1/2)}{20}t} \Rightarrow \ln 0.25 = \frac{\ln(1/2)}{20}t \Rightarrow$

$t = \frac{\ln 0.25}{\frac{\ln(1/2)}{20}} = 40$

It will take 40 hours to decay to 25% of its original value.

86. $Q = Q_0 e^{-5\cdot 10^{-3}t} \Rightarrow \frac{Q}{Q_0} = \frac{1}{2} = e^{-5\cdot 10^{-3}t} \Rightarrow$

$\ln\left(\frac{1}{2}\right) = -5\cdot 10^{-3}t \Rightarrow t = \frac{\ln(1/2)}{-5\cdot 10^{-3}} \approx 138.6$

The half-life of plutonium-210 is about 138.6 days.

87. At 5% compounded yearly,

$A = 7000\left(1 + \frac{0.05}{1}\right)^{7(1)} = 9849.70$.

At 4.75% compounded monthly,

$A = 7000\left(1 + \frac{0.0475}{12}\right)^{7(12)} = 9754.74$.

The 5% investment provides the greater return.

88. $20{,}000 = 8000e^{0.07t} \Rightarrow \frac{5}{2} = e^{0.07t} \Rightarrow$

$\ln\left(\frac{5}{2}\right) = 0.07t \Rightarrow t = \frac{\ln\left(\frac{5}{2}\right)}{0.07} \approx 13.09$ years

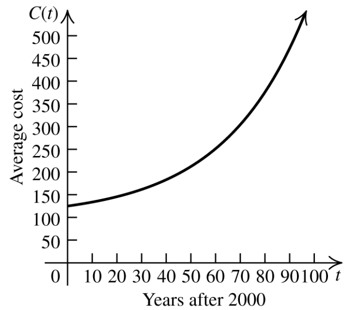
89. a. $t = 10; P(10) = 33e^{0.003(10)} \approx 34$ million

b. $60 = 33e^{0.003t} \Rightarrow \frac{60}{33} = e^{0.003t} \Rightarrow$

$\ln\left(\frac{60}{33}\right) = 0.003t \Rightarrow t = \frac{\ln\left(\frac{60}{33}\right)}{0.003} \approx 199.3$ yr

The population will be 60 million sometime during the year 2206 (after 199.3 years).

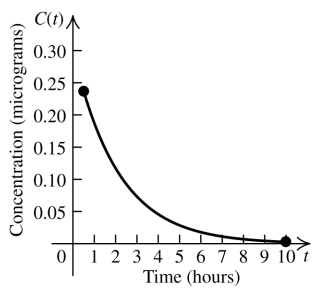
90. a.



b. $C(10) = 100 + 25e^{0.03(10)}$
$\approx \$133.75$ thousand

c. $250 = 100 + 25e^{0.03t} \Rightarrow 6 = e^{0.03t} \Rightarrow$
$\ln 6 = 0.03t \Rightarrow t = \dfrac{\ln 6}{0.03} \approx 59.73$ years
The average cost of a house will be \$250,000 after 59.73 years, in 2060.

91. a.



b. $0.029 = 0.3e^{-0.47t} \Rightarrow \dfrac{0.029}{0.3} = e^{-0.47t} \Rightarrow$

$$\ln\left(\frac{0.029}{0.3}\right) = -0.47t \Rightarrow t = \frac{\ln\left(\frac{0.029}{0.3}\right)}{-0.47} \Rightarrow$$

$t \approx 4.97 \approx 5$ hours

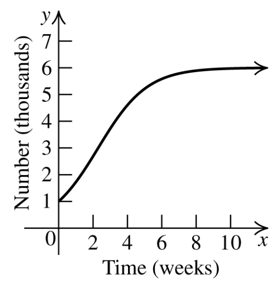
92. a. $x = \dfrac{1}{152}\log\left(\dfrac{I_0}{0.1I_0}\right) = \dfrac{1}{152}\log 10 = \dfrac{1}{152}$ cm

b. $x = \dfrac{1}{152}\log\left(\dfrac{I_0}{I_0/40}\right) = \dfrac{1}{152}\log 40$
≈ 0.01054 cm

c. Note that 0.2 mm = 0.02 cm
$0.02 = \dfrac{1}{152}\log\left(\dfrac{I_0}{I}\right) \Rightarrow 3.04 = \log\left(\dfrac{I_0}{I}\right) \Rightarrow$
$\dfrac{I_0}{I} = 10^{3.04} \approx 1096$ times

93. First find a and k:
$f(0) = 212 = 75 + ae^{-k(0)} \Rightarrow 137 = a$
$f(1) = 200 = 75 + 137e^{-k(1)} \Rightarrow \dfrac{125}{137} = e^{-k} \Rightarrow$
$\ln(125/137) = -k \Rightarrow k \approx 0.0917$
$150 = 75 + 137e^{-0.0917t} \Rightarrow \dfrac{75}{137} = e^{-0.0917t} \Rightarrow$
$t = \dfrac{\ln(75/137)}{-0.0917} \approx 6.57$ minutes

94. a.



b. $P(0) = \dfrac{6}{1 + 5e^{-0.7(0)}} = \dfrac{6}{1+5} = 1$ thousand
$= 1000$ people

c. $P(4) = \dfrac{6}{1 + 5e^{-0.7(4)}} \approx 4.601$ thousand
$= 4601$ people

d. The numerator is the upper limit, so 6000 people will fall ill.

95. a. $D(0) = 5e^{0.08(0)} \approx 5$ thousand = 5000 people per square mile

b. $D(5) = 5e^{0.08(5)} \approx 7.459$ thousand = 7459 people per square mile

c. $15 = 5e^{0.08t} \Rightarrow 3 = e^{0.08t} \Rightarrow$
$t = \dfrac{\ln 3}{0.08} \approx 13.73$ miles

96. Using the continuous compounding formula, we have
$280e^{0.01t} = 200e^{0.03t} \Rightarrow \dfrac{280}{200} = \dfrac{e^{0.03t}}{e^{0.01t}} \Rightarrow$
$\dfrac{7}{5} = e^{0.02t} \Rightarrow t = \dfrac{\ln(7/5)}{0.02} \approx 16.82$ years.
There will be an average of one vehicle per person in the year 2017.

97. $I = I_0e^{-0.73(2)} \approx 0.2322I_0$

98. $I = I_0 e^{-0.003(10)} \Rightarrow \frac{I}{I_0} = e^{-0.03} \approx 0.97 \approx 97\%.$

So, approximately 3% of the signal was lost at 10 miles.

$I = I_0 e^{-0.003(20)} \Rightarrow \frac{I}{I_0} = e^{-0.06} \approx 0.94 \approx 94\%.$

So, approximately 6% of the signal was lost at 20 miles.

99. a. $s(470,000) = 0.04 + 0.86\ln(470,000) \approx 11.27$ feet per second

b. $s(450) = 0.04 + 0.86\ln(450) \approx 5.29$ feet per second

c. $4.6 = 0.04 + 0.86\ln p \Rightarrow \frac{4.56}{0.86} = \ln p \Rightarrow$
$p = e^{4.56/0.86} \approx 200.8 \approx 201$ people

100. a. $A(0) = 0.26 = A_0 e^{-k(0)} \Rightarrow A_0 = 0.26$

b. $A(0.5) = 0.18 = 0.26e^{-k(0.5)} \Rightarrow$
$\frac{18}{26} = e^{-k(0.5)} \Rightarrow k = -\frac{\ln\left(\frac{18}{26}\right)}{0.5} \approx 0.74$

c. $0.08 = 0.26e^{-0.74t} \Rightarrow \frac{0.08}{0.26} = e^{-0.74t} \Rightarrow$
$t = \frac{\ln\left(\frac{0.08}{0.26}\right)}{-0.74} \approx 1.59$ hours or 1 hour 36 min

101. a. $m(0) = \frac{6}{1+5e^{-0.7(0)}} = \frac{6}{1+5} = 1$ gram

b. The mass approaches 6 grams

c. $5 = \frac{6}{1+5e^{-0.7t}} \Rightarrow 5 + 25e^{-0.7t} = 6 \Rightarrow$
$e^{-0.7t} = \frac{1}{25} \Rightarrow t = \frac{\ln\left(\frac{1}{25}\right)}{-0.7} \approx 4.6$ days

102. a. $20 = 60\left(1 - e^{-k(1)}\right) = 60 - 60e^{-k} \Rightarrow$
$\frac{2}{3} = e^{-k} \Rightarrow k = -\ln\left(\frac{2}{3}\right) \approx 0.41$

b. $n(10) = 60\left(1 - e^{-0.41(10)}\right) \approx 59$ units

c. $40 = 60\left(1 - e^{-0.41t}\right) = 60 - 60e^{-0.41t} \Rightarrow$
$\frac{1}{3} = e^{-0.41t} \Rightarrow t = \frac{\ln\left(\frac{1}{3}\right)}{-0.41} \approx 2.7 = 3$ days

103. $P(f) = 1200\log_2 \frac{f}{f_0}$

$P(f) = 1200\log_2\left(\frac{1.2f_0}{f_0}\right) = 1200\log_2(1.2)$
$= 1200\left(\frac{\log 1.2}{\log 2}\right) \approx 315.6 \approx 316$

The difference, 316 − 300 = 16 cents, is noticeable.

104. Let the magnitude and brightness of star *A* be m_1 and b_1, and let the magnitude of star *B* be $m_2 = m_1 - 3$ and b_2.

$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$

$(m_1 - 3) - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$-3 = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow -1.2 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$

$\frac{b_1}{b_2} = 10^{-1.2} = \frac{1}{10^{1.2}} \approx \frac{1}{15.8} \Rightarrow 15.8b_1 = b_2$

Star *B* is about 15.8 times brighter than star *A*.

105. $\text{pH}_{\text{very acid rain}} = 2.4 = -\log\left[\text{H}^+\right] \Rightarrow$
$\left[\text{H}^+\right] = 10^{-2.4} \Rightarrow$
$\left[\text{H}^+\right] = 10^{0.6} \times 10^{-3} \approx 4.0 \times 10^{-3}$

The average concentration of hydrogen ions in the very acid rain is 4.0×10^{-3} moles per liter.

$\text{pH}_{\text{acid rain}} = 5.6 = -\log\left[\text{H}^+\right] \Rightarrow$
$\left[\text{H}^+\right] = 10^{-5.6} = 10^{0.4} \times 10^{-6} \approx 2.5 \times 10^{-6}$

$\frac{\left[\text{H}^+\right]_{\text{very acid rain}}}{\left[\text{H}^+\right]_{\text{acid rain}}} = \frac{10^{-2.4}}{10^{-5.6}} = 10^{3.2} = 10^{0.2} \times 10^3$
$\approx 1.585 \times 10^3 = 1585$

The very acid rain is 1585 times more acidic than acid rain.

106. a. Let I_{1997} denote the intensity of the 1997 Iran earthquake and let I_{2010} denote the intensity of the 2010 Chile earthquake. Then,

$7.5 = \log\left(\frac{I_{1997}}{I_0}\right) \Rightarrow \frac{I_{1997}}{I_0} = 10^{7.5} \Rightarrow$
$I_{1997} = 10^{7.5} I_0$

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(*continued*)

$$8.8 = \log\left(\frac{I_{2010}}{I_0}\right) \Rightarrow \frac{I_{2010}}{I_0} = 10^{8.8} \Rightarrow$$
$$I_{2010} = 10^{8.8} I_0$$
$$\frac{I_{2010}}{I_{1997}} = \frac{10^{8.8} I_0}{10^{7.5} I_0} = 10^{1.3} \approx 20.2 \Rightarrow$$
$$I_{2010} \approx 20.0 I_{1997}$$

The intensity of the 2010 Iran earthquake was about 20.0 times that of the 1997 Chile earthquake.

b. Let M_{2010} and E_{2010} represent the magnitude and energy of the 2010 Chile earthquake. Let M_{1997} and E_{1997} represent the magnitude and energy of the 1977 Iran earthquake.

$$\frac{E_{2010}}{E_{1997}} = 10^{1.5(M_{2010} - M_{1997})} = 10^{1.5(8.8-7.5)}$$
$$= 10^{1.95} \approx 89.1$$

The energy released by the 2010 Chile earthquake was about 89.1 times that released by the 1997 Iran earthquake.

107. $$I = I_0 \times 10^{L/10}$$
$$I_{115} = I_0 \times 10^{115/10}$$
$$I_{95} = I_0 \times 10^{95/10}$$
$$\frac{I_{115}}{I_{95}} = \frac{I_0 \times 10^{115/10}}{I_0 \times 10^{95/10}} = 10^{20/10} = 10^2 = 100$$

The 115 dB sound is 100 times as intense as the 95 dB sound.

108. $I_2 = 5000 I_1$

$$L_2 = 10\log\left(\frac{I_2}{I_0}\right) = 10\log\left(\frac{5000 I_1}{I_0}\right)$$
$$= 10\log\left(5000 \times \frac{I_1}{I_0}\right)$$
$$= 10\left(\log 5000 + \log\left(\frac{I_1}{I_0}\right)\right)$$
$$= 10\log 5000 + 10\log\left(\frac{I_1}{I_0}\right) \approx 37.0 + L_1$$

The more intense sound is about 37 dB louder than the less intense sound.

109. Let m_1 represent the magnitude of Sirius and let m_2 represent the magnitude of Saturn.

$$m_2 - m_1 = 2.5\log\left(\frac{b_1}{b_2}\right)$$
$$1.47 - (-1) = 2.5\log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{2.47}{2.5} = 0.988 = \log\left(\frac{b_1}{b_2}\right) \Rightarrow$$
$$\frac{b_1}{b_2} = 10^{0.988} \approx 9.7 \Rightarrow b_1 = 9.7 b_2$$

Sirius is about 9.7 times brighter than Saturn.

110. $$\log\left(5 \times 10^{15}\right) = 4.4 + 1.5M$$
$$M = \frac{\log 5 + 15 - 4.4}{1.5} \approx 7.53$$

An earthquake of magnitude 7.53 releases as energy equivalent to that released by a one megaton nuclear bomb.

Chapter 4 Practice Test A

1. $5^{-x} = 125 \Rightarrow 5^{-x} = 5^3 \Rightarrow x = -3$

2. $\log_2 x = 5 \Rightarrow x = 2^5 = 32$

3. Range: $(-\infty, 1)$; asymptote: $y = 1$

4. $\log_2 \frac{1}{8} \Rightarrow 2^x = \frac{1}{8} = 2^{-3} \Rightarrow x = -3$

5. $\left(\frac{1}{4}\right)^{2-x} = 4 \Rightarrow 4^{x-2} = 4 \Rightarrow x - 2 = 1 \Rightarrow x = 3$

6. $\log 0.001 \Rightarrow 10^x = 0.001 = 10^{-3} \Rightarrow x = -3$

7. $\ln 3 + 5\ln x = \ln 3 + \ln x^5 = \ln\left(3x^5\right)$

8. $2^{x+1} = 5 \Rightarrow (x+1)\ln 2 = \ln 5 \Rightarrow$
$x\ln 2 + \ln 2 = \ln 5 \Rightarrow x\ln 2 = \ln 5 - \ln 2 \Rightarrow$
$x\ln 2 = \ln\left(\frac{5}{2}\right) \Rightarrow x = \frac{\ln(5/2)}{\ln 2}$

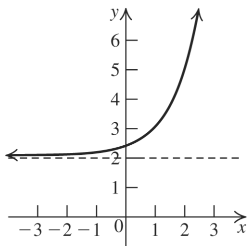
9. $e^{2x} + e^x - 6 = 0 \Rightarrow (e^x + 3)(e^x - 2) = 0 \Rightarrow$
$e^x = -3$ (reject this) or $e^x - 2 = 0 \Rightarrow$
$e^x = 2 \Rightarrow x = \ln 2$

10. $$\ln\frac{2x^3}{(x+1)^5} = \ln(2x^3) - \ln(x+1)^5$$
$$= \ln 2 + 3\ln x - 5\ln(x+1)$$

11. $\ln e^{-5} = -5$

12. $y = \ln(x-1) + 3$

13.



14. Domain: $(-\infty, 0)$

15. $3\ln x + \ln(x^3+2) - \frac{1}{2}\ln(3x^2+2)$
$= \ln x^3 + \ln(x^3+2) - \ln\sqrt{3x^2+2}$
$= \ln\frac{x^3(x^3+2)}{\sqrt{3x^2+2}}$

16. $\log x = \log 6 - \log(x-1) \Rightarrow \log x = \log\frac{6}{x-1} \Rightarrow$
$x = \frac{6}{x-1} \Rightarrow x^2 - x = 6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow$
$(x-3)(x+2) = 0 \Rightarrow x = -2$ (reject this) or $x = 3$

17. $\log_x 9 = 2 \Rightarrow x^2 = 9 \Rightarrow x = 3$

18. $A = 15,000\left(1+\frac{0.07}{4}\right)^{4t} = 15,000(1.0175)^{4t}$

19. $1,500,000 = 15,000e^{0.2t} \Rightarrow 100 = e^{0.2t} \Rightarrow$
$\ln 100 = 0.2t \Rightarrow t = \frac{\ln 100}{0.2} \approx 23.03$
The Hispanic population reached 1.5 million in 1983, 23.03 years after 1960.

20. Let I_{1960} denote the intensity of the 1960 Morocco earthquake and let I_{1972} denote the intensity of the 1972 Nicaragua earthquake. Then,
$5.8 = \log\left(\frac{I_{1960}}{I_0}\right) \Rightarrow \frac{I_{1960}}{I_0} = 10^{5.8} \Rightarrow$
$I_{1960} = 10^{5.8} I_0$
$6.2 = \log\left(\frac{I_{1972}}{I_0}\right) \Rightarrow \frac{I_{1972}}{I_0} = 10^{6.2} \Rightarrow$
$I_{1972} = 10^{6.2} I_0$
$\frac{I_{1972}}{I_{1960}} = \frac{10^{6.2} I_0}{10^{5.8} I_0} = 10^{0.4} \approx 2.5 \Rightarrow I_{1972} \approx 2.5 I_{1960}$
The intensity of the 1972 Nicarague earthquake was about 2.5 times that of the 1960 Morocco earthquake.

Chapter 4 Practice Test B

1. $3^{-x} = 9 \Rightarrow 3^{-x} = 3^2 \Rightarrow x = -2$.
The answer is B.

2. $\log_5 x = 2 \Rightarrow 5^2 = 25 = x$. The answer is B.

3. The answer is B.

4. $\log_4 64 \Rightarrow 4^x = 64 = 4^3 \Rightarrow x = 3$.
The answer is D.

5. $\left(\frac{1}{3}\right)^{1-x} = 3 \Rightarrow 3^{x-1} = 3 \Rightarrow x - 1 = 1 \Rightarrow x = 2$.
The answer is D.

6. $\log 0.01 \Rightarrow 10^x = 10^{-2} \Rightarrow x = -2$.
The answer is B.

7. $\ln 7 + 2\ln x = \ln(7x^2)$. The answer is B.

8. $2^{x^2} = 3^x \Rightarrow x^2 \ln 2 = x\ln 3 \Rightarrow$
$x^2 \ln 2 - x\ln 3 = 0 \Rightarrow x(x\ln 2 - \ln 3) = 0 \Rightarrow$
$x = 0$ or $x\ln 2 - \ln 3 = 0 \Rightarrow x = \frac{\ln 3}{\ln 2}$
The answer is C.

9. $e^{2x} - e^x - 6 = 0 \Rightarrow (e^x - 3)(e^x + 2) = 0 \Rightarrow$
$e^x = -2$ (reject this) or $e^x - 3 = 0 \Rightarrow$
$e^x = 3 \Rightarrow x = \ln 3$.
The answer is D.

10. $\ln\frac{3x^2}{(x+1)^{10}} = \ln(3x^2) - \ln(x+1)^{10}$
$= \ln 3 + 2\ln x - 10\ln(x+1)$.
The answer is D.

11. $\ln e^{3x} = 3x$. The answer is B.

12. The answer is D.

13. The answer is B.

14. The answer is A.

15. $\ln x - 2\ln(x^2+1) + \frac{1}{2}\ln(x^4+1)$
$= \ln x - \ln(x^2+1)^2 + \ln\sqrt{x^4+1}$
$= \ln\frac{x\sqrt{x^4+1}}{(x^2+1)^2}$.
The answer is A.

16. $\log x = \log 12 - \log(x+1) \Rightarrow \log x = \log \dfrac{12}{x+1} \Rightarrow$
$x = \dfrac{12}{x+1} \Rightarrow x^2 + x = 12 \Rightarrow x^2 + x - 12 = 0 \Rightarrow$
$(x+4)(x-3) = 0 \Rightarrow x = -4$ (reject this) or
$x = 3$
The answer is D.

17. $\log_x 16 = 4 \Rightarrow x^4 = 16 = 2^4 \Rightarrow x = 2$
The answer is B.

18. $A = 12,000\left(1+\dfrac{0.105}{12}\right)^{12t} = 12,000(1.00875)^{12t}$
The answer is D.

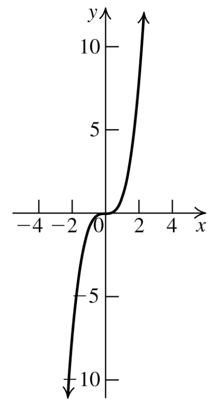
19. $t = 2020 - 2000 = 20$
$P(20) = 10,000 \log_5(20+5) = 10,000 \log_5 25$
$= 20,000$. The answer is B.

20. Let I_{1994} denote the intensity of the 1994 Northridge, CA earthquake and let I_{1988} denote the intensity of the 1988 Armenia earthquake. Then,
$6.7 = \log\left(\dfrac{I_{1994}}{I_0}\right) \Rightarrow \dfrac{I_{1994}}{I_0} = 10^{6.7} \Rightarrow$
$I_{1994} = 10^{6.7} I_0$
$7.0 = \log\left(\dfrac{I_{1988}}{I_0}\right) \Rightarrow \dfrac{I_{1988}}{I_0} = 10^{7.0} \Rightarrow$
$I_{1988} = 10^{7.0} I_0$
$\dfrac{I_{1988}}{I_{1994}} = \dfrac{10^{7.0} I_0}{10^{6.7} I_0} = 10^{0.3} \approx 2.0 \Rightarrow$
$I_{1988} \approx 2.0 I_{1994}$
The intensity of the 1988 Armenia earthquake was about twice that of the 1994 Northridge, CA earthquake. The answer is B.

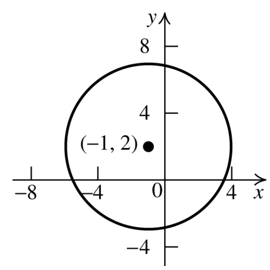
Cumulative Review Exercises (Chapters P–4)

1. $x^2 + (0-1)^2 = 1 \Rightarrow x^2 = 0 \Rightarrow x = 0.$
$0^2 + (y-1)^2 = 1 \Rightarrow y^2 - 2y + 1 = 1 \Rightarrow$
$y^2 - 2y = 0 \Rightarrow y(y-2) = 0 \Rightarrow y = 0$ or $y = 2$.
The x-intercept is 0.
The y-intercepts are 0 and 2.
$x^2 + (y-1)^2 = 1 \Rightarrow (y-1)^2 = 1 - x^2 \Rightarrow$
$y - 1 = \pm\sqrt{1-x^2} \Rightarrow y = 1 \pm \sqrt{1-x^2} = f(x)$
$f(-x) = 1 \pm \sqrt{1-(-x)^2} = 1 \pm \sqrt{1-x^2} = f(x) \Rightarrow$
$f(x)$ is even $\Rightarrow f$ is symmetric about the y-axis.

2.



3. $x^2 + 2x + y^2 - 4y - 20 = 0$
Group the terms and complete the squares.
$\left(x^2 + 2x + 1\right) + \left(y^2 - 4y + 4\right) = 20 + 1 + 4$
$(x+1)^2 + (y-2)^2 = 25$
The center of the circle is (−1, 2) and its radius is 5.



4. $2x + 3y = 17 \Rightarrow y = -\dfrac{2}{3}x + \dfrac{17}{3} \Rightarrow$ the slope of the perpendicular line is
$\dfrac{3}{2}.4 = \dfrac{3}{2}(-2) = b \Rightarrow 7 = b.$
The equation is $y = \dfrac{3}{2}x + 7.$

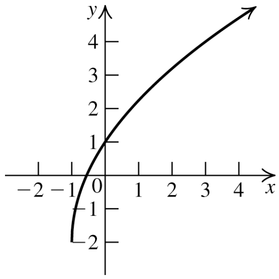
5. Logarithms are ≥ 0, so the domain is $(-\infty, -3) \cup (2, \infty)$.

6. a. $f(-2) = 2(-2) - 3 = -7$

b. $f(0) = 2(0) - 3 = -3$

c. $f(3) = 2(3^2) + 1 = 19$

7. Shift the graph of $f(x) = \sqrt{x}$ one unit left, stretch vertically by a factor of 3, then shift the graph two units down.



8. $(f \circ g)(x) = f\left(-\frac{1}{x}\right) = 3\sqrt{-\frac{1}{x}} \Rightarrow$ the domain is $(-\infty, 0)$.

9. a. The graph of $f(x)$ passes the horizontal line test, so there is an inverse. $y = 2x^3 + 1$ becomes $x = 2y^3 + 1 \Rightarrow \frac{x-1}{2} = y^3 \Rightarrow$

$$y = \sqrt[3]{\frac{x-1}{2}} = f^{-1}(x).$$

b. The graph of $f(x)$ does not pass the horizontal line test, so there is no inverse.

c. The graph of $f(x)$ passes the horizontal line test, so there is an inverse. $y = \ln x$ becomes $x = \ln y \Rightarrow e^x = y = f^{-1}(x)$.

10. a.

$$\begin{array}{r} 2x \\ x^2+1 \overline{\smash{)}\, 2x^3 + 0x^2 + 3x + 1} \\ \underline{2x^3 + 0x^2 + 2x} \\ x + 1 \end{array}$$

The quotient is $2x + \frac{x+1}{x^2+1}$.

b.

$$\begin{array}{r|rrrrr} -3 & 2 & 7 & 0 & -2 & 3 \\ & & -6 & -3 & 9 & -21 \\ \hline & 2 & 1 & -3 & 7 & -18 \end{array}$$

$$\frac{2x^4 + 7x^3 - 2x + 3}{x+3} = 2x^3 + x^2 - 3x + 7 - \frac{18}{x+3}$$

11. a. $(x-1)(x-2)(x+3) = x^3 - 7x + 6$

b. $(x-1)(x+1)(x-i)(x+i) = x^4 - 1$

12. a. $\log_2 5x^3 = \log_2 5 + 3\log_2 x$

b. $\log_a \sqrt[3]{\frac{xy^2}{z}} = \frac{1}{3}\log_a \frac{xy^2}{z}$
$= \frac{1}{3}(\log_a x + 2\log_a y - \log_a z)$

c. $\ln \frac{3\sqrt{x}}{5y} = \ln 3\sqrt{x} - \ln 5y$
$= \ln 3 + \frac{1}{2}\ln x - (\ln 5 + \ln y)$
$= \ln 3 + \frac{1}{2}\ln x - \ln 5 - \ln y$

13. a. $\frac{1}{2}(\log x + \log y) = \frac{1}{2}\log(xy) = \log\sqrt{xy}$

b. $3\ln x - 2\ln y = \ln x^3 - \ln y^2 = \ln \frac{x^3}{y^2}$

14. a. $\log_9 17 = \frac{\log 17}{\log 9} \approx 1.289$

b. $\log_3 25 = \frac{\log 25}{\log 3} \approx 2.930$

c. $\log_{1/2} 0.3 = \frac{\log 0.3}{\log(1/2)} \approx 1.737$

15. a. $\log_6(x+3) + \log_6(x-2) = 1 \Rightarrow$
$\log_6((x+3)(x-2)) = 1 \Rightarrow$
$(x+3)(x-2) = 6 \Rightarrow x^2 + x - 6 = 6 \Rightarrow$
$x^2 + x - 12 = 0 \Rightarrow (x+4)(x-3) = 0 \Rightarrow$
$x = -4$ (reject this) or $x = 3$

b. $5^{x^2-4x+5} = 25 = 5^2 \Rightarrow x^2 - 4x + 5 = 2 \Rightarrow$
$x^2 - 4x + 3 = 0 \Rightarrow (x-3)(x-1) = 0 \Rightarrow$
$x = 1$ or $x = 3$

c. $3.1^{x-1} = 23 \Rightarrow (x-1)\ln 3.1 = \ln 23 \Rightarrow$
$x - 1 = \frac{\ln 23}{\ln 3.1} \Rightarrow x = 1 + \frac{\ln 23}{\ln 3.1} \approx 3.771$

16. The possible rational zeros are $\{\pm1, \pm2, \pm4\}$. Using synthetic division, we find that one zero is 1:

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -2 & -2 & 4 \\ & & 1 & 0 & -2 & -4 \\ \hline & 1 & 0 & -2 & -4 & 0 \end{array}$$

Using synthetic division again, we find that 2 is a root of the depressed equation $f(x) = x^3 - 2x - 4$:

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2⌋	1	0	−2	−4
		2	4	4
	1	2	2	0

There are no rational zeros of the depressed equation $f(x) = x^2 + 2x + 2$. So the only rational zeros are {1, 2}.
Find the upper bound for the zeros by testing each possible rational zero using synthetic division. The smallest value that makes all the terms in the bottom row nonnegative is the upper bound. The upper bound is 3.

3⌋	1	−1	−2	−2	4
		3	6	12	30
	1	2	4	10	34

Find the lower bound for the zeros by testing each possible rational zero using synthetic division. The largest value that makes the terms in the bottom row alternate signs is the lower bound. The lower bound is –1.

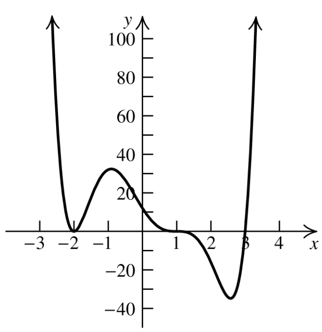
−1⌋	1	−1	−2	−2	4
		−1	2	0	2
	1	−2	0	−2	6

17. a. The zeros are 1 (multiplicity 3), –2 (multiplicity 2), and 3 (multiplicity 1).

b. At $x = 1$ and $x = 3$, the graph crosses the x-axis. At $x = –2$, the graph touches, but does not cross, the x-axis.

c. As $x \to -\infty, f(x) \to \infty$.
As $x \to \infty, f(x) \to \infty$.

d.



18. a. The vertical asymptotes are $x = 3$ and $x = –4$.

b. $f(x) = \dfrac{(x-1)(x+2)}{(x-3)(x+4)} = \dfrac{x^2 + x - 2}{x^2 + x - 12} \Rightarrow$ the horizontal asymptote is $y = 1$.

c. $f(x)$ lies above the horizontal asymptote on $(-\infty, -4) \cup (3, \infty)$. $f(x)$ lies below the horizontal asymptote on $(–4, 3)$.

d.