In Exercises 33–36, find the function of the form $f(x) = ca^x$ that contains the two given graph points.

33. a. $(0, 1)$ and $(2, 16)$ b. $(0, 1)$ and $\left(-2, \frac{1}{9}\right)$

34. a. $(0, 3)$ and $(2, 12)$ b. $(0, 5)$ and $(1, 15)$

35. a. $(1, 1)$ and $(2, 5)$ b. $(1, 1)$ and $\left(2, \frac{1}{5}\right)$

36. a. $(1, 5)$ and $(2, 125)$ b. $(-1, 4)$ and $(1, 16)$

In Exercises 37–44, sketch the graph of the given function by making a table of values. (Use a calculator if necessary.)

37. $f(x) = 4^x$

38. $g(x) = 10^x$

39. $g(x) = \left(\frac{3}{2}\right)^{-x}$

40. $h(x) = 7^{-x}$

41. $h(x) = \left(\frac{1}{4}\right)^x$

42. $f(x) = \left(\frac{1}{10}\right)^x$

43. $f(x) = (1.3)^{-x}$

44. $g(x) = (0.7)^{-x}$

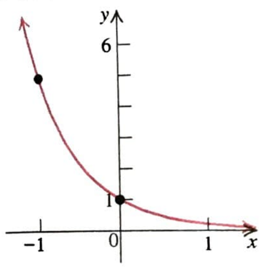
Match each exponential function given in Exercises 45–48 with one of the graphs labeled (a), (b), (c), and (d).

45. $f(x) = 5^x$

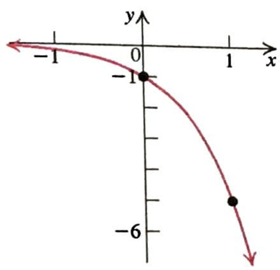
46. $f(x) = -5^x$

47. $f(x) = 5^{-x}$

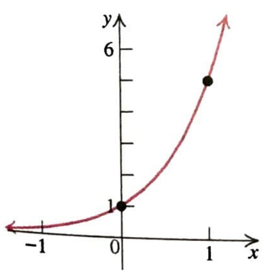
48. $f(x) = 5^{-x} + 1$



(a)



(b)



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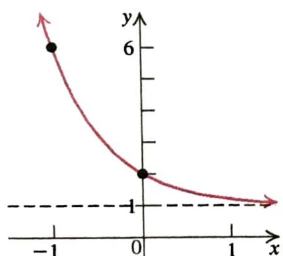


(d)

In Exercises 49–56, start with the graph of the appropriate basic exponential function f and use transformations to sketch the graph of the function g. State the domain and range of g and the horizontal asymptote of its graph.

49. $g(x) = 3^{x-1}$

50. $g(x) = 3^x - 1$

51. $g(x) = 4^{-x}$

52. $g(x) = -4^x$

53. $g(x) = -2 \cdot 5^{x-1} + 4$

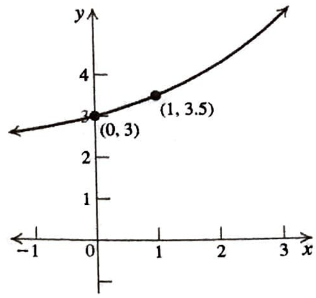
54. $g(x) = \frac{1}{2} \cdot 5^{1-x} - 2$

55. $g(x) = -e^{x-2} + 3$

56. $g(x) = 3 + e^{2-x}$

In Exercises 57–60, write an equation of the form $f(x) = a^x + b$ from the given graph. Then compute $f(2)$.

57.



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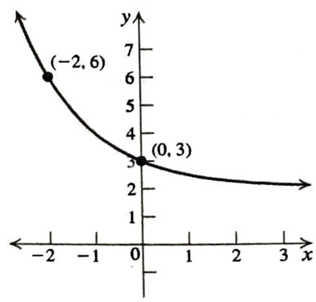

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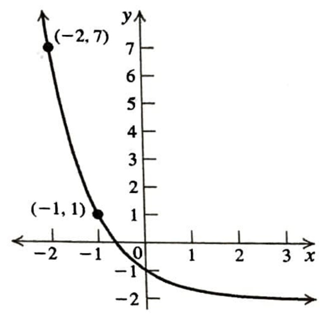
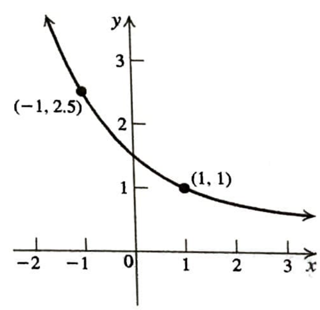


60.



In Exercises 61–64, write an equation of each graph in the final position.

61. The graph of $y = 2^x$ is shifted 2 units left and then 5 units up.

62. The graph of $y = 3^x$ is shifted 3 units right and then is reflected about the y-axis.

63. The graph of $y = \left(\frac{1}{2}\right)^x$ is stretched vertically by a factor of 2 and then is shifted 5 units down.

64. The graph of $y = 2^{-x}$ is reflected about the x-axis and then is shifted 3 units up.

In Exercises 65–68, find the simple interest for each value of principal P, rate r per year, and time t.

65. $P = \$5000$, $r = 10\%$, $t = 5$ years
66. $P = \$10{,}000$, $r = 5\%$, $t = 10$ years
67. $P = \$7800$, $r = 6\frac{7}{8}\%$, $t = 10$ years and 9 months
68. $P = \$8670$, $r = 4\frac{1}{8}\%$, $t = 6$ years and 8 months

In Exercises 69–72, find (a) the future value of the given principal P and (b) the interest earned in the given period.

69. $P = \$3500$ at 6.5% compounded annually for 13 years
70. $P = \$6240$ at 7.5% compounded monthly for 12 years
71. $P = \$7500$ at 5% compounded continuously for 10 years
72. $P = \$8000$ at 6.5% compounded daily for 15 years

In Exercises 73–76, find the principal P that will generate the given future value A.

73. $A = \$10{,}000$ at 8% compounded annually for 10 years
74. $A = \$10{,}000$ at 8% compounded quarterly for 10 years
75. $A = \$10{,}000$ at 8% compounded daily for 10 years
76. $A = \$10{,}000$ at 8% compounded continuously for 10 years

In Exercises 77–84, starting with the graph of $y = e^x$, use transformations to sketch the graph of each function and state its horizontal asymptote.

77. $f(x) = e^{-x}$
78. $f(x) = -e^x$
79. $f(x) = e^{x-2}$
80. $f(x) = e^{2-x}$
81. $f(x) = 1 + e^x$
82. $f(x) = 2 - e^{-x}$
83. $f(x) = -e^{x-2} + 3$
84. $g(x) = 3 + e^{2-x}$

Applying the Concepts

85. **Metal cooling.** Suppose a metal block is cooling so that its temperature T (in °C) is given by $T = 200 \cdot 4^{-0.1t} + 25$, where t is in hours.
    a. Find the temperature after
       (i) 2 hours. (ii) 3.5 hours.
    b. How long has the cooling been taking place if the block now has a temperature of 125°C?
    c. Find the eventual temperature ($t \to \infty$).

86. **Ethnic population.** The population (in thousands) of people of East Indian origin in the United States is approximated by the function

$$p(t) = 1600(2)^{0.1047t},$$

where t is the number of years since 2010.
    a. Find the population of this group in
       (i) 2010. (ii) 2018.
    b. Predict the population in 2025.

87. **Price appreciation.** In 2010, the median price of a house in Miami was \$190,000. Assuming a rate of increase of 3% per year, what can we expect the price of such a house to be in 2015?

88. **Investment.** How much should a mother invest at the time her son is born to provide him with \$80,000 at age 21? Assume that the interest is 7% compounded quarterly.

89. **Depreciation.** Trans Trucking Co. purchased a truck for \$80,000. The company depreciated the truck at the end of each year at the rate of 15% of its current value. What is the value of the truck at the end of the fifth year?

90. **Manhattan Island purchase.** In 1626, Peter Du Minuit purchased Manhattan Island from the Native Americans for 60 Dutch guilders (about \$24). Suppose the \$24 was invested in 1626 at a 6% rate. How much money would that investment be worth in 2006 if the interest was
    a. Simple interest.
    b. Compounded annually.
    c. Compounded monthly.
    d. Compounded continuously.

91. **Investment.** Ms. Ann Scheiber retired from government service in 1941 with a monthly pension of \$83 and \$5000 in savings. At the time of her death in January 1995 at the age of 101, Ms. Scheiber had turned the \$5000 into \$22 million through shrewd investments in the stock market. She bequeathed all of it to Yeshiva University in New York City. What annual rate of return compounded annually would turn \$5000 to a whopping \$22 million in 54 years?

92. **Population.** The population of Sometown, USA, was 12,000 in 2000 and grew to 15,000 in 2010. Assume that the population will continue to grow exponentially at the same constant rate. What will be the population of Sometown in 2020? $\left[\textit{Hint: } \text{Show that } e^k = \left(\frac{5}{4}\right)^{1/10}.\right]$

93. **Medicine.** Tests show that a new ointment X helps heal wounds. If A_0 square millimeters is the area of the original wound, then the area A of the wound after n days of application of the ointment X is given by $A = A_0e^{-0.43n}$. If the area of the original wound was 10 square millimeters, find the area of the wound after ten days of application of the new ointment.

94. **Cooling.** The temperature T (in °C) of coffee at time t minutes after its removal from the microwave is given by the equation

$$T = 25 + 73e^{-0.28t}.$$

Find the temperature of the coffee at each time listed.
    a. $t = 0$
    b. $t = 10$
    c. $t = 20$
    d. after a long time

95. **Interest rate.** Mary purchases a 12-year bond for \$60,000.00. At the end of 12 years, she will redeem the bond for approximately \$95,600.00. If the interest is compounded quarterly, what was the interest rate on the bond?

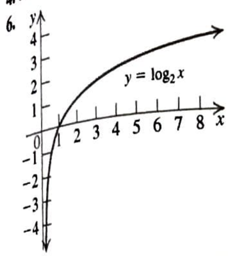
96. **Best deal.** Fidelity Federal offers three types of investments: (i) 9.7% compounded annually, (ii) 9.6% compounded monthly, and (iii) 9.5% compounded continuously. Which investment is the best deal?

97. **Paper stacking.** Suppose we have a large sheet of paper 0.015 centimeter thick and we tear the paper in half and put the pieces on top of each other. We keep tearing and stacking in this manner, always tearing each piece in half. How high will the resulting pile of paper be if we continue the process of tearing and stacking
    a. 30 times?
    b. 40 times?
    c. 50 times? [*Hint:* Use a calculator.]

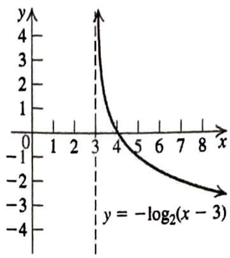
Answers to Practice Problems

1. a. $\log_2 1024 = 10$ **b.** $\log_9\left(\frac{1}{3}\right) = -\frac{1}{2}$ **c.** $\log_a p = q$ **2. a.** $2^6 = 64$ **b.** $v^w = u$ **3. a.** 2 **b.** $-1/2$ **c.** -5 **4. a.** 0 **b.** 5 **c.** 5 **5.** $(-\infty, 1)$

6.



7. Shift the graph of $y = \log_2 x$ 3 units right and reflect the graph about the x-axis.



8. $P_2 = \log 3 - \log 2 \approx 0.176.$ About 17.6% of the data is expected to have 2 as the first digit. **9. a.** -1 **b.** ≈ 0.693 **10. a.** ≈ 17 years **b.** 21.97% **11.** ≈ 5.4 min

SECTION 4.2 Exercises

Basic Concepts and Skills

1. The domain of the function $y = \log_a x$ is __________, and its range is __________.
2. The logarithmic form $y = \log_a x$ is equivalent to the exponential form __________.
3. The logarithm with base 10 is called the __________ logarithm, and the logarithm with base e is called the __________ logarithm.
4. $a^{\log_a x} =$ __________, and $\log_a a^x =$ __________.
5. **True or False.** The graph of $y = \log_a x$, $a > 0$, $a \neq 1$, is the graph of an increasing function.
6. **True or False.** The graph of $y = \log_a x$, $a > 0$, and $a \neq 1$, has no horizontal asymptote.

In Exercises 7–18, write each exponential equation in logarithmic form.

7. $5^2 = 25$ **8.** $(49)^{-1/2} = \frac{1}{7}$

9. $\left(\frac{1}{16}\right)^{-1/2} = 4$ **10.** $(a^2)^2 = a^4$

11. $10^0 = 1$ **12.** $10^4 = 10{,}000$

13. $(10)^{-1} = 0.1$ **14.** $3^x = 5$

15. $a^2 + 2 = 7$ **16.** $a^e = \pi$

17. $2a^3 - 3 = 10$ **18.** $5 \cdot 2^{ct} = 11$

In Exercises 19–30, write each logarithmic equation in exponential form.

19. $\log_2 32 = 5$ **20.** $\log_7 49 = 2$

21. $\log_{10} 100 = 2$ **22.** $\log_{10} 10 = 1$

23. $\log_{10} 1 = 0$ **24.** $\log_a 1 = 0$

25. $\log_{10} 0.01 = -2$ **26.** $\log_{1/5} 5 = -1$

27. $3 \log_8 2 = 1$ **28.** $1 + \log 1000 = 4$

29. $\ln 2 = x$ **30.** $\ln \pi = a$

In Exercises 31–40, evaluate each expression without using a calculator.

31. $\log_5 125$ **32.** $\log_9 81$

33. $\log 10{,}000$ **34.** $\log_3 \frac{1}{3}$

35. $\log_2 \frac{1}{8}$ **36.** $\log_4 \frac{1}{64}$

37. $\log_3 \sqrt{27}$ **38.** $\log_{27} 3$

39. $\log_{16} 2$ **40.** $\log_5 \sqrt{125}$

In Exercises 41–52, evaluate each expression.

41. $\log_3 1$ **42.** $\log_{1/2} 1$

43. $\log_7 7$ **44.** $\log_{1/9} \frac{1}{9}$

45. $\log_6 6^7$ **46.** $\log_{1/2}\left(\frac{1}{2}\right)^5$

47. $3^{\log_3 5}$ **48.** $7^{\log_7 \frac{1}{2}}$

49. $2^{\log_2 7} + \log_5 5^{-3}$ **50.** $3^{\log_3 5} - \log_2 2^{-3}$

51. $4^{\log_4 6} - \log_4 4^{-2}$ **52.** $10^{\log x} - e^{\ln y}$

In Exercises 53–60, find the domain of each function.

53. $f(x) = \log_2 (x + 1)$

54. $g(x) = \log_3 (x - 8)$

55. $f(x) = \log_3 \sqrt{x - 1}$

56. $g(x) = \log_4 \sqrt{3 - x}$

57. $f(x) = \log(x - 2) + \log(2x - 1)$

58. $g(x) = \ln \sqrt{x + 5} - \ln(x + 1)$

SECTION 4.3 **Exercises**

Basic Concepts and Skills

1. $\log_a MN =$ ______ + ______
2. ______ $= \log_a M - \log_a N$
3. $\log_a M^r =$ ______
4. The change-of-base formula using base e is $\log_a M =$ ______.
5. True or False. $\log_a (u + v) = \log_a u + \log_a v$
6. True or False. $\log \frac{x}{10} = \log x - 1$

In Exercises 7–18, given that $\log x = 2$, $\log y = 3$, $\log 2 \approx 0.3$, and $\log 3 \approx 0.48$, evaluate each expression without using a calculator.

7. $\log 6$
8. $\log 4$
9. $\log 5$ $\left[\textit{Hint: } 5 = \frac{10}{2}\right]$
10. $\log (3x)$
11. $\log\left(\frac{2}{x}\right)$
12. $\log x^2$
13. $\log (2x^2y)$
14. $\log xy^3$
15. $\log \sqrt[3]{x^2y^4}$
16. $\log (\log x^2)$
17. $\log \sqrt[3]{48}$
18. $\log_2 3$

In Exercises 19–32, write each expression in expanded form. Assume that all expressions containing variables represent positive numbers.

19. $\ln [x(x - 1)]$
20. $\ln \frac{x(x + 1)}{(x - 1)^2}$
21. $\log_a \sqrt{xy^3}$
22. $\log_a \frac{3x^2}{\sqrt{y}}$
23. $\log_a \sqrt[3]{\frac{x}{y}}$
24. $\log_a \sqrt[3]{\frac{x^2}{y^5}}$
25. $\log_2 \sqrt[4]{\frac{xy^2}{8}}$
26. $\log \sqrt[3]{\frac{x^2y}{100}}$
27. $\log \frac{\sqrt{x^2 + 1}}{x + 3}$
28. $\log_4 \left(\frac{x^2 - 9}{x^2 - 6x + 8}\right)^{2/3}$
29. $\log_b x^2y^3z$
30. $\log_b \sqrt{xyz}$
31. $\ln \left[\frac{x\sqrt{x - 1}}{x^2 + 2}\right]$
32. $\ln \left[\frac{\sqrt{x} - 2\sqrt[3]{x + 1}}{x^2 + 3}\right]$
33. $\ln \left[\frac{(x + 1)^2}{(x - 3)\sqrt{x + 4}}\right]$
34. $\ln \left[\frac{2x + 3}{(x + 4)^2(x - 3)^4}\right]$
35. $\ln \left[(x + 1)\sqrt{\frac{x^2 + 2}{x^2 + 5}}\right]$
36. $\ln \left[\frac{\sqrt[3]{2x + 1}\,(x + 1)}{(x - 1)^2(3x + 2)}\right]$
37. $\ln \left[\frac{x^3(3x + 1)^4}{\sqrt{x^2 + 1}\,(x + 2)^{-5}(x - 3)^2}\right]$
38. $\ln \left[\frac{(x + 1)^{1/2}(x^2 - 2)^{2/5}}{(2x - 1)^{3/2}(x^2 + 2)^{-4/5}}\right]$

In Exercises 39–48, write each expression in condensed form.

39. $\log_2 x + \log_2 7$
40. $\log_2 x - \log_2 3$
41. $\frac{1}{2} \log x - \log y + \log z$
42. $\frac{1}{2}(\log x + \log y)$
43. $\frac{1}{5}(\log_2 z + 2 \log_2 y)$
44. $\frac{1}{3}(\log x - 2 \log y + 3 \log z)$
45. $\ln x + 2 \ln y + 3 \ln z$
46. $2 \ln x - 3 \ln y + 4 \ln z$
47. $2 \ln x - \frac{1}{2} \ln (x^2 + 1)$
48. $2 \ln x + \frac{1}{2} \ln (x^2 - 1) - \frac{1}{2} \ln (x^2 + 1)$

In Exercises 49–52, write an estimate of each number in scientific notation.

49. e^{500}
50. π^{650}
51. 324^{756}
52. 723^{416}
53. Which is larger, 234^{567} or 567^{234}?
54. Which is larger, 4321^{8765} or 8765^{4321}?
55. Find the number of digits in $17^{200} \cdot 53^{67}$.
56. Find the number of digits in $67^{200} \div 23^{150}$.

In Exercises 57–64, use the change-of-base formula and a calculator to evaluate each logarithm.

57. $\log_2 5$
58. $\log_4 11$
59. $\log_{1/2} 3$
60. $\log_{\sqrt{3}} 12.5$
61. $\log_{\sqrt{5}} \sqrt{17}$
62. $\log_{15} 123$
63. $\log_2 7 + \log_4 3$
64. $\log_2 9 - \log_{\sqrt{2}} 5$

In Exercises 65–72, find the value of each expression without using a calculator.

65. $\log_3 \sqrt{3}$
66. $\log_{1/4} 4$
67. $\log_3 (\log_2 8)$
68. $2^{\log_2 2}$
69. $5^{2 \log_5 3 + \log_5 2}$
70. $e^{3 \ln 2 - 2 \ln 3}$
71. $\log 4 + 2 \log 5$
72. $\log_2 160 - \log_2 5$

Answers to Practice Problems

1. a. $x = 5$ **b.** $x = \frac{2}{3}$ **2.** $x = \frac{\ln\left(\frac{11}{7}\right)}{\ln 3} - 1 \approx -0.589$

3. $x = \frac{\ln 3}{2\ln 2 - \ln 3} \approx 3.819$ **4.** $x = \ln 5 \approx 1.609$

5. a. United States: 343.61 million; Pakistan: 255.96 million

b. Sometime in 2022 (after 11.69 yr) **c.** In 23.68 yrs (sometime in 2033) **6.** $x = e^{3/2}$ **7.** $x = 9$ **8.** $\varnothing$

9. The average growth rate was approximately 1.74%.

10. $x < -2$ **11.** $x < \frac{1 - e^2}{3}$

SECTION 4.4 Exercises

Basic Concepts and Skills

1. An equation that contains terms of the form a^x is called a(n) ________ equation.
2. An equation that contains terms of the form $\log_a x$ is called a(n) ________ equation.
3. The equation $y = \frac{M}{1 + ae^{-bx}}$ represents a(n) ________ model.
4. **True or False.** A logistic curve always has two horizontal asymptotes.
5. **True or False.** Because the domain of $f(x) = \log_a x$ is the interval $(0, \infty)$, a logarithmic equation cannot have a negative solution.
6. **True or False.** The equation $8^{2x} = 4^{3x}$ is true for all real values of x.

In Exercises 7–22, solve each equation.

7. $2^x = 16$
8. $3^x = 243$
9. $8^x = 32$
10. $5^{x-1} = 1$
11. $4^{|x|} = 128$
12. $9^{|x|} = 243$
13. $5^{-|x|} = 625$
14. $3^{-|x|} = 81$
15. $\ln x = 0$
16. $\ln(x - 1) = 1$
17. $\log_2 x = -1$
18. $\log_2(x + 1) = 3$
19. $\log_3 |x| = 2$
20. $\log_2 |x + 1| = 3$
21. $\frac{1}{2}\log x - 2 = 0$
22. $\frac{1}{3}\log(x + 1) - 1 = 0$

In Exercises 23–52, solve each exponential equation. Write the exact answer with natural logarithms and then approximate the result correct to three decimal places.

23. $2^x = 3$
24. $3^x = 5$
25. $2^{2x+3} = 15$
26. $3^{2x+5} = 17$
27. $5 \cdot 2^x - 7 = 10$
28. $3 \cdot 5^x + 4 = 11$
29. $3 \cdot 4^{2x-1} + 4 = 14$
30. $2 \cdot 3^{4x-5} - 7 = 10$
31. $5^{1-x} = 2^x$
32. $3^{2x-1} = 2^{x+1}$
33. $2^{1-x} = 3^{4x+6}$
34. $5^{2x+1} = 3^{x-1}$
35. $2 \cdot 3^{x-1} = 5^{x+1}$
36. $5 \cdot 2^{2x+1} = 7 \cdot 3^{x-1}$
37. $(1.065)^t = 2$
38. $(1.0725)^t = 2$
39. $2^{2x} - 4 \cdot 2^x = 21$
40. $4^x - 4^{-x} = 2$
41. $9^x - 6 \cdot 3^x + 8 = 0$
42. $\frac{3^x + 5 \cdot 3^{-x}}{3} = 2$
43. $3^{3x} - 4 \cdot 3^{2x} + 2 \cdot 3^x = 8$
44. $2^{3x} + 3 \cdot 2^{2x} - 2^x = 3$
45. $\frac{3^x - 3^{-x}}{3^x + 3^{-x}} = \frac{1}{4}$
46. $\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{3}$
47. $\frac{4}{2 + 3^x} = 1$
48. $\frac{7}{2^x - 1} = 3$
49. $\frac{17}{5 - 3^x} = 7$
50. $\frac{15}{3 + 2 \cdot 5^x} = 4$
51. $\frac{5}{2 + 3^x} = 4$
52. $\frac{7}{3 + 5 \cdot 2^x} = 4$

In Exercises 53–70, solve each logarithmic equation.

53. $3 + \log(2x + 5) = 2$
54. $1 + \log(3x - 4) = 0$
55. $\log(x^2 - x - 5) = 0$
56. $\log(x^2 - 6x + 9) = 0$
57. $\log_4(x^2 - 7x + 14) = 1$
58. $\log_4(x^2 + 5x + 10) = 1$
59. $\ln(2x - 3) - \ln(x + 5) = 0$
60. $\log(x + 8) + \log(x - 1) = 1$
61. $\log x + \log(x + 9) = 1$
62. $\log_5(3x - 1) - \log_5(2x + 7) = 0$
63. $\log_a(5x - 2) - \log_a(3x + 4) = 0$
64. $\log(x - 1) + \log(x + 2) = 1$
65. $\log_6(x + 2) + \log_6(x - 3) = 1$
66. $\log_2(3x - 2) - \log_2(5x + 1) = 3$
67. $\log_3(2x - 7) - \log_3(4x - 1) = 2$
68. $\log_4\sqrt{x + 3} - \log_4\sqrt{2x - 1} = \frac{1}{4}$
69. $\log_7 3x + \log_7(2x - 1) = \log_7(16x - 10)$
70. $\log_3(x + 1) + \log_3 2x = \log_3(3x + 1)$