EXAMPLE 6 Tracking an

A television camera at ground level is filming the lift-off of a space shuttl that is rising vertically according to the position equation $s = 50t^2$, where is measured in feet and t is measured in seconds. The camera is 2000 fe from the launch pad. Find the rate of change in the distance between th camera and the base of the shuttle 10 seconds after lift-off. (Assume that t camera and the base of the shuttle are level with each other when $t = 0$.)

SOLUTION

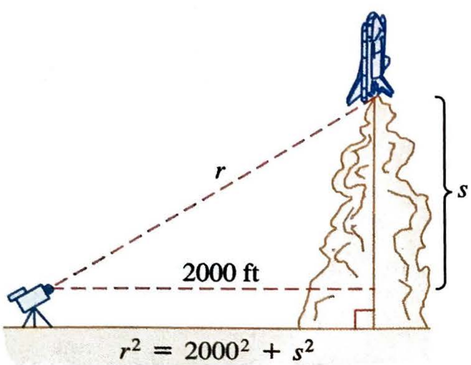


FIGURE 3.31

1. We let r be the distance between the camera and the base of the shu as shown in Figure 3.31. Then we can find the velocity of the rocke differentiating s with respect to t to obtain $ds/dt = 100t$. Thus, we h the following model.

   Given: $\dfrac{ds}{dt} = 100t = \text{velocity}$

   Find: $\dfrac{dr}{dt}$ when $t = 10$

2. Using Figure 3.31 we relate s and r by the equation
$$r^2 = 2000^2 + s^2.$$

3. Implicit differentiation with respect to t yields
$$2r\frac{dr}{dt} = 2s\frac{ds}{dt}$$
$$\frac{dr}{dt} = \frac{s}{r}\cdot\frac{ds}{dt} = \frac{s}{r}(10t).$$

4. Now, when $t = 10$, we know that $s = 50(10^2) = 5000$, and we
$$r = \sqrt{2000^2 + 5000^2} = 1000\sqrt{29}.$$
Finally, the rate of change of r when $t = 10$ is
$$\frac{dr}{dt} = \frac{5000}{1000\sqrt{29}}(100)(10) = 928.48 \text{ ft/sec}.$$

EXERCISES for Section 3.7

In Exercises 1–4, assume that x and y are both differentiable functions of t and find the indicated values of dy/dt and dx/dt.

Equation	*Find*	*Given*
1. $y = \sqrt{x}$	(a) $\dfrac{dy}{dt}$ when $x = 4$	$\dfrac{dx}{dt} = 3$
	(b) $\dfrac{dx}{dt}$ when $x = 25$	$\dfrac{dy}{dt} = 2$

Equation	*Find*
2. $y = x^2 - 3x$	(a) $\dfrac{dy}{dt}$ when $x = 3$
	(b) $\dfrac{dx}{dt}$ when $x = 1$
3. $xy = 4$	(a) $\dfrac{dy}{dt}$ when $x = 8$
	(b) $\dfrac{dx}{dt}$ when $x = 1$

Equation	Find	Given
4. $x^2 + y^2 = 25$	(a) $\dfrac{dy}{dt}$ when $x = 3, y = 4$	$\dfrac{dx}{dt} = 8$
	(b) $\dfrac{dx}{dt}$ when $x = 4, y = 3$	$\dfrac{dy}{dt} = -2$

5. The radius r of a circle is increasing at a rate of 2 inches per minute. Find the rate of change of the area when (a) $r = 6$ inches and (b) $r = 24$ inches.

6. The radius r of a sphere is increasing at a rate of 2 inches per minute. Find the rate of change of the volume when (a) $r = 6$ inches and (b) $r = 24$ inches.

7. Let A be the area of a circle of radius r that is changing with respect to time. If dr/dt is constant, is dA/dt constant? Explain why or why not.

8. Let V be the volume of a sphere of radius r that is changing with respect to time. If dr/dt is constant, is dV/dt constant? Explain why or why not.

9. A spherical balloon is inflated with gas at the rate of 20 cubic feet per minute. How fast is the radius of the balloon increasing at the instant the radius is (a) 1 foot and (b) 2 feet?

10. The formula for the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h.$$

Find the rate of change of the volume if dr/dt is 2 inches per minute and $h = 3r$ when (a) $r = 6$ inches and (b) $r = 24$ inches.

11. At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at the rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when it is 15 feet high?

12. A conical tank (with vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at the rate of 10 cubic feet per minute, find the rate of change of the depth of the water the instant it is 8 feet deep.

13. All edges of a cube are expanding at the rate of 3 centimeters per second. How fast is the volume changing when each edge is (a) 1 centimeter and (b) 10 centimeters?

14. The conditions are the same as in Exercise 13. Now measure how fast the *surface* area is changing when each edge is (a) 1 centimeter and (b) 10 centimeters.

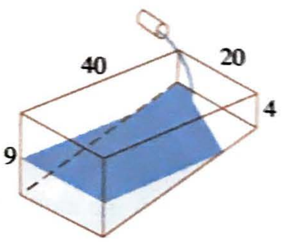
15. A point is moving along the graph of $y = x^2$ so that dx/dt is 2 centimeters per minute. Find dy/dt when (a) $x = 0$ and (b) $x = 3$.

16. The conditions are the same as in Exercise 15, but now measure the rate of change of the distance between the point and the origin.

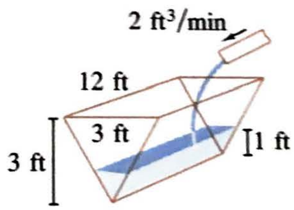
17. A point is moving along the graph of $y = 1/(1 + x^2)$ so that $dx/dt = 2$ centimeters per minute. Find dy/dt for the following values of x.
(a) $x = -2$
(b) $x = 0$
(c) $x = 2$
(d) $x = 10$

18. A point is moving along the graph of $y = x^3$ so that $dx/dt = 2$ centimeters per minute. Find dy/dt for the following values of x.
(a) $x = -2$
(b) $x = 1$
(c) $x = 0$
(d) $x = 3$

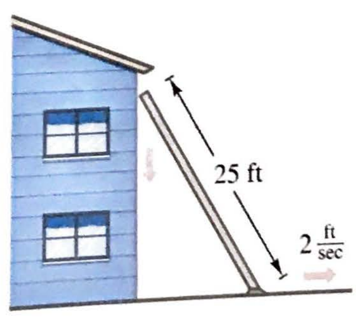
19. A swimming pool is 40 feet long, 20 feet wide, 4 feet deep at the shallow end, and 9 feet deep at the deep end (see figure). Water is being pumped into the pool at 10 cubic feet per minute, and there is 4 feet of water at the deep end.
(a) What percentage of the pool is filled?
(b) At what rate is the water level rising?



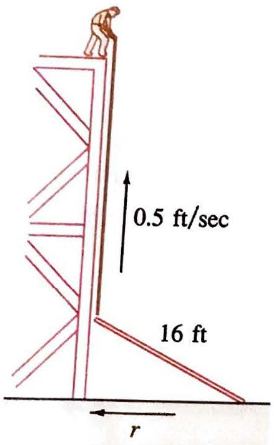
20. A trough is 12 feet long and 3 feet across the top (see figure). Its ends are isosceles triangles with an altitude of 3 feet. If water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when it is 1 foot deep?



21. A ladder 25 feet long is leaning against the wall of a house (see figure). The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top moving down the wall when the base of the ladder is (a) 7 feet, (b) 15 feet, and (c) 24 feet from the wall?

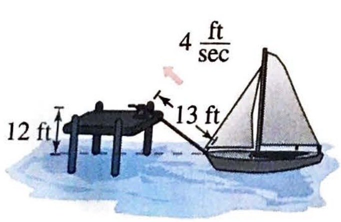


22. A construction worker pulls a 16-foot plank up the side of a building under construction by means of a rope tied to the end of the plank (see figure). Assume the opposite end of the plank follows a path perpendicular to the wall of the building and the worker pulls the rope at the rate of 0.5 feet per second. How fast is the end of the plank sliding along the ground when it is 8 feet from the wall of the building?



23. Consider the right triangle formed by the moving ladder, the side of the house, and the ground in Exercise 21. When the base is 7 feet from the wall, find the rate at which the area of the triangle is changing.

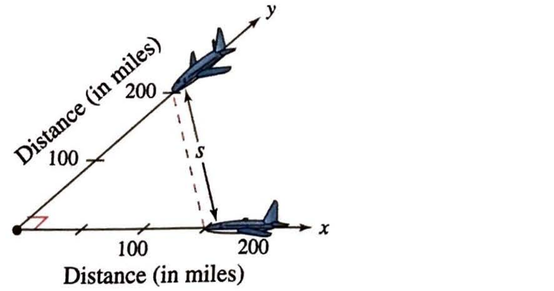
24. A boat is pulled in by means of a winch on the dock 12 feet above the deck of the boat (see figure). The winch pulls in rope at the rate of 4 feet per second. Determine the speed of the boat when there is 13 feet of rope out. What happens to the speed of the boat as it gets closer to the dock?



25. An air traffic controller spots two planes at the same altitude converging on a point as they fly at right angles to each other (see figure). One plane is 150 miles from the point and is moving at 450 miles per hour. The other plane is 200 miles from the point and has a speed of 600 miles per hour.

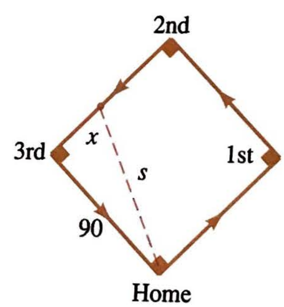
(a) At what rate is the distance between the planes decreasing?

(b) How much time does the traffic controller have t get one of the planes on a different flight path?



26. The point $(0, y)$ moves along the y-axis at a constant rate of R feet per second, while the point $(x, 0)$ moves along the x-axis at a constant rate of r feet per second. Find an expression for the rate of change of the distance between the two points.

27. A baseball diamond has the shape of a square with sides 90 feet long (see figure). A player 30 feet from third base is running at a speed of 28 feet per second. At what rate is the player's distance from home plate changing?



28. For the baseball diamond in Exercise 27, suppose the player is running from first to second at a speed of 28 feet per second. Find the rate at which the distance from home plate is changing when the player is 30 feet from second.

29. A man 6 feet tall walks at a rate of 5 feet per second away from a light that is 15 feet above the ground (see figure). When he is 10 feet from the base of the light,

(a) at what rate is the tip of his shadow moving?

(b) at what rate is the length of his shadow changing?