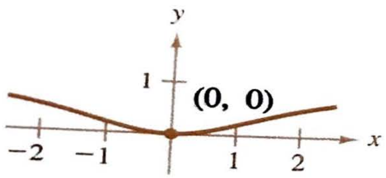
TABLE 4.2

Left endpoint	Critical number	Critical number	Right endpoint
$f(-1) = -5$ Minimum	$f(0) = 0$ Maximum	$f(1) = -1$	$f(3) = 6 - 3\sqrt[3]{9} \approx -0.24$

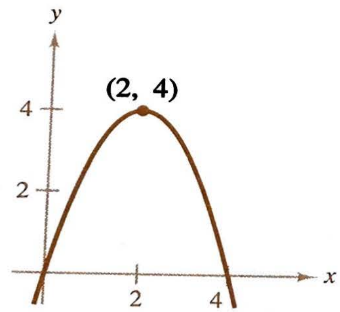
EXERCISES for Section 4.1

In Exercises 1–6, find the value of the derivative (if it exists) at the indicated extrema.

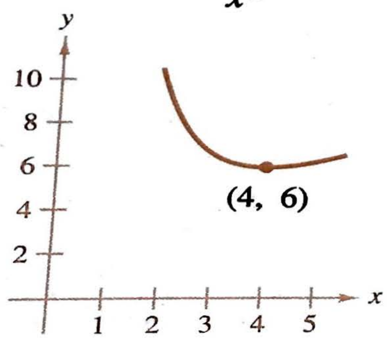
1. $f(x) = \dfrac{x^2}{x^2 + 4}$



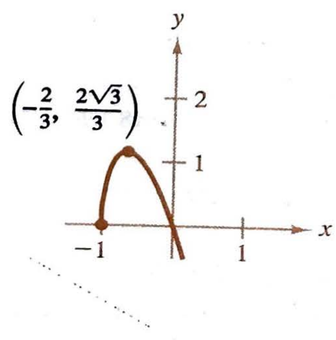
2. $f(x) = -x^2 + 4x$



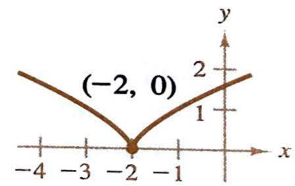
3. $f(x) = x + \dfrac{32}{x^2}$



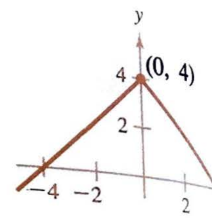
4. $f(x) = -3x\sqrt{x + 1}$



5. $f(x) = (x + 2)^{2/3}$



6. $f(x) = 4 - |x|$



In Exercises 7–18, locate the absolute extrema given function on the indicated interval.

Function	*Interval*
7. $f(x) = 2(3 - x)$	$[-1, 2]$
8. $f(x) = \dfrac{2x + 5}{3}$	$[0, 5]$
9. $f(x) = -x^2 + 3x$	$[0, 3]$
10. $f(x) = x^2 + 2x - 4$	$[-1, 1]$
11. $f(x) = x^3 - 3x^2$	$[-1, 3]$
12. $f(x) = x^3 - 12x$	$[0, 4]$
13. $f(x) = 3x^{2/3} - 2x$	$[-1, 1]$
14. $g(x) = \sqrt[3]{x}$	$[-1, 1]$
15. $h(t) = 4 - \lvert t - 4 \rvert$	$[1, 6]$

16. $g(t) = \dfrac{t^2}{t^2 + 3}$ $\quad [-1, 1]$

17. $h(s) = \dfrac{1}{s - 2}$ $\quad [0, 1]$

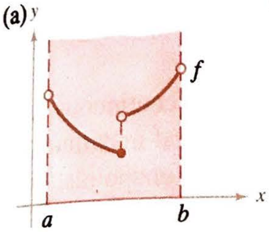
18. $h(t) = \dfrac{t}{t - 2}$ $\quad [3, 5]$

19. Explain why the function $f(x) = 1/x^2$ has a maximum on $[1, 2]$ but not on $(0, 2]$.

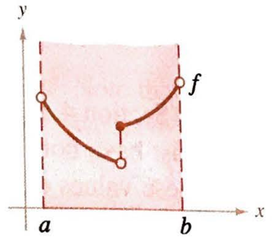
20. Explain why the function $y = 1/(x + 1)$ has a minimum on $[0, 2]$, but not on $[-2, 0]$.

In Exercises 21–24, determine from the graph whether f possesses a minimum in the interval (a, b).

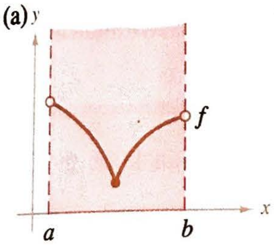
21. (a)



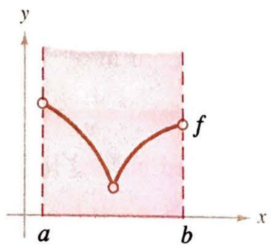
(b)



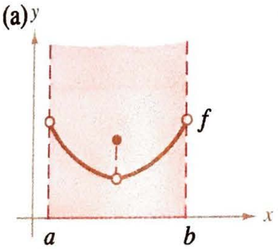
22. (a)



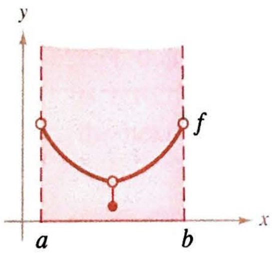
(b)



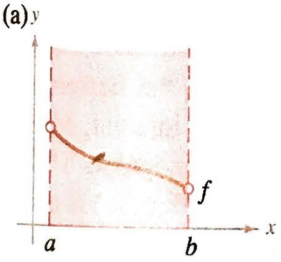
23. (a)



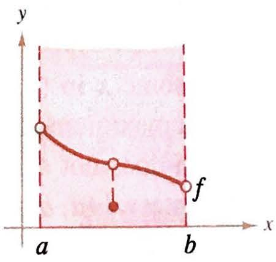
(b)



24. (a)



(b)



In Exercises 25 and 26, locate the absolute extrema of the function (if any exist) over the indicated interval.

25. $f(x) = 2x - 3$
(a) $[0, 2]$
(b) $[0, 2)$
(c) $(0, 2]$
(d) $(0, 2)$

26. $f(x) = 5 - x$
(a) $[1, 4]$
(b) $[1, 4)$
(c) $(1, 4]$
(d) $(1, 4)$

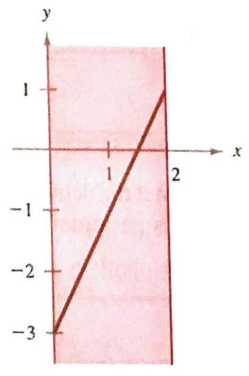


FIGURE FOR 25

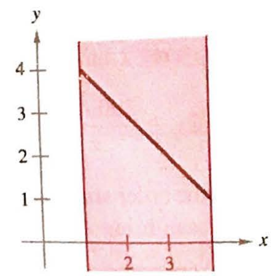


FIGURE FOR 26

In Exercises 27–30, find the maximum value of $|f''(x)|$ on the indicated interval. (This value is used in the error estimate for the Trapezoidal Rule, as discussed in Section 5.6.)

Function	*Interval*
27. $f(x) = \dfrac{1}{x^2 + 1}$	$[0, 3]$
28. $f(x) = \dfrac{1}{x^2 + 1}$	$\left[\dfrac{1}{2}, 3\right]$
29. $f(x) = \sqrt{1 + x^3}$	$[0, 2]$
30. $f(x) = x^3(3x^2 - 10)$	$[0, 1]$

In Exercises 31–34, find the maximum value of $|f^{(4)}(x)|$ on the indicated interval. (This value is used in the error estimate for Simpson's Rule, as discussed in Section 5.6.)

Function	*Interval*
31. $f(x) = 15x^4 - \left(\dfrac{2x - 1}{2}\right)^6$	$[0, 1]$
32. $f(x) = x^5 - 5x^4 + 20x^3 + 600$	$\left[0, \dfrac{3}{2}\right]$
33. $f(x) = (x + 1)^{2/3}$	$[0, 2]$
34. $f(x) = \dfrac{1}{x^2 + 1}$	$[-1, 1]$

35. The formula for the power output P of a battery is given by

$$P = VI - RI^2$$

where V is the electromotive force in volts, R is the resistance, and I is the current. Find the current (measured in amperes) that corresponds to a maximum value of P in a battery for which $V = 12$ volts and $R = 0.5$ ohms. (Assume that a 15-amp fuse bounds the output in the interval $0 \le I \le 15$.)

37. Tangent line: $3x - y + 7 = 0$
Normal line: $x + 3y - 1 = 0$

39. Tangent line: $x + 2y - 10 = 0$
Normal line: $2x - y = 0$

41. Tangent line: $2x - 3y - 3 = 0$
Normal line: $3x + 2y - 11 = 0$

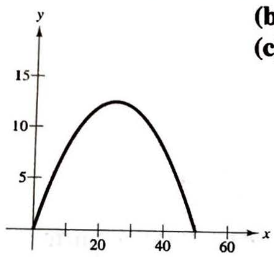
43. (a) $(0, -1), \left(-2, \frac{7}{3}\right)$ (b) $(-3, 2), \left(1, -\frac{2}{3}\right)$
(c) $\left(-1 + \sqrt{2}, \frac{2[1 - 2\sqrt{2}]}{3}\right), \left(-1 - \sqrt{2}, \frac{2[1 + 2\sqrt{2}]}{3}\right)$

45. $f'(x) = -\frac{2}{x^3}$ **47.** $f'(x) = \frac{1}{2\sqrt{x + 2}}$

49. $v(t) = 1 - \frac{1}{(t + 1)^2}$, $a(t) = \frac{2}{(t + 1)^3}$

51. (a) -18.667 (b) -7.284 (c) -3.240
(d) -0.747 **53.** 56 ft/sec

55. (a)



(b) 50
(c) $x = 25$

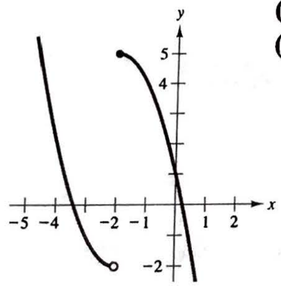
(d) $y' = 1 - 0.04x$

x	0	10	25	30	50
y'	1	0.6	0	-0.2	-1

(e) $y'(25) = 0$

59. (a) $2\sqrt{2}$ units/sec (b) 4 units/sec
(c) 8 units/sec **61.** $\frac{2}{25}$ ft/min

65. (a)



(b) No
(c) No

Chapter 4

Section 4.1

1. $f'(0) = 0$ **3.** $f'(4) = 0$

5. $f'(-2)$ is undefined

7. Minimum: $(2, 2)$
Maximum: $(-1, 8)$

9. Minima: $(0, 0)$ and $(3, 0)$
Maximum: $\left(\frac{3}{2}, \frac{9}{4}\right)$

11. Minima: $(-1, -4)$ and $(2, -4)$
Maxima: $(0, 0)$ and $(3, 0)$

13. Minimum: $(0, 0)$
Maximum: $(-1, 5)$

15. Minimum: $(1, 1)$
Maximum: $(4, 4)$

17. Minimum: $(1, -1)$
Maximum: $\left(0, -\frac{1}{2}\right)$

19. f is bounded on $[1, 2]$ but not bounded on $(0, 2]$.

21. (a) Yes
(b) No

23. (a) No
(b) Yes

25. (a) Minimum: $(0, -3)$
Maximum: $(2, 1)$
(b) Minimum: $(0, -3)$
(c) Maximum: $(2, 1)$
(d) No extrema

27. Maximum: $|f''(0)| = 2$

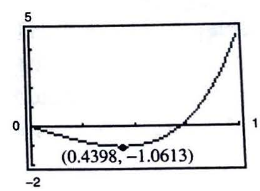
29. Maximum: $\left|f''\left(\sqrt[3]{-10 + \sqrt{108}}\right)\right| \approx 1.47$

31. Maximum: $\left|f^{(4)}\left(\frac{1}{2}\right)\right| = 360$

33. Maximum: $\left|f^{(4)}(0)\right| = \frac{56}{81}$

35. Maximum: $P(12) = 72$

37. 0.4398



Section 4.2

1. $f(0) = f(2) = 0$
f is not differentiable on $(0, 2)$

3. $f'(1) = 0$ **5.** $f'\left(\frac{6 - \sqrt{3}}{3}\right) = 0$
$f'\left(\frac{6 + \sqrt{3}}{3}\right) = 0$

7. Not differentiable at $x = 0$

9. Not differentiable at $x = 0$

11. $f'(-2 + \sqrt{5}) = 0$ **13.** $f'\left(-\frac{1}{2}\right) = -1$

15. $f'\left(\frac{8}{27}\right) = 1$ **17.** $f'\left(\frac{-2 + \sqrt{6}}{2}\right) = \frac{2}{3}$

19. $f'\left(\frac{\sqrt{3}}{3}\right) = 1$ **21.** (a) $f(1) = f(2) = 64$
(b) Velocity $= 0$ for some t

23. (a) -48 ft/sec (b) $t = \frac{3}{2}$ sec

25. $f(x)$ is not continuous on $[2, 6]$

33. (a) $x - 4y + 3 = 0$ (b) $c = 4$, $x - 4y + 4$