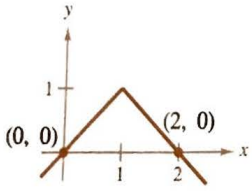
REMARK A useful alternative form of the Mean Value Theorem is as follows: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) such that $f(b) = f(a) + (b - a)f'(c)$.

When working the exercises for this section, keep in mind that polynomial functions and rational functions are differentiable at all points in their domain.

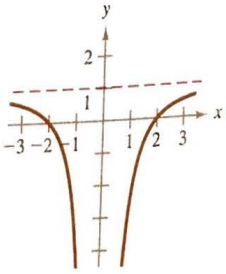
EXERCISES for Section 4.2

In Exercises 1 and 2, state why Rolle's Theorem does not apply to the function even though there exist a and b such that $f(a) = f(b) = 0$.

1. $f(x) = 1 - |x - 1|$



2. $f(x) = \dfrac{x^2 - 4}{x^2}$



In Exercises 3–12, determine whether Rolle's Theorem can be applied to f on the indicated interval. If Rolle's Theorem can be applied, find all values of c in the interval such that $f'(c) = 0$.

Function	*Interval*
3. $f(x) = x^2 - 2x$	$[0, 2]$
4. $f(x) = x^2 - 3x + 2$	$[1, 2]$
5. $f(x) = (x - 1)(x - 2)(x - 3)$	$[1, 3]$
6. $f(x) = (x - 3)(x + 1)^2$	$[-1, 3]$
7. $f(x) = \|x\| - 1$	$[-1, 1]$
8. $f(x) = 3 - \|x - 3\|$	$[0, 6]$
9. $f(x) = x^{2/3} - 1$	$[-8, 8]$
10. $f(x) = x - x^{1/3}$	$[0, 1]$
11. $f(x) = \dfrac{x^2 - 2x - 3}{x + 2}$	$[-1, 3]$
12. $f(x) = \dfrac{x^2 - 1}{x}$	$[-1, 1]$

In Exercises 13–20, apply the Mean Value Theorem to f on the indicated interval. In each case, find all values of c in the interval (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Function	*Interval*
13. $f(x) = x^2$	$[-2, 1]$
14. $f(x) = x(x^2 - x - 2)$	$[-1, 1]$
15. $f(x) = x^{2/3}$	$[0, 1]$
16. $f(x) = \dfrac{x + 1}{x}$	$\left[\dfrac{1}{2}, 2\right]$
17. $f(x) = \dfrac{x}{x + 1}$	$\left[-\dfrac{1}{2}, 2\right]$
18. $f(x) = \sqrt{x - 2}$	$[2, 6]$
19. $f(x) = x^3$	$[0, 1]$
20. $f(x) = x^3 - 2x$	$[0, 2]$

21. The height of a ball t seconds after it is thrown is given by

$$f(t) = -16t^2 + 48t + 32.$$

(a) Verify that $f(1) = f(2)$.

(b) According to Rolle's Theorem, what must be the velocity at some time in the interval $[1, 2]$?

22. The ordering and transportation cost C of components used in a manufacturing process is approximated by

$$C(x) = 10\left(\frac{1}{x} + \frac{x}{x + 3}\right)$$

where C is measured in thousands of dollars and x is the order size in hundreds.

(a) Verify that $C(3) = C(6)$.

(b) According to Rolle's Theorem, the rate of change of cost must be zero for some order size in the interval $[3, 6]$. Find that order size.

23. The height of an object t seconds after it was dropped from a height of 500 feet is given by

$$s(t) = -16t^2 + 500.$$

(a) Find the average velocity of the object during the first 3 seconds.

(b) Use the Mean Value Theorem to verify that at some time during the first three seconds of fall the instantaneous velocity equals the average velocity. Find that time.