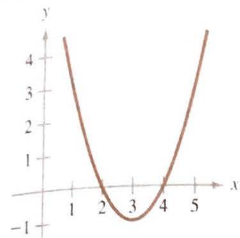
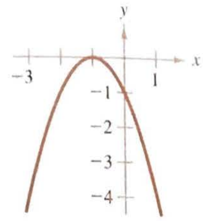
EXERCISES for Section 4.3

In Exercises 1–6, identify the open intervals on which the function is increasing or decreasing.

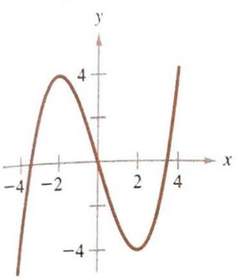
1. $f(x) = x^2 - 6x + 8$



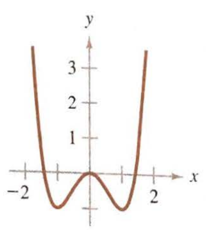
2. $y = -(x + 1)^2$



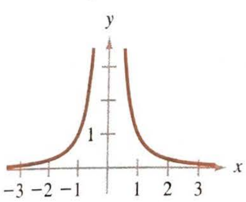
3. $y = \dfrac{x^3}{4} - 3x$



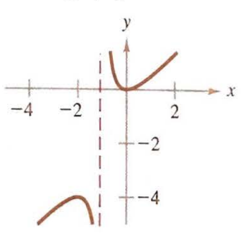
4. $f(x) = x^4 - 2x^2$



5. $f(x) = \dfrac{1}{x^2}$



6. $y = \dfrac{x^2}{x + 1}$



In Exercises 7–18, find the critical numbers of f (if any), find the open intervals on which f is increasing or decreasing, and locate all relative extrema.

7. $f(x) = -2x^2 + 4x + 3$
8. $f(x) = x^2 + 8x + 10$
9. $f(x) = x^2 - 6x$
10. $f(x) = (x - 1)^2(x + 2)$
11. $f(x) = 2x^3 + 3x^2 - 12x$
12. $f(x) = (x - 3)^3$
13. $f(x) = x^{1/3} + 1$
14. $f(x) = x^{2/3}(x - 5)$
15. $f(x) = \dfrac{x^2}{x^2 - 9}$
16. $f(x) = \dfrac{x + 3}{x^2}$
17. $f(x) = \dfrac{x^5 - 5x}{5}$
18. $f(x) = x^4 - 32x + 4$

In Exercises 19–26, find the critical numbers of f (if any), find the open intervals on which the algebraic function is increasing or decreasing, and locate all relative extrema. Use a graphing utility to confirm your results.

19. $f(x) = x^3 - 6x^2 + 15$
20. $f(x) = x^4 - 2x^3$
21. $f(x) = (x - 1)^{2/3}$
22. $f(x) = (x - 1)^{1/3}$
23. $f(x) = x + \dfrac{1}{x}$
24. $f(x) = \dfrac{x}{x + 1}$
25. $f(x) = \dfrac{x^2 - 2x + 1}{x + 1}$
26. $f(x) = \dfrac{x^2 - 3x - 4}{x - 2}$

In Exercises 27 and 28, determine whether the given function is strictly monotonic on the indicated interval.

27. $f(x) = x^2$
    (a) $(-\infty, \infty)$ (b) $(-\infty, 0)$ (c) $(0, \infty)$
28. $f(x) = x^3 - x$
    (a) $(-1, 0)$ (b) $\left(-1, -\frac{1}{2}\right)$ (c) $(-1, 1)$

29. The height (in feet) of a ball at time t (in seconds) is given by the position function

$$s(t) = 96t - 16t^2.$$

Find the open interval on which the ball is moving up and the open interval on which it is moving down. What is the maximum height of the ball?

30. Repeat Exercise 29 using the position function

$$s(t) = -16t^2 + 64t.$$

31. Coughing forces the trachea (windpipe) to contract, which affects the velocity v of the air passing through the trachea. Suppose the velocity of the air during coughing is

$$v = k(R - r)r^2$$

where k is a constant, R is the normal radius of the trachea, and r is the radius during coughing. What radius will produce the maximum air velocity?

32. The concentration C of a certain chemical in the bloodstream t hours after injection into muscle tissue is given by

$$C = \frac{3t}{27 + t^3}.$$

When is the concentration greatest?

33. After a drug is administered to a patient, the drug concentration in the patient's bloodstream over a two-hour period is given by

$$C = 0.29483t + 0.04253t^2 - 0.00035t^3$$

where C is measured in milligrams and t is the time in minutes. Find the open interval on which C is increasing or decreasing.

34. A fast-food restaurant sells x hamburgers to make a profit P given by

$$P = 2.44x - \frac{x^2}{20{,}000} - 5000, \quad 0 \le x \le 35{,}000.$$

Find the open interval on which P is increasing or decreasing.

35. After birth, an infant normally will lose weight for a few days and then start gaining. A model for the average weight W of infants over the first two weeks following birth is

$$W = 0.033t^2 - 0.3974t + 7.3032.$$

Find the open intervals on which W is increasing or decreasing.

36. The electric power P in watts in a direct-current circuit with two resistors R_1 and R_2 connected in series is

$$P = \frac{vR_1R_2}{(R_1 + R_2)^2}$$

where v is the voltage. If v and R_1 are held constant, what resistance R_2 produces maximum power?

37. The resistance R of a certain type of resistor is given by

$$R = \sqrt{0.001T^4 - 4T + 100}$$

where R is measured in ohms and the temperature T is measured in degrees Celsius. What temperature produces a minimum resistance for this type of resistor?

38. Consider the functions $f(x) = x$ and $g(x) = x^3$ on the interval $(0, 1)$.
(a) Prove that $f(x) > g(x)$. [Hint: Show that $h(x) > 0$ where $h = f - g$.]
(b) Sketch the graphs of f and g on the same set of axes.

39. Find a, b, c, and d so that the function given by

$$f(x) = ax^3 + bx^2 + cx + d$$

has a relative minimum at $(0, 0)$ and a relative maximum at $(2, 2)$.

40. Find a, b, and c so that the function given by

$$f(x) = ax^2 + bx + c$$

has a relative maximum at $(5, 20)$ and passes through the point $(2, 10)$.

In Exercises 41–46, assume that f is differentiable for all x. The sign of f' is as follows.

$f'(x) > 0$ on $(-\infty, -4)$
$f'(x) < 0$ on $(-4, 6)$
$f'(x) > 0$ on $(6, \infty)$

In each exercise, supply the appropriate inequality for the indicated value of c.

	Function	*Sign of $g'(c)$*
41.	$g(x) = f(x) + 5$	$g'(0)$ __ 0
42.	$g(x) = 3f(x) - 3$	$g'(-5)$ __ 0
43.	$g(x) = -f(x)$	$g'(-6)$ __ 0
44.	$g(x) = -f(x)$	$g'(0)$ __ 0
45.	$g(x) = f(x - 10)$	$g'(0)$ __ 0
46.	$g(x) = f(x - 10)$	$g'(8)$ __ 0

47. Sketch the graph of a function f such that

$$f'(x) = \begin{cases} > 0, & x < 4 \\ \text{undefined}, & x = 4 \\ < 0 & x > 4 \end{cases}$$

48. A differentiable function f has only one critical number $x = 5$. Identify the relative extrema of f at the critical number if $f'(4) = -2.5$ and $f'(6) = 3$.

In Exercises 49 and 50, use a computer or graphics calculator (a) to sketch the graph of f and f' on the same coordinate axes over the specified interval, (b) to find the critical numbers of f, and (c) to find the interval(s) on which f' is positive and the interval(s) on which it is negative. Note the behavior of f in relation to the sign of f'.

	Function	*Interval*
49.	$f(x) = 2x\sqrt{9 - x^2}$	$[-3, 3]$
50.	$f(x) = 10(5 - \sqrt{x^2 - 3x + 16})$	$[0, 5]$

51. Prove the second case of Theorem 4.5.

52. Prove the second and third cases of Theorem 4.6.

4.4 Concavity and the Second Derivative Test

Concavity ▪ Points of inflection ▪ The Second Derivative Test

We have already seen that locating the intervals in which a function f incr[eases] or decreases helps to determine its graph. In this section we show th[at] locating the intervals in which f' increases or decreases, we can dete[rmine] where the graph of f is *curving upward* or *curving downward*. We r[efer to] this notion of curving u[pward] ...

Section 4.3

1. Increasing on $(3, \infty)$
Decreasing on $(-\infty, 3)$

3. Increasing on $(-\infty, -2)$ and $(2, \infty)$
Decreasing on $(-2, 2)$

5. Increasing on $(-\infty, 0)$
Decreasing on $(0, \infty)$

7. Critical number: $x = 1$
Increasing on $(-\infty, 1)$
Decreasing on $(1, \infty)$
Relative maximum: $(1, 5)$

9. Critical number: $x = 3$
Increasing on $(3, \infty)$
Decreasing on $(-\infty, 3)$
Relative minimum: $(3, -9)$

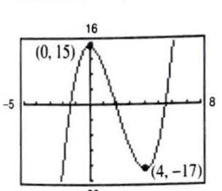
11. Critical numbers: $x = -2, 1$
Increasing on $(-\infty, -2)$ and $(1, \infty)$
Decreasing on $(-2, 1)$
Relative maximum: $(-2, 20)$
Relative minimum: $(1, -7)$

13. Critical number: $x = 0$
Increasing on $(-\infty, \infty)$
No relative extrema

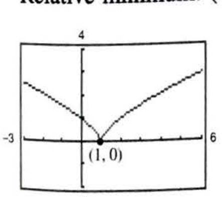
15. Critical number: $x = 0$
Discontinuities: $x = -3, 3$
Increasing on $(-\infty, -3)$ and $(-3, 0)$
Decreasing on $(0, 3)$ and $(3, \infty)$
Relative maximum: $(0, 0)$

17. Critical numbers: $x = -1, 1$
Increasing on $(-\infty, -1)$ and $(1, \infty)$
Decreasing on $(-1, 1)$
Relative maximum: $\left(-1, \frac{4}{5}\right)$
Relative minimum: $\left(1, -\frac{4}{5}\right)$

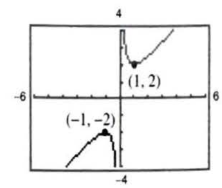
19. Critical numbers: $x = 0, 4$
Increasing on $(-\infty, 0)$ and $(4, \infty)$
Decreasing on $(0, 4)$
Relative maximum: $(0, 15)$
Relative minimum: $(4, -17)$



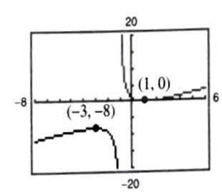
21. Critical number: $x = 1$
Increasing on $(1, \infty)$
Decreasing on $(-\infty, 1)$
Relative minimum: $(1, 0)$



23. Critical numbers: $x = -1, 1$
Discontinuity: $x = 0$
Increasing on $(-\infty, -1)$ and $(1, \infty)$
Decreasing on $(-1, 0)$ and $(0, 1)$
Relative maximum: $(-1, -2)$
Relative minimum: $(1, 2)$



25. Critical numbers: $x = -3, 1$
Discontinuity: $x = -1$
Increasing on $(-\infty, -3)$ and $(1, \infty)$
Decreasing on $(-3, -1)$ and $(-1, 1)$
Relative maximum: $(-3, -8)$
Relative minimum: $(1, 0)$



27. **(a)** Not monotonic
(b) Strictly monotonic
(c) Strictly monotonic

29. Moving upward when $0 < t < 3$
Moving downward when $3 < t < 6$
Maximum height: $s(3) = 144$ ft

31. $r = \dfrac{2R}{3}$

33. Increasing when $0 < t < 84.3388$ minutes
Decreasing when $84.3388 < t < 120$ minutes

35. Increasing when $6.02 < t < 14$ days
Decreasing when $0 < t < 6.02$ days

37. $T = 10°$ **39.** $f(x) = -\frac{1}{2}x^3 + \frac{3}{2}x^2$

41. $g'(0) < 0$ **43.** $g'(-6) < 0$ **45.** $g'(0) > 0$

47.