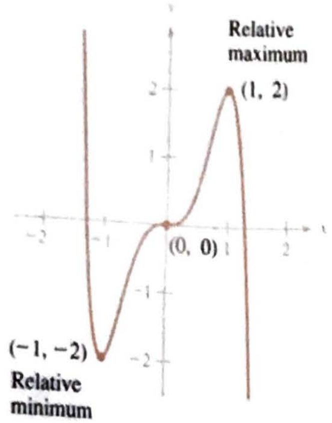


$f(x) = -3x^5 + 5x^3$

FIGURE 4.28

SOLUTION

We begin by finding the critical numbers of f.

$$f'(x) = -15x^4 + 15x^2 = 15x^2(1 - x^2) = 0$$

$$x = -1, 0, 1 \qquad \text{Critical numbers}$$

Using $f''(x) = 30(-2x^3 + x)$, we apply the Second Derivative Test as follo

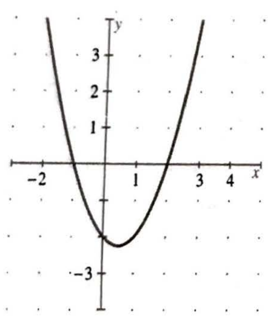
Point	*Sign of f''*	*Conclusion*
$(-1, -2)$	$f''(-1) = 30 > 0$	Relative minimum
$(1, 2)$	$f''(1) = -30 < 0$	Relative maximum
$(0, 0)$	$f''(0) = 0$	Test fails

Since the Second Derivative Test fails at $(0, 0)$, we use the First Derivat Test and observe that f increases to the left and right of $x = 0$. Thus, $(0,$ is neither a relative minimum nor a relative maximum (even though the gra has a horizontal tangent line at this point). The graph of f is shown in Figu 4.28.

EXERCISES for Section 4.4

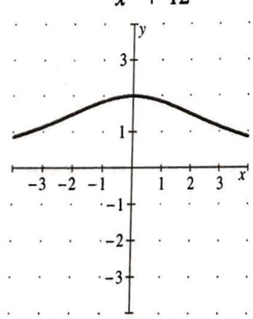
In Exercises 1–6, find the open intervals on which the graph of the given function is concave upward and those on which it is concave downward.

1. $y = x^2 - x - 2$



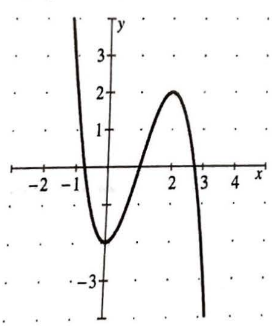
Generated by Derive

2. $f(x) = \dfrac{24}{x^2 + 12}$



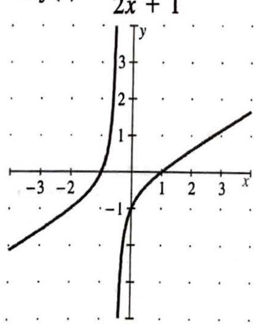
Generated by Derive

3. $y = -x^3 + 3x^2 - 2$



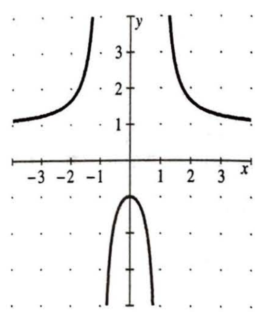
Generated by Derive

4. $f(x) = \dfrac{x^2 - 1}{2x + 1}$



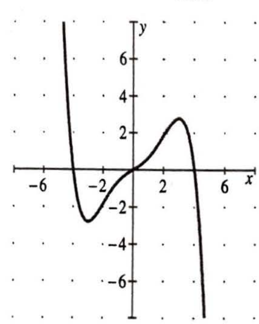
Generated by Derive

5. $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$



Generated by Derive

6. $y = \dfrac{-3x^5 + 40x^3 + 135x}{270}$



Generated by Derive

In Exercises 7–18, identify all relative extrema. Us Second Derivative Test where applicable.

7. $f(x) = 6x - x^2$

8. $f(x) = x^2 + 3x -$

9. $f(x) = (x - 5)^2$

10. $f(x) = -(x - 5)^2$

11. $f(x) = x^3 - 3x^2 + 3$ **12.** $f(x) = 5 + 3x^2 - x^3$
13. $f(x) = x^4 - 4x^3 + 2$
14. $f(x) = x^3 - 9x^2 + 27x - 26$
15. $f(x) = x^{2/3} - 3$ **16.** $f(x) = \sqrt{x^2 + 1}$
17. $f(x) = x + \dfrac{4}{x}$ **18.** $f(x) = \dfrac{x}{x - 1}$

In Exercises 19–34, sketch the graph of the given function and identify all relative extrema and points of inflection.

19. $f(x) = x^3 - 12x$ **20.** $f(x) = x^3 + 1$
21. $f(x) = x^3 - 6x^2 + 12x - 8$
22. $f(x) = 2x^3 - 3x^2 - 12x + 8$
23. $f(x) = \dfrac{1}{4}x^4 - 2x^2$ **24.** $f(x) = 2x^4 - 8x + 3$
25. $f(x) = x(x - 4)^3$ **26.** $f(x) = x^3(x - 4)$
27. $f(x) = x^2 + \dfrac{1}{x^2}$ **28.** $f(x) = \dfrac{x^2}{x^2 - 1}$
29. $f(x) = x\sqrt{x + 3}$ **30.** $f(x) = x\sqrt{x + 1}$
31. $f(x) = \dfrac{x}{x^2 - 4}$ **32.** $f(x) = \dfrac{1}{x^2 - x - 2}$
33. $f(x) = \dfrac{x - 2}{x^2 - 4x + 3}$ **34.** $f(x) = \dfrac{x + 1}{x^2 + x + 1}$

In Exercises 35 and 36, use a symbolic differentiation utility to analyze the function over the specified interval. (a) Find the first- and second-order derivatives of the function, (b) find any relative extrema and points of inflection, and (c) sketch the graphs of f, f', and f'' on the same coordinate axes and state the relationship between the behavior of f and the signs of f' and f''.

Function	*Interval*
35. $f(x) = 0.2x^2(x - 3)^3$	$[-1, 4]$
36. $f(x) = x^2\sqrt{6 - x^2}$	$[-\sqrt{6}, \sqrt{6}]$

37. Sketch the graph of a function f having the following characteristics.

(a) $f(2) = f(4) = 0$
$f'(x) < 0$ if $x < 3$
$f'(3)$ is undefined
$f'(x) > 0$ if $x > 3$
$f''(x) < 0, x \neq 3$

(b) $f(0) = f(2) = 0$
$f'(x) > 0$ if $x < 1$
$f'(1) = 0$
$f'(x) < 0$ if $x > 1$
$f''(x) < 0$

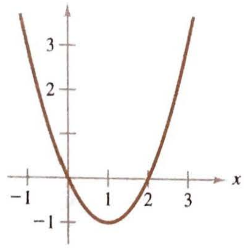
38. Sketch the graph of a function f having the following characteristics.

(a) $f(2) = f(4) = 0$
$f'(x) > 0$ if $x < 3$
$f'(3)$ is undefined
$f'(x) < 0$ if $x > 3$
$f''(x) > 0, x \neq 3$

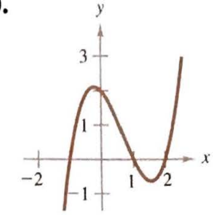
(b) $f(0) = f(2) = 0$
$f'(x) < 0$ if $x < 1$
$f'(1) = 0$
$f'(x) > 0$ if $x > 1$
$f''(x) > 0$

In Exercises 39 and 40, trace the given graph of f. On the same set of axes sketch the graph of f' and f''.

39.



40.



41. Find the inflection points (if any) and sketch the graph of $f(x) = (x - c)^n$ for $n = 1, 2, 3,$ and 4. What conclusion can be made about the value of n and the existence of an inflection point on the graph of f?

42. (a) Sketch the graph of $f(x) = \sqrt[3]{x}$ and identify the inflection point.
(b) Does $f''(x)$ exist at the inflection point?

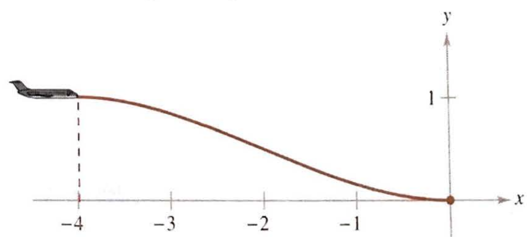
43. Show that the point of inflection of

$$f(x) = x(x - 6)^2$$

lies midway between the relative extrema of f.

44. Prove that a cubic function with three distinct real zeros has a point of inflection whose x-coordinate is the average of the three zeros.

45. A small aircraft starts its descent from an altitude of 1 mile, 4 miles west of the runway (see figure).
(a) Find the cubic polynomial function $f(x) = ax^3 + bx^2 + cx + d$ on the interval $[-4, 0]$, which describes a smooth glide path for the landing.
(b) If the glide path of the plane is described by the function of part (a), when would the plane be descending at the greatest rate?



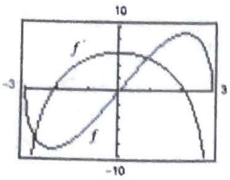
(For more information on this type of modeling, see the article "How Not to Land at Lake Tahoe!" by Richard Barshinger in the May, 1992, issue of *The American Mathematical Monthly*.)

46. The equation

$$E = \frac{kqx}{(x^2 + a^2)^{3/2}}$$

gives the electric field intensity on the axis of a uniformly charged ring, where q is the total charge on the ring, k is a constant, and a is the radius of the ring. At what value of x is E maximum?

49. (a)



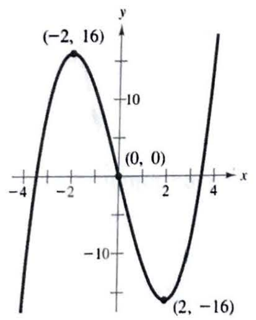
(b) Critical numbers: $x = \pm\dfrac{3\sqrt{2}}{2}$

(c) $f' > (0)$ on $\left(-\dfrac{3\sqrt{2}}{2}, \dfrac{3\sqrt{2}}{2}\right)$

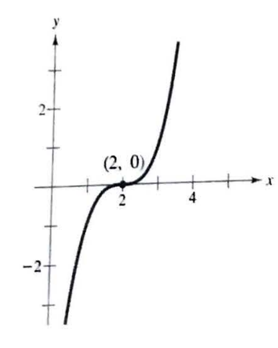
$f' < (0)$ on $\left(-3, -\dfrac{3\sqrt{2}}{2}\right), \left(\dfrac{3\sqrt{2}}{2}, 3\right)$

Section 4.4

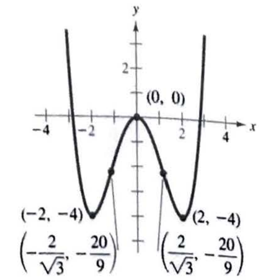
1. Concave upward: $(-\infty, \infty)$
3. Concave upward: $(-\infty, 1)$
Concave downward: $(1, \infty)$
5. Concave upward: $(-\infty, -1), (1, \infty)$
Concave downward: $(-1, 1)$
7. Relative maximum: $(3, 9)$
9. Relative minimum: $(5, 0)$
11. Relative maximum: $(0, 3)$
Relative minimum: $(2, -1)$
13. Relative minimum: $(3, -25)$
15. Relative minimum: $(0, -3)$
17. Relative maximum: $(-2, -4)$
Relative minimum: $(2, 4)$
19. Relative maximum: $(-2, 16)$
Relative minimum: $(2, -16)$
Point of inflection: $(0, 0)$



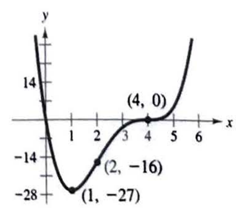
21. Point of inflection: $(2, 0)$



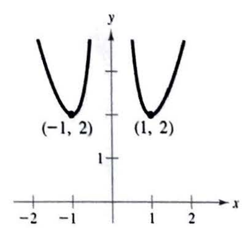
23. Relative minima: $(\pm 2, -4)$
Relative maximum: $(0, 0)$
Points of inflection: $\left(\pm\dfrac{2}{\sqrt{3}}, -\dfrac{20}{9}\right)$



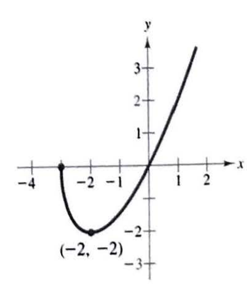
25. Relative minimum: $(1, -27)$
Points of inflection: $(2, -16), (4, 0)$



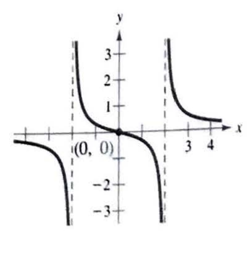
27. Relative minima: $(-1, 2), (1, 2)$



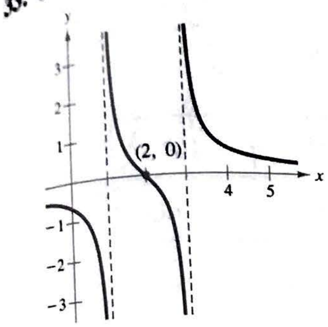
29. Relative minimum: $(-2, -2)$



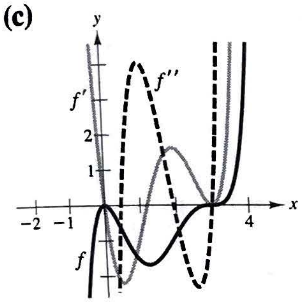
31. Point of inflection: $(0, 0)$



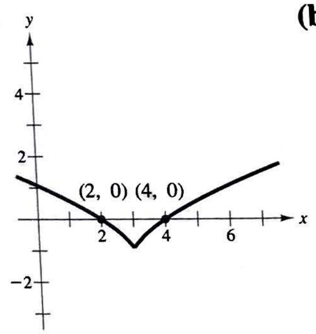
33. Point of inflection: $(2, 0)$



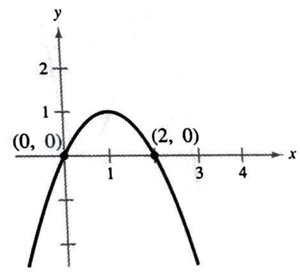
35. (a) $f'(x) = 0.2x(x - 3)^2(5x - 6)$
$f''(x) = 0.4(x - 3)(10x^2 - 24x + 9)$
(b) Relative maximum: $(0, 0)$
Relative minimum: $(1.2, -1.6796)$
Points of inflection: $(0.4652, -0.7049)$, $(1.9348, -0.9049$, $(3, 0)$
(c)



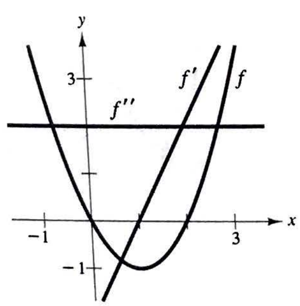
37. (a)



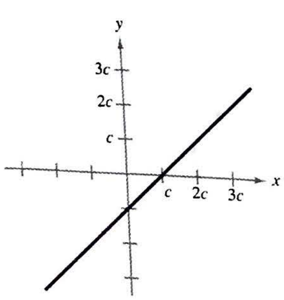
(b)

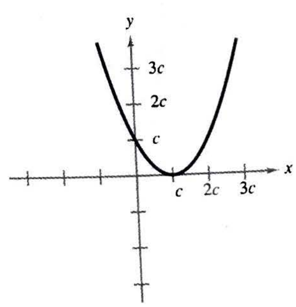


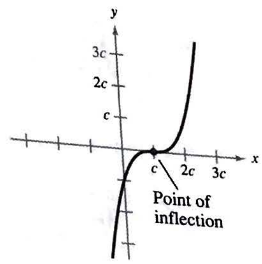
39.

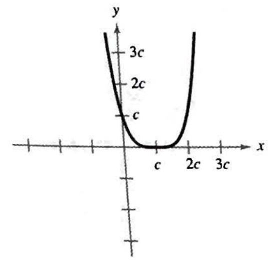


41. $f(x) = (x - c)^n$ has a point of inflection at $(c, 0)$ if n is odd and $n \geq 3$.





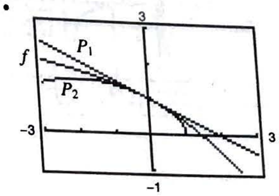




43. Relative extrema: $(2, 32)$, $(6, 0)$
Point of inflection: $(4, 16)$

45. (a) $f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$
(b) Two miles from touchdown

47.



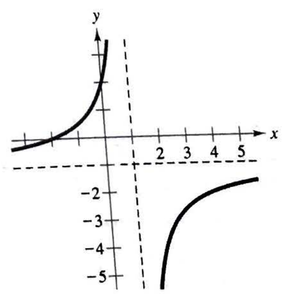
The values of f, P_1, and P_2, and their first derivatives, are equal at $x = 0$.

49. (a) $S'' > 0$
(b) $S'' < 0$
(c) $S' = C$, $S'' = 0$
(d) $S' = 0$, $S'' = 0$
(e) $S' < 0$, $S'' > 0$
(f) $S' > 0$

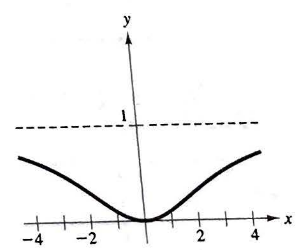
Section 4.5

1. h **2.** c **3.** e **4.** a **5.** d **6.** g
7. b **8.** f **9.** $\frac{2}{3}$ **11.** 0
13. Limit does not exist.
15. Limit does not exist. **17.** 5 **19.** -1
21. 2 **23.** 1 **25.** 0 **27.** $-\frac{1}{2}$

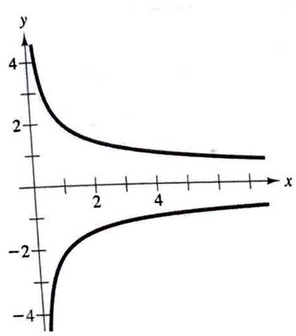
29.



31.



33.



35.