Math 15, Dr. Sturm

Summary of Graphing Techniques (chapters 11 - 15)

We graph $y = f(x)$ by obtaining information from $f(x), f'(x), and\ f''(x)$,

Information from $f(x)$:

1. $y - intercept, put\ x = 0$; $x - intercepts,\ put\ y = 0$.
2. End behavior: $\lim_{x\to\infty} \frac{ax^n+\cdots}{bx^m+\cdots} = \lim_{x\to\infty} \frac{ax^n}{bx^m}$

   $\lim_{x\to\infty} x^{odd} = \infty$, $\lim_{x\to-\infty} x^{odd} = -\infty$; $\lim_{x\to\infty} x^{even} = \infty$, $\lim_{x\to-\infty} x^{even} = \infty$;

   $\lim_{x\to\infty} \frac{1}{x^n} = 0, n \geq 1$; $\lim_{x\to\infty} \frac{2x}{x} = 2$;
3. Find the vertical asymptotes of $f(x)$ by setting the denominator equal to zero.

Information from $f'(x)$:

$\pm analysis\ of\ f'(x) -$ place zeros of f' and places where f' DNE on the number line.

$f' > 0 \ \rightarrow f\ increasing$; $f' < 0 \ \rightarrow \ f\ decreasing$

The first derivative test determines if $(c, f(c))$, is a relative maximum, relative minimum, or neither: If the derivative of $f(x)$ changes sign on either side of $x = c$, and if $f(c)$ exists, then $(c, f(c))$ is a relative max (pos on the left, neg on the right) or relative min (neg on the right , pos on the left).

Information from $f''(x)$:

$\pm analysis\ of\ f''(x) -$ place zeros of f'' and places where f'' DNE on the number line.

$f'' > 0 \ \rightarrow f\ concave\ up$; $f'' < 0 \ \rightarrow \ f\ concave\ down$

Inflection points occur where $f(x)$ is continuous and $f(x)$ changes from concave up to concave down, or from concave down to concave up.

(Note: Often places where $f' or\ f''\ DNE$ are zeros of the denominator.)