REMARK What would the answer be if Example 5 asked for the dimensions needed to enclose the *minimum* total area?

The feasible domain is $0 \le x \le 1$. Since $dA/dx = [2(\pi + 4)x - 8]/\pi$, the only critical number is $x = 4/(\pi + 4) \approx 0.56$. Therefore, since

$$A(0) \approx 1.273, \qquad A(0.56) \approx 0.56, \qquad \text{and} \qquad A(1) = 1$$

we conclude that the maximum area occurs when $x = 0$. That is, *all* the wire is used for the circle. ▭

Let's review the primary equations developed in the first five examples. As applications go, these five examples are fairly simple, and yet the resulting primary equations are quite complicated.

$$V = 27x - \frac{x^3}{4} \qquad W = \sqrt{x^2 + 144} + \sqrt{x^2 - 60x + 1684}$$

$$d = \sqrt{x^4 - 3x^2 + 4} \qquad A = \frac{1}{\pi}[(\pi + 4)x^2 - 8x + 4]$$

$$A = 30 + 2x + \frac{72}{x}$$

You must expect that real applications often involve equations that are at least as complicated as these five. Remember that one of the main goals of this course is to learn to use calculus to analyze equations that initially seem formidable.

EXERCISES for Section 4.7

1. Find two positive numbers whose sum is 110 and whose product is a maximum.
   (a) Use paper and pencil to complete six rows of a table like the one below. (The first two rows are shown.) Use the result to guess the two numbers required.

First number, x	*Second number*	*Product, P*
10	$110 - 10$	$10(110 - 10) = 1000$
20	$110 - 20$	$20(110 - 20) = 1800$

   (b) Write the product P as a function of x.
   (c) Use calculus to find the critical number of the function of part (b). Then find the two numbers.
   (d) Use a computer to generate additional rows of the table. Use the table to estimate the solution.
   (e) Use a graphing utility to graph the function in part (b) and estimate the solution from the graph.

In Exercises 2–6, find two positive numbers that satisfy the given requirements.

2. The sum is S and the product is maximum.
3. The product is 192 and the sum is minimum.
4. The product is 192 and the sum of the first plus three times the second is minimum.
5. The second number is the reciprocal of the first and their sum is minimum.
6. The sum of the first and twice the second is 100 and the product is maximum.

In Exercises 7 and 8, find the length and width of a rectangle of maximum area for the given perimeter.

7. Perimeter: 100 feet
8. Perimeter: P units

In Exercises 9 and 10, find the length and width of a rectangle of minimum perimeter for the given area.

9. Area: 64 square feet
10. Area: A square feet

In Exercises 11 and 12, find the point on the graph of the function closest to the given point.

	Function	*Point*
11.	$f(x) = \sqrt{x}$	$(4, 0)$
12.	$f(x) = x^2$	$\left(2, \frac{1}{2}\right)$

13. A dairy farmer plans to fence in a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

14. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?

15. An open box is to be made from a square piece of material, 12 inches on a side, by cutting equal squares from each corner and turning up the sides (see figure). Find the volume of the largest box that can be made.

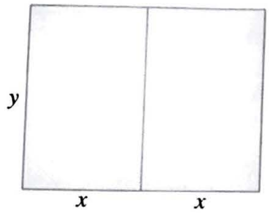


FIGURE FOR 14

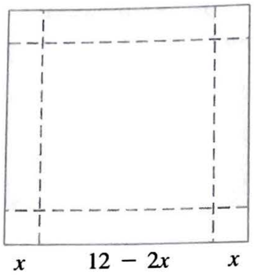


FIGURE FOR 15

16. (a) Solve Exercise 15 given that the square of material is s inches on a side.
    (b) If the dimensions of the square piece of material are doubled, how does the volume change?

17. An open box is to be made from a rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the dimensions of the box of maximum volume if the material has dimensions of 2 feet by 3 feet.

18. A net enclosure for practicing golf is open at one end (see figure). Find the dimensions that require the least amount of netting if the volume of the enclosure is to be $83\frac{1}{3}$ cubic meters.

19. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.

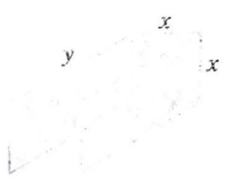


FIGURE FOR 18

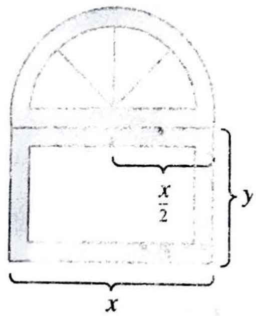


FIGURE FOR 19

20. A physical fitness room consists of a rectangular region with a semicircle on each end. If the perimeter of the room is a 200-meter track, find the dimensions that make the rectangular region as large as possible.

21. A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point (2, 3). Find the vertices of the triangle so that its area is minimum.

22. Find the dimensions of the largest isosceles triangle that can be inscribed in a circle of radius 4 (see figure).

23. A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$ (see figure). What length and width should the rectangle have so that its area is a maximum?

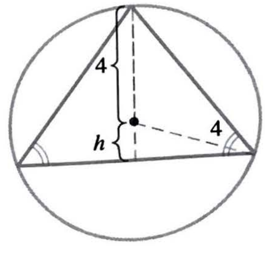


FIGURE FOR 22

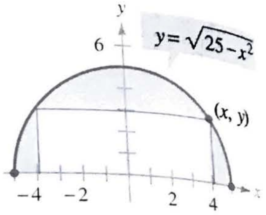


FIGURE FOR 23

24. Find the dimensions of the largest rectangle that can be inscribed in a semicircle of radius r (see Exercise 23).

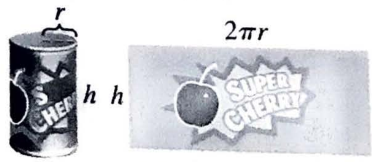
25. Find the dimensions of the trapezoid of greatest area that can be inscribed in a semicircle of radius r.

26. A page is to contain 30 square inches of print. The margins at the top and bottom of the page are 2 inches. The margins on each side are only 1 inch. Find the dimensions of the page so that the least paper is used.

27. A right circular cylinder is to be designed to hold 22 cubic inches of soft drink (approximately 12 fluid ounces) and to use a minimum of material in its construction (see figure).
    (a) Use paper and pencil to complete four rows of a table like the one below. (The first two rows are shown.) Use the result to guess the minimum amount of material.

Radius	*Height*	*Surface area*
0.2	$\dfrac{22}{\pi(0.2)^2}$	$2\pi(0.2)\left[0.2 + \dfrac{22}{\pi(0.2)^2}\right] \approx 220.3$
0.4	$\dfrac{22}{\pi(0.4)^2}$	$2\pi(0.4)\left[0.4 + \dfrac{22}{\pi(0.4)^2}\right] \approx 111.1$

    (b) Write the surface area S as a function of radius r.
    (c) Use calculus to find the critical number of the function in part (b) and find the dimensions that will yield the minimum surface area.
    (d) Use a computer to generate additional rows of the table in part (a) and estimate the minimum surface area from the table.
    (e) Use a graphing utility to obtain the graph of the function in part (b) and estimate the required dimensions from the graph.



Lateral side of cylinder (unrolled)

FIGURE FOR 27

28. Use calculus to find the dimensions of the cylinder in Exercise 27 if its volume is V_0 cubic units.
29. A rectangular package to be sent by a postal service can have a maximum combined length and girth (perimeter of a cross section) of 108 inches (see figure). Find the dimensions of the package of maximum volume that can be sent. (Assume the cross section is square.)
30. Rework Exercise 29 for a cylindrical package. (The cross sections are circular.)
31. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius r (see figure).

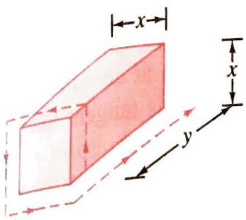


FIGURE FOR 29

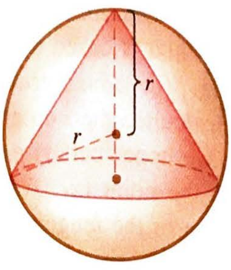


FIGURE FOR 31

32. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius r.
33. A solid is formed by adjoining two hemispheres to each end of a right circular cylinder. The total volume of the figure is 12 cubic inches. Find the radius of the cylinder that produces the minimum surface area.
34. An industrial tank of the shape described in Exercise 33 must have a volume of 3000 cubic feet. If the construction cost of the hemispherical ends is twice as much per square foot of surface area as the sides, find the dimensions that will minimize cost.
35. The combined perimeter of an equilateral triangle and a square is 10. Find the dimensions of the triangle and square that produce a minimum total area.
36. The combined perimeter of a circle and a square is 16. Find the dimensions of the circle and square that produce a minimum total area.
37. Ten feet of wire is to be used to form an isosceles right triangle and a circle. How much of the wire should be used for the circle if the total area enclosed is to be (a) minimum and (b) maximum?
38. Twenty feet of wire is to be used to form two figures. In each of the following cases, how much should be used for each figure so that the total enclosed area is a maximum?
    (a) equilateral triangle and square
    (b) square and regular pentagon
    (c) regular pentagon and hexagon
39. A wooden beam has a rectangular cross section of height h and width w (see figure). The strength S of the beam is directly proportional to the width and the square of the height. What are the dimensions of the strongest beam that can be cut from a round log of diameter 24 inches? [Hint: $S = kh^2w$, where k is the proportionality constant.]
40. The illumination from a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. Two light sources of intensities I_1 and I_2 are d units apart. At what point on the line segment joining the two sources is the illumination least?
41. A man is in a boat 2 miles from the nearest point on the coast. He is to go to a point Q, 3 miles down the coast and 1 mile inland (see figure). If he can row at 2 miles per hour and walk at 4 miles per hour, toward what point on the coast should he row in order to reach point Q in the least time?
42. The conditions are the same as in Exercise 41 except that the man can row at 4 miles per hour (see figure). How does this change the solution?

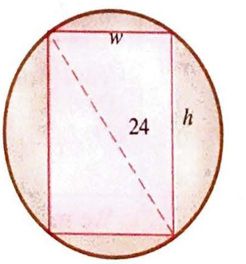


FIGURE FOR 39

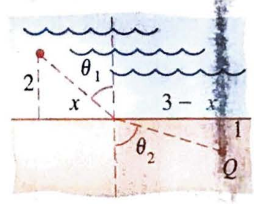


FIGURE FOR 41–42

In Exercises 43 and 44, use a computer or graphics calculator to sketch the graphs of the primary equation and its first derivative in the given applied extrema problem. From the graphs, find the required extrema.

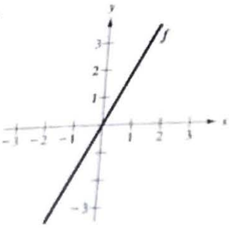
43. Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius 10.
44. Find the length of the longest pipe that can be carried level around a right-angle corner if the two intersecting corridors are of width 5 feet and 8 feet, respectively.

4.8 Newton's Method

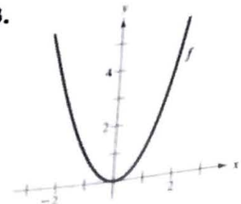
Newton's Method ▪ Algebraic solutions of polynomial equations

In the first seven sections of this chapter, we frequently needed to find the zeros of a function. Until now our functions have been chosen carefully so that elementary algebraic techniques suffice for finding the zeros. For instance,

51.



53.



Section 4.7

1. (a)

First number, x	Second number	Product, P
10	$110 - 10$	$10(110 - 10) = 1000$
20	$110 - 20$	$20(110 - 20) = 1800$
30	$110 - 30$	$30(110 - 30) = 2400$
40	$110 - 40$	$40(110 - 40) = 2800$
50	$110 - 50$	$50(110 - 50) = 3000$
60	$110 - 60$	$60(110 - 60) = 3000$

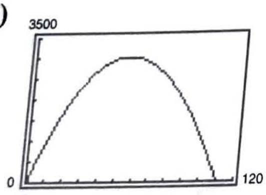
(b) $P = x(110 - x)$

(c) 55 and 55

(d)

First number, x	Second number	Product, P
10	$110 - 10$	$10(110 - 10) = 1000$
20	$110 - 20$	$20(110 - 20) = 1800$
30	$110 - 30$	$30(110 - 30) = 2400$
40	$110 - 40$	$40(110 - 40) = 2800$
50	$110 - 50$	$50(110 - 50) = 3000$
60	$110 - 60$	$60(110 - 60) = 3000$
70	$110 - 70$	$70(110 - 70) = 2800$
80	$110 - 80$	$80(110 - 80) = 2400$
90	$110 - 90$	$90(110 - 90) = 1800$
100	$110 - 100$	$100(110 - 100) = 1000$

(e)



3. $\sqrt{192}$ and $\sqrt{192}$ **5.** 1 and 1

7. $l = w = 25$ ft **9.** $l = w = 8$ ft

11. $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$ **13.** $600m \times 300m$

15. $V = 128$ when $x = 2$

17. $\frac{5 - \sqrt{7}}{6}$ ft $\times \frac{1 + \sqrt{7}}{3}$ ft $\times \frac{4 + \sqrt{7}}{3}$ ft

19. Rectangular portion: $\frac{16}{\pi + 4}$ ft $\times \frac{32}{\pi + 4}$ ft

21. $(0, 0)$, $(4, 0)$, $(0, 6)$

23. Length: $\frac{5\sqrt{2}}{2}$; width: $5\sqrt{2}$

25. Bases: r and $2r$; altitude: $\frac{\sqrt{3}r}{2}$

27. (a)

Radius, r	Height	Surface area, S
0.2	$\frac{22}{\pi(0.2)^2}$	$2\pi(0.2)\left[0.2 + \frac{22}{\pi(0.2)^2}\right] \approx 220.3$
0.4	$\frac{22}{\pi(0.4)^2}$	$2\pi(0.4)\left[0.4 + \frac{22}{\pi(0.4)^2}\right] \approx 111.0$
0.6	$\frac{22}{\pi(0.6)^2}$	$2\pi(0.6)\left[0.6 + \frac{22}{\pi(0.6)^2}\right] \approx 75.6$
0.8	$\frac{22}{\pi(0.8)^2}$	$2\pi(0.8)\left[0.8 + \frac{22}{\pi(0.8)^2}\right] \approx 59.0$

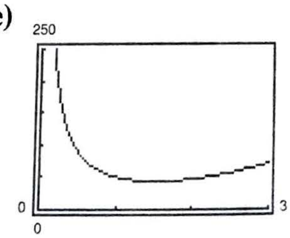
(b) $S = 2\pi r\left(r + \frac{22}{\pi r^2}\right)$

(c) $r = \sqrt[3]{\frac{11}{\pi}}$, $h = 2r$

(d)

Radius, r	Height	Surface area, S
0.2	$\frac{22}{\pi(0.2)^2}$	$2\pi(0.2)\left[0.2 + \frac{22}{\pi(0.2)^2}\right] \approx 220.3$
0.4	$\frac{22}{\pi(0.4)^2}$	$2\pi(0.4)\left[0.4 + \frac{22}{\pi(0.4)^2}\right] \approx 111.0$
0.6	$\frac{22}{\pi(0.6)^2}$	$2\pi(0.6)\left[0.6 + \frac{22}{\pi(0.6)^2}\right] \approx 75.6$
0.8	$\frac{22}{\pi(0.8)^2}$	$2\pi(0.8)\left[0.8 + \frac{22}{\pi(0.8)^2}\right] \approx 59.0$
1.0	$\frac{22}{\pi(1.0)^2}$	$2\pi(1.0)\left[1.0 + \frac{22}{\pi(1.0)^2}\right] \approx 50.3$
1.2	$\frac{22}{\pi(1.2)^2}$	$2\pi(1.2)\left[1.2 + \frac{22}{\pi(1.2)^2}\right] \approx 45.7$
1.4	$\frac{22}{\pi(1.4)^2}$	$2\pi(1.4)\left[1.4 + \frac{22}{\pi(1.4)^2}\right] \approx 43.7$
1.6	$\frac{22}{\pi(1.6)^2}$	$2\pi(1.6)\left[1.6 + \frac{22}{\pi(1.6)^2}\right] \approx 43.6$
1.8	$\frac{22}{\pi(1.8)^2}$	$2\pi(1.8)\left[1.8 + \frac{22}{\pi(1.8)^2}\right] \approx 44.8$
2.0	$\frac{22}{\pi(2.0)^2}$	$2\pi(2.0)\left[2.0 + \frac{22}{\pi(2.0)^2}\right] \approx 47.1$

(e)



29. 18 in. $\times$ 18 in. $\times$ 36 in. **31.** $\frac{32\pi r^3}{81}$

33. $r = \sqrt[3]{\frac{9}{\pi}} \approx 1.42$ in.

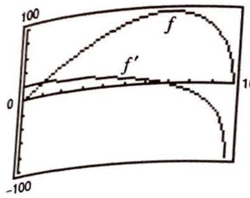
35. Side of square: $\frac{10\sqrt{3}}{9 + 4\sqrt{3}}$; side of triangle: $\frac{30}{9 + 4\sqrt{3}}$

37. (a) $\dfrac{10\pi}{\pi + 3 + 2\sqrt{2}} \approx 3.5$ ft **(b)** 10 ft

39. $w = 8\sqrt{3}$, $h = 8\sqrt{6}$

41. 1 mile from the nearest point on the coast

43. 14.1421×7.071



Section 4.8

1.

n	x_n	$f(x_n)$	$f'(x_n)$	$\dfrac{f(x_n)}{f'(x_n)}$	$x_n - \dfrac{f(x_n)}{f'(x_n)}$
1	1.700	−0.110	3.400	−0.032	1.732

3.

n	x_n	$f(x_n)$	$f'(x_n)$	$\dfrac{f(x_n)}{f'(x_n)}$	$x_n - \dfrac{f(x_n)}{f'(x_n)}$
1	1	1.000	9.000	0.111	0.889

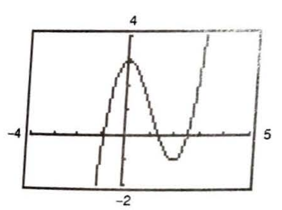
5. 0.682 **7.** 1.146 **9.** 3.317 **11.** 0.569

13. $f'(x_1) = 0$ **15.** $1 = x_1 = x_3 = \ldots$
$0 = x_2 = x_4 = \ldots$

17. $x_{i+1} = \dfrac{x_i^2 + a}{2x_i}$ **19.** 2.646 **21.** 1.565

25. (1.939, 0.240) **27.** $x \approx 1.563$ mi

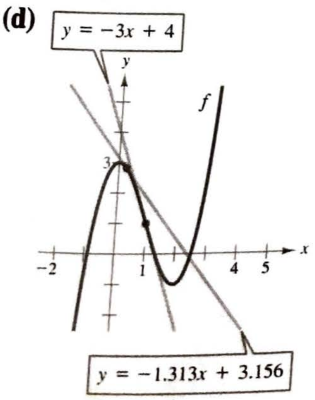
29. (a)



(b) 1.333

(c) 2.405

(d)



(e) If the initial estimate $x = x_1$ is not sufficiently close to the desired zero of a function, the x-intercept of the corresponding tangent line to the function may approximate a second zero of the function.

Section 4.9

1. $6x\,dx$ **3.** $12x^2\,dx$ **5.** $-\dfrac{3}{(2x-1)^2}\,dx$

7. $\dfrac{1}{2\sqrt{x}}\,dx$ **9.** $\dfrac{1-2x^2}{\sqrt{1-x^2}}\,dx$

11.

$dx = \Delta x$	dy	Δy	$\Delta y - dy$	$\dfrac{dy}{\Delta y}$
1.000	1.000	1.000	0.000	1.000
0.500	0.500	0.500	0.000	1.000
0.100	0.100	0.100	0.000	1.000
0.010	0.010	0.010	0.000	1.000
0.001	0.001	0.001	0.000	1.000

13.

$dx = \Delta x$	dy	Δy	$\Delta y - dy$	$\dfrac{dy}{\Delta y}$
1.000	4.000	5.000	1.000	0.800
0.500	2.000	2.250	0.250	0.889
0.100	0.400	0.410	0.010	0.976
0.010	0.040	0.040	0.000	1.000
0.001	0.004	0.004	0.000	1.000

15.

$dx = \Delta x$	dy	Δy	$\Delta y - dy$	$\dfrac{dy}{\Delta y}$
1.000	80.000	211.000	131.000	0.379
0.500	40.000	65.656	25.656	0.609
0.100	8.000	8.841	0.841	0.905
0.010	0.800	0.808	0.008	0.990
0.001	0.080	0.080	0.000	1.000