

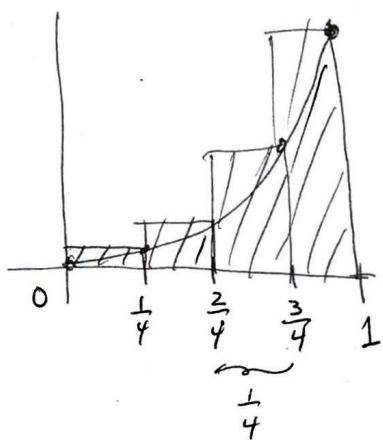
Honors Problem ! Fundamental Thm

① Find the area under $f(x) = x^2$ from $x=0$ to $x=1$ two ways: using calculus ($\int_0^1 x^2 dx$) and without calculus.

Without calc

use this formula

$$\sum_{i=1}^n i^2 = 1 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$



$$\begin{aligned} \text{Area} &\approx \left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right) + \left(\frac{2}{4}\right)^2 \left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right) \\ &\quad + \left(\frac{4}{4}\right)^2 \left(\frac{1}{4}\right) \\ &= \frac{1}{4} \cdot \frac{1}{4^2} (1^2 + 2^2 + 3^2 + 4^2) \end{aligned}$$

Now divide into n rectangles, use the formula to get an expression for the sum of the areas of the n rectangles. Take the limit as $n \rightarrow \infty$.