

1. Find the point on the graph of  $y = \sqrt{x+2}$  closest to  $(6,0)$ .
2. An open box is made from a square piece of material, 8 inches on a side, by cutting squares from each corner and turning up the sides. Find the volume of the largest box that can be made.
3. The sum of 4 times a first number and 3 times a second number is 12. Maximize the product of the two numbers.

4. Use the method of differentials to approximate  $\sqrt{9.01}$ .

5. Find  $\frac{dy}{dx}$

a)  $y = \sin x \cos x$  b)  $y = [\cos(x^2) + 10]^3$  c)  $y = \sec(\sin x)$  d)  $y = \tan(xy) + \sec x$

6.

a) $\int (3x^5 - 4x^3 - 7x + 2) dx =$	b) $\int \frac{5x^3 - 7x}{\sqrt{x}} dx =$
c) $\int (3x - 2)(x + 5) dx =$	d) $\int \frac{\sin x}{\cos^2 x} dx$
e) $\int x \sec(x^2) \tan(x^2) dx$	f) $\int \frac{x}{\sqrt{5x^2 + 4}} dx$
g) $\int \frac{1}{t^2} (3 + \frac{5}{t})^{10} dt$	h) $\int \frac{(5 - x^{\frac{2}{3}})^{10}}{\sqrt[3]{x}} dx$
i) $\int \frac{x}{\sqrt{x-1}} dx$	j) $\int \frac{x}{(x+1)^3} dx$

7. Evaluate the definite integrals: a)  $\int_0^{\pi/2} \sin x dx$  b)

$\int_0^1 (2x^2 - 5x) dx$

8. Find the area between:  $y = -x^2 + 4x$  and the  $x$ -axis.

9. Find the average of function  $y = 3x^2 + 1$  on the interval  $[1,3]$ .

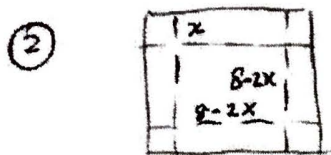
10. Find a)  $\frac{d}{dx} \int_1^x \sin t dt$  b)  $\frac{d}{dx} \int_1^{x^2} t^3 dt$

1.  $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$

(Answers posted  
on mathbreeze)

NTS Review exam 3 - solutions

① Minimize  $D = \sqrt{(x-6)^2 + (y-0)^2} = \sqrt{(x-6)^2 + x + 2}$   
 $D' = \frac{1}{2} ((x-6)^2 + x + 2)^{-1/2} \cdot (2(x-6) + 1)$   
 $2(x-6) + 1 = 0 \Rightarrow x = \frac{7}{2}, y = \sqrt{\frac{7}{2} + 2}$



② minimize  $V = x(8-2x)(8-2x)$   
 $= 4x(4-x)(4-x)$   
 $= 4x(16 - 8x + x^2) = 4(16x - 8x^2 + x^3)$   
 $V' = 4(16 - 16x + 3x^2) = 4(3x - 4)(x - 4)$   
 $x = \frac{4}{3}$  (circled),  ~~$x = 4$~~

③  $4x + 3y = 12 \Rightarrow 3y = 12 - 4x, y = 4 - \frac{4}{3}x$   
 $P = xy = x(4 - \frac{4}{3}x) = 4x - \frac{4}{3}x^2$   
 $\frac{dP}{dx} = 4 - \frac{8}{3}x = 0 \quad 4 = \frac{8}{3}x$   
 $\left(\frac{3}{2}\right) = \frac{3}{8} \cdot 4 = x$   
 $y = 4 - \frac{4}{3} \cdot \frac{3}{2} = 2$

④  $f(x) = \sqrt{x} \quad x_0 = 9 \quad dx = .01$   
 $f(x_0 + dx) \approx f(x_0) + f'(x_0) dx = \sqrt{9} + \frac{1}{2\sqrt{9}} (.01)$   
 $= 3 + \frac{1}{6} (.01) = 3.0016667$

- ⑤ a)  $\cos^2 x - \sin^2 x$   
 b)  $3 [\cos(x^2) + 10]^2 (-\sin(x^2) \cdot 2x)$   
 c)  $\sec(\sin x) \tan(\sin x) \cdot \cos x$

$$d) y' = \sec^2(xy) \cdot [xy' + y] + \sec x \tan x$$

$$y' - \sec^2(xy) \cdot xy' = \sec^2(xy) \cdot y + \sec x \tan x$$

$$y' = \frac{\sec^2(xy) \cdot y + \sec x \tan x}{(1 - \sec^2(xy) \cdot x)}$$

$$8) a) \frac{3 \cdot x^6}{6} - \frac{4 \cdot x^4}{4} - \frac{7 \cdot x^2}{2} + 2x + C$$

$$b) \int 5x^{5/2} - 7x^{1/2} dx = 5x^{7/2} \cdot \frac{2}{7} - 7x^{3/2} \cdot \frac{2}{3} + C$$

$$c) \int (3x^2 + 13x - 10) dx = \frac{3 \cdot x^3}{3} + \frac{13 \cdot x^2}{2} - 10x + C$$

$$d) \frac{(\cos x)^{-1}}{+1} + C$$

$$e) \frac{1}{2} \sec(x^2) + C$$

$$f) 2(5x^2 + 4)^{1/2} \cdot \frac{1}{10} + C$$

$$g) \frac{(3 + \frac{x}{t})''}{''} \cdot \left(-\frac{1}{5}\right) + C$$

$$h) \frac{(5 - x^{1/3})''}{''} \left(-\frac{3}{2}\right) + C$$

$$i) \frac{2}{3} (x-1)^{3/2} + 2(x-1)^{1/2} + C$$

$$j) \frac{(x+1)^{-1}}{-1} + \frac{(x+1)^{-2}}{-2} + C$$

$$\textcircled{7} \text{ a) } \int_0^{\pi/2} \sin x \, dx = -\cos x \Big|_0^{\pi/2} = -\cos \frac{\pi}{2} + \cos 0 = \boxed{1}$$

$$\text{b) } \int_0^1 (2x^2 - 5x) \, dx = \left[ \frac{2x^3}{3} - \frac{5x^2}{2} \right]_0^1 = \left[ \frac{2}{3} - \frac{5}{2} \right] - [0] = \boxed{-\frac{11}{6}}$$

$$\textcircled{8} \quad -x^2 + 4x = 0$$

$$x = 0, x = 4$$

$$-x(x - 4) = 0$$

$$\int_0^4 -x^2 + 4x \, dx = \left[ -\frac{x^3}{3} + 4 \cdot \frac{x^2}{2} \right]_0^4$$

$$= -\frac{64}{3} + 32 = \boxed{\frac{32}{3}}$$

$$\textcircled{9} \quad \frac{1}{3-1} \int_1^3 (3x^2 + 1) \, dx = \frac{1}{2} \left[ \frac{3x^3}{3} + x \right]_1^3 = \frac{1}{2} ([27 + 3] - [1 + 1])$$

$$= \frac{1}{2} (27 + 3 - 2) = \boxed{14}$$

$$\textcircled{10} \text{ a) } \boxed{\sin x}$$

$$\text{b) } (x^2)^3 \cdot 2x = 2x^6 \cdot x = \boxed{2x^7}$$