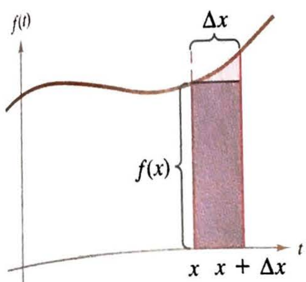
Now, from the Mean Value Theorem for Integrals, we know there exists a number c in the interval $[x, x + \Delta x]$ such that the integral in the above expression is equal to $f(c)\Delta x$. Moreover, since $x \le c \le x + \Delta x$, it follows that $c \to x$ as $\Delta x \to 0$. Thus, we have

$$F'(x) = \lim_{\Delta x \to 0} \left[\frac{1}{\Delta x} f(c)\Delta x\right] = \lim_{\Delta x \to 0} f(c) = f(x).$$



$$f(x)\Delta x \approx \int_x^{x+\Delta x} f(t)\,dt$$

FIGURE 5.29

REMARK Using the area model for definite integrals, we can view the approximation

$$f(x)\Delta x \approx \int_x^{x+\Delta x} f(t)\,dt$$

as saying that the area of the rectangle of height $f(x)$ and width Δx is approximately equal to the area of the region lying between the graph of f and the x-axis on the interval $[x, x + \Delta x]$, as shown in Figure 5.29.

Note that the Second Fundamental Theorem of Calculus tells us that if a function is continuous, then we can be sure that it has an antiderivative. This antiderivative need not, however, be an elementary function. (Recall the discussion of elementary functions in Section 1.5.)

EXAMPLE 6 Applying the Second Fundamental Theorem of Calculus

Evaluate

$$\frac{d}{dx}\int_0^x \sqrt{t^2 + 1}\,dt.$$

SOLUTION

Note that $f(t) = \sqrt{t^2 + 1}$ is continuous on the entire real line. Thus, using the Second Fundamental Theorem of Calculus, we can write

$$\frac{d}{dx}\int_0^x \sqrt{t^2 + 1}\,dt = \sqrt{x^2 + 1}.$$

EXERCISES for Section 5.4

In Exercises 1–24, evaluate the definite integral.

1. $\int_0^1 2x\,dx$
2. $\int_2^7 3\,dv$
3. $\int_{-1}^0 (x - 2)\,dx$
4. $\int_2^5 (-3v + 4)\,dv$
5. $\int_{-1}^1 (t^2 - 2)\,dt$
6. $\int_0^3 (3x^2 + x - 2)\,dx$
7. $\int_0^1 (2t - 1)^2\,dt$
8. $\int_{-1}^1 (t^3 - 9t)\,dt$
9. $\int_1^2 \left(\frac{3}{x^2} - 1\right) dx$
10. $\int_0^1 (3x^3 - 9x + 7)\,dx$
11. $\int_1^2 (5x^4 + 5)\,dx$
12. $\int_{-3}^3 v^{1/3}\,dv$
13. $\int_{-1}^1 (\sqrt[3]{t} - 2)\,dt$
14. $\int_1^2 \sqrt{\frac{2}{x}}\,dx$
15. $\int_1^4 \frac{u - 2}{\sqrt{u}}\,du$
16. $\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$
17. $\int_0^1 \frac{x - \sqrt{x}}{3}\,dx$
18. $\int_0^2 (2 - t)\sqrt{t}\,dt$

19. $\int_{-1}^{1} (t^{1/3} - t^{2/3})\, dt$

20. $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\, dx$

21. $\int_{-1}^{1} |x|\, dx$

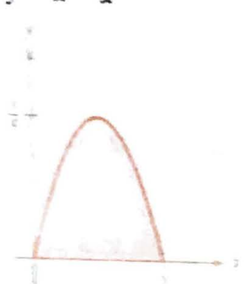
22. $\int_{0}^{3} |2x - 3|\, dx$

23. $\int_{0}^{4} |x^2 - 4x + 3|\, dx$

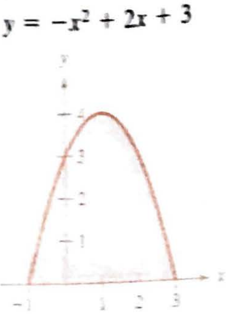
24. $\int_{-1}^{1} |x^3|\, dx$

In Exercises 25–30, determine the area of the indicated region.

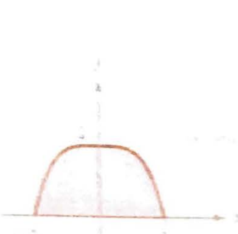
25. $y = x - x^2$



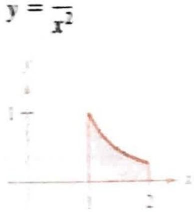
26. $y = -x^2 + 2x + 3$



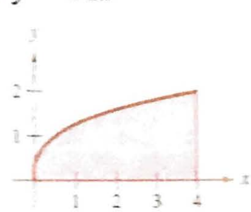
27. $y = 1 - x^4$



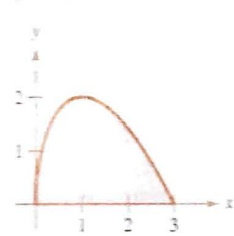
28. $y = \frac{1}{x^2}$



29. $y = \sqrt[3]{2x}$



30. $y = (3 - x)\sqrt{x}$



In Exercises 31–34, find the area of the region bounded by the graphs of the given equations.

31. $y = 3x^2 + 1,\ x = 0,\ x = 2,\ y = 0$
32. $y = 1 + \sqrt{x},\ x = 0,\ x = 4,\ y = 0$
33. $y = x^3 + x,\ x = 2,\ y = 0$
34. $y = -x^2 + 3x,\ y = 0$

In Exercises 35–38, find the values of c guaranteed by the *Mean Value Theorem for Integrals* for the given function over the specified interval.

Function	*Interval*
35. $f(x) = x^3$	$[0, 2]$
36. $f(x) = \frac{9}{x^3}$	$[1, 3]$
37. $f(x) = -x^2 + 4x$	$[0, 3]$
38. $f(x) = \sqrt{x}$	$[1, 9]$

In Exercises 39–42, sketch the graph of the given function over the specified interval. Find the average value of the function over the interval and all values of x where the function equals its average value.

Function	*Interval*
39. $f(x) = 4 - x^2$	$[-2, 2]$
40. $f(x) = \frac{x^2 + 1}{x^2}$	$\left[\frac{1}{2}, 2\right]$
41. $f(x) = x - 2\sqrt{x}$	$[0, 4]$
42. $f(x) = \frac{1}{(x - 3)^2}$	$[0, 2]$

In Exercises 43–48, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result of part (a).

43. $F(x) = \int_0^x (t + 2)\, dt$

44. $F(x) = \int_0^x t(t^2 + 1) \ldots$

45. $F(x) = \int_8^x \sqrt[3]{t}\, dt$

46. $F(x) = \int_4^x \sqrt{t}\, dt$

47. $F(x) = \int_1^x \frac{1}{t^2}\, dt$

48. $F(x) = \int_0^x t^{3/2}\, dt$

In Exercises 49–52, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

49. $F(x) = \int_{-2}^{x} (t^2 - 2t + 5)\, dt$

50. $F(x) = \int_{1}^{x} \sqrt[4]{t}\, dt$

51. $F(x) = \int_{-1}^{x} \sqrt{t^4 + 1}\, dt$

52. $F(x) = \int_{1}^{x} \frac{t^2}{t^2 + 1}\, dt$

53. The volume V in liters of air in the lungs during a 5-second respiratory cycle is approximated by the model

$$V = 0.1729t + 0.1522t^2 - 0.0374t^3$$

where t is the time in seconds. Approximate the average volume of air in the lungs during one cycle.

54. The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is given by

$$v = k(R^2 - r^2)$$

where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use zero and R as the limits of integration.)