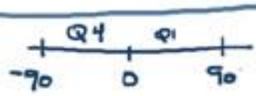
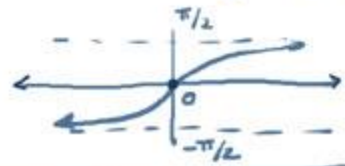
Math 16

085 Inverse trig functions and differentiation

$$\boxed{y = \arcsin x\ ; \quad -1 \le x \le 1 \qquad -\frac{\pi}{2} \le y \le \frac{\pi}{2}}$$



$$y = \arctan x\ ; \quad -\infty < x < \infty \qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$$



ex:

S	A
T	C

① $\arcsin\left(\frac{\sqrt{3}}{2}\right) = 60°$

② $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -60°$

③ $\arctan(\sqrt{3}) = 60°$

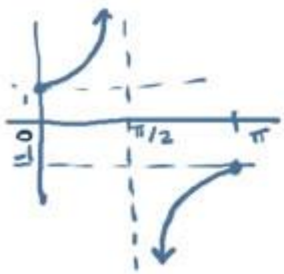
④ $\sin\Big(\underbrace{\arctan\left(\tfrac{3}{2}\right)}_{\theta}\Big) = \frac{3}{\sqrt{13}}$

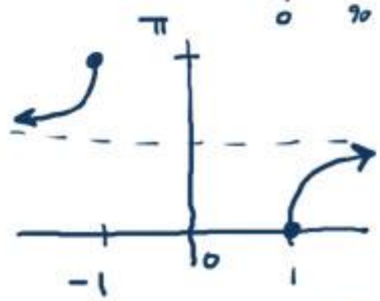
$$\frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$\frac{1}{\cos x}$

$y = \sec x$; $0 \le x \le \pi$, $x \ne \frac{\pi}{2}$ $\xrightarrow{inv}$ $y = \text{arc sec}\, x$; $x \le -1,\ x \ge 1$

$y \le -1,\ y \ge 1$ $\qquad$ $0 \le y \le \pi$, $y \ne \frac{\pi}{2}$





<u>ex</u> : ① $\text{arc sec}(2) = 60°$

angle with sec of 2
$\Rightarrow$ ∡ with cos of $\frac{1}{2}$

S | A
T | C

② $\text{arc sec}(-2) = 120°$

Deriv of $y = \arcsin x$

$$y = \arcsin x$$

$$\sin y = x$$

diff

$$(\cos y)\, y' = 1$$

$$y' = \frac{1}{\cos y} = \frac{1}{\cos(\underbrace{\arcsin x}_{\theta})}$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

$$\boxed{y = \arcsin x \qquad \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$$

$$\boxed{y = \arcsin u \qquad \frac{dy}{dx} = \frac{u'}{\sqrt{1-u^2}}}$$

$$\boxed{\begin{aligned} y &= \ln x \\ e^y &= x \\ e^y \cdot y' &= 1 \\ y' &= \frac{1}{e^y} = \frac{1}{x} \end{aligned}}$$

$$\boxed{\begin{aligned} y &= \arcsin x \\ \sin y &= x \\ y &= \arcsin x \\ \sin y &= \sin(\arcsin x) \\ \sin y &= x \end{aligned}}$$

$$a^2 + x^2 = 1$$

$$a^2 = 1 - x^2$$

$$a = \sqrt{1-x^2}$$

$$\cos\theta = \frac{\sqrt{1-x^2}}{1}$$

Derivative of $y = \arctan x$

$y = \arctan x$

$\tan(y) = \tan(\arctan x)$

$\tan y = x$

$(\sec^2 y) \cdot y' = 1$

$$y' = \frac{1}{\sec^2 y} = \frac{1}{\sec^2(\underbrace{\arctan x}_{\theta})}$$

$$= \frac{1}{1+x^2}$$

$c^2 = x^2 + 1^2$

$\sec\theta = \sqrt{1+x^2}$

$\sec^2\theta = 1+x^2$

$$\boxed{\begin{aligned} &y = \arctan x \\ &\frac{dy}{dx} = \frac{1}{1+x^2} \end{aligned}}$$

$$\boxed{\begin{aligned} &y = \arctan u \\ &\frac{dy}{dx} = \frac{u'}{1+u^2} \end{aligned}}$$

$y = \arcsin x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

ex: Find $\frac{dy}{dx}$

① $y = \arctan(x^2)$

$$y' = \frac{2x}{1+(x^2)^2}$$

$$= \frac{2x}{1+x^4}$$

② $y = \arcsin(e^x)$

$$y' = \frac{e^x}{\sqrt{1-(e^x)^2}}$$

$$= \frac{e^x}{\sqrt{1-e^{2x}}}$$

③ $y = \arctan(2x+3)$

$$y' = \frac{2}{1+(2x+3)^2}$$

④ $y = \arctan(x+3)$

$$y' = \frac{1}{1+(x+3)^2}$$

$$\boxed{\int \frac{1}{x^2+6x+10}\,dx}$$

⑤ $y = \arcsin(x^2+1)$

$$y' = \frac{2x}{\sqrt{1-(x^2+1)^2}}$$

$$\cancel{\int \frac{1}{x^2+6x+8}\,dx}$$

$$\cancel{\int \frac{1}{1-(x+3)^2}\,dx}$$

Deriv of $y = \operatorname{arcsec} x$ $\quad (x \le -1,\ x \ge 1)$

Assume $x \ge 1$

$y = \operatorname{arcsec} x$

$\sec y = \sec(\operatorname{arcsec} x)$

$\sec\theta = x$

$\sec y = x$

$\tan\theta = \sqrt{x^2-1}$

$(\sec y \tan y)\, y' = 1$

$$y' = \frac{1}{\sec y \tan y}$$

$$= \frac{1}{\sec(\underbrace{\operatorname{arcsec} x}_{\theta})\tan(\operatorname{arcsec} x)} = \frac{1}{x\sqrt{x^2-1}} \quad (\text{for } x \ge 1)$$

$$\boxed{\begin{aligned} y &= \operatorname{arcsec} x \\ y' &= \frac{1}{|x|\sqrt{x^2-1}} \end{aligned}} \quad (x \le -1,\ x \ge 1)$$

$$\boxed{\begin{aligned} y &= \operatorname{arcsec} u \\ y' &= \frac{u'}{|u|\sqrt{u^2-1}} \end{aligned}}$$

ex: Find $\frac{dy}{dx}$

① $y = \operatorname{arcsec}(x^2)$

$$y' = \frac{2x}{|x^2|\sqrt{(x^2)^2 - 1}} = \frac{2x}{|x^2|\sqrt{x^4 - 1}}$$

② $y = \arcsin(2x + 5)$

$$y' = \frac{2}{|2x+5|\sqrt{(2x+5)^2 - 1}}$$

$$\int \frac{1}{1+x^2}\,dx \qquad \int \frac{1}{\underbrace{x^2+6x+10}_{(x+3)^2+1}}\,dx$$

$$\int \frac{1}{\sqrt{1-(x+5)^2}}\,dx \qquad \int \frac{1}{x\sqrt{x^2-1}}\,dx$$