

	Math 21 – Review for final	Answers
1	Find the arc length of the parametric curve: $x(t) = e^{-t} \cos t$ and $y(t) = e^{-t} \sin t$, $0 \leq t \leq \frac{\pi}{2}$	$\sqrt{2}(1 - e^{-\frac{\pi}{2}})$
2	$\mathbf{u} = \langle 1, -2, 3 \rangle$ and $\mathbf{v} = \langle 2, 5, -1 \rangle$ find A) $\mathbf{u} \cdot \mathbf{v}$ B) angle between \mathbf{u} and \mathbf{v} C) $proj_{\mathbf{u}} \mathbf{v}$	A) -11 B) 122° C) $-\frac{11}{14} \langle 1, -2, 3 \rangle$
3	Area of the triangle with vertices: P(1,3,5), Q(3,3,0), R(-2,0,5)	$\frac{9\sqrt{6}}{2}$
4	Equation of line through (1,2,3) and parallel to vector (with direction vector) $\langle 2, 0, -1 \rangle$.	$x = 2t + 1$ $y = 2$ $z = -t + 3$
5	Equation of plane through points (0,0,0), (1,2,3), (-2,3,3)	$3x + 9y - 7z = 0$
6	Distance from P(1,5,-4) to plane $3x - y + 2z = 6$	$\frac{16}{\sqrt{14}}$
7	Distance from Q(10,3,-2) to line $x=4t-2$; $y=3$; $z=-t+1$	0
8	Which of the following statements about surface $\frac{x^2}{4} + \frac{z^2}{16} - \frac{y^2}{9} = -1$ are true? A) $y \geq 3$ or $y \leq -3$ B) the trace with the yz-plane is a hyperbola with axis equal to the y-axis C) the trace with the yz-plane is a hyperbola with axis equal to the z-axis D) $z \geq 4$ or $z \leq -4$	A and B
9	What is the domain of $f(x, y) = \frac{1}{\sqrt{(x-1)(y+3)}}$?	$x > 1$ and $y > -3$ or $x < 1$ and $y < -3$
10	For $f(x, y) = xy^3 - x^2 + y^2$ A) $f_{xy} =$ *B) The tangent line to $f(x, y)$ at point (1,2), in the plane $y = 2$, has equation _____. (*this won't be on the final C) The differential at $f(1,2)$, $dx = .1$ and $dy = .2$ D) The slope of the curve at point (1,2) in the direction of $\mathbf{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$, has slope _____. (directional derivative)	A) $3y^2$ B) slope: $f_x(1,2) = 6$ Line with direction vector $\langle 1, 0, 6 \rangle$ through (1,2,11) is $x = 1 + t$; $y = 2$; $z = 11 + 6t$ C) $f_x(.1) + f_y(.2)$ at (1,2) is $6(.1) + 16(.2) = 3.8$ D) since $\ \mathbf{u}\ = 1$, $D_{\mathbf{u}}f(1,2) = \frac{3}{5}(6) + \frac{4}{5}(16)$
11	Find max/min/saddle point(s) of $f(x, y) = 2x^2 + 2xy + y^2 + 2x - 3$	Rel min at (-1,1,-4)
12	For, $f(x, y) = x^2 + y^2 + xy$, if A) $x(t) = 5t + 1$; $y(t) = 7t^2$ find $\frac{df}{dt}$ B) $x(s, t) = s^2 + 3t^2$; $y(s, t) = 7s + 5t$ find $\frac{\partial f}{\partial s}$	A) $(2x + y)(5) + (2y + x)(14t)$ B) $(2x + y)(2s) + (2y + x)(7)$

13	A cone is increasing in size such that the height is given by $h(t) = 5t$ and the radius is given by $r(t) = 7t^2$. Find the rate of change of the volume when $t = 1$. $V = \frac{1}{3}\pi r^2 h$	$\frac{315\pi}{3}$
14	Find two integrals that both represent the volume under $f(x, y)$ and above the triangular region in the xy -plane with vertices $(0,0)$, $(4,0)$, $(0,2)$.	$\int_0^4 \int_0^{-\frac{1}{2}x+2} f(x, y) dy dx$ $\int_0^2 \int_0^{-2y+4} f(x, y) dx dy$
15	Find two double integrals (dydx and dx dy) that represents the area of the region under $y = x^2$, from $x = 0$ to $x = 1$.	$\int_0^1 \int_0^{x^2} 1 dy dx$ $\int_0^1 \int_{\sqrt{y}}^1 1 dx dy$
16	The volume under surface $z = x^2 + y^2$ and above the triangle with vertices $(0,0)$, $(2,0)$, $(1,1)$.	$\frac{4}{3}$
17	Converge or diverge and by which test? A) $\sum_1^\infty \frac{n^2}{\sqrt{n^8+1}}$ B) $\sum_1^\infty n/(n^3 - 1)$ C) $\sum_1^\infty \frac{(-1)^n}{2n+1}$ D) $\sum_1^\infty n^2 \left(\frac{2}{3}\right)^n$	A) converge; direct comp with $\sum_1^\infty 1/n^2$ B) converge; limit comp with $\sum_1^\infty 1/n^2$ C) Alt series; $a_n \rightarrow 0$; $a_{n+1} \leq a_n$ ($f' < 0$) D) conv; ratio test
18	Sum of $\sum_4^\infty \left(\frac{2}{3}\right)^n$	0.59259
19	Interval of convergence? A) $\sum_0^\infty \frac{(x-2)^n}{10^n}$ B) $\sum_0^\infty \frac{nx^n}{n+2}$ C) $\sum_1^\infty \frac{x^n}{n\sqrt{n} 3^n}$ D) $\sum_0^\infty \frac{x^n}{\sqrt{n^2+3}}$ E) $\sum_1^\infty \frac{(-1)^{n+1}}{n2^n}$	A) $(-8,12)$ B) $(-1,1)$ C) $[-3,3]$ D) $[-1,1)$ E) $(-2,2]$
20	Use the alternating series to obtain an estimate of $\sum_0^\infty (-.3)^n$ with an accuracy of .001 .	0.763
21	Find power series for $\frac{1}{5x+3}$ and the interval of convergence.	$\frac{1}{3} \sum_0^\infty (-1)^n \left(\frac{5x}{3}\right)^n ; \left(-\frac{3}{5}, \frac{3}{5}\right)$
22	For $f(x) = \frac{x^n}{n^2}$, find $F(x)$, where $F'(x) = f(x)$ and $F(0) = 2$.	$2 + \sum_1^\infty \frac{x^{n+1}}{n^2(n+1)}$
23	Find the first 4 terms of the power series for $f(x) = e^x \sin x$	$x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5$