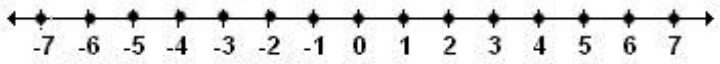
CHAPTER 2 : Integers, Exponents and Order of Operations

2A – Adding Integers

The **integers** are as follows: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

The integers on the number line:

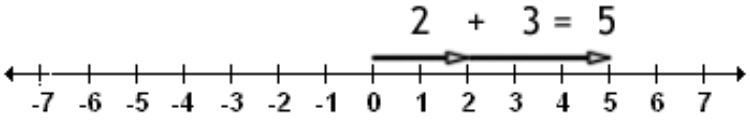


EXAMPLE 1: Adding on the number line.

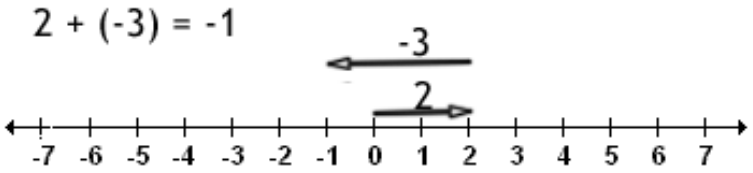
A) $2 + 3 =$ **B)** $2 + (-3) =$ **C)** $-2 + (-3) =$ **D)** $-3 + 2 =$ **E)** $3 + (-2) =$

SOLUTION:

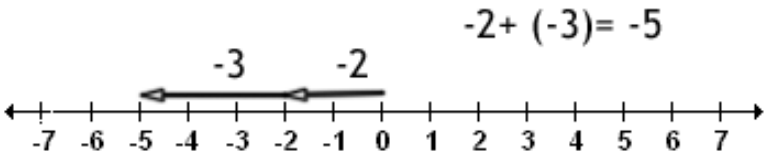
A)



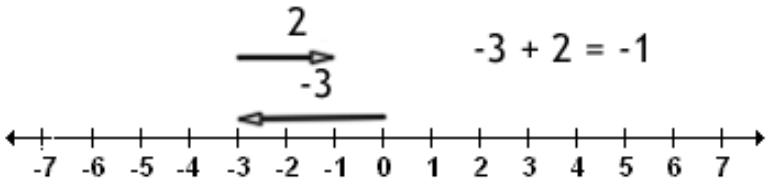
B)



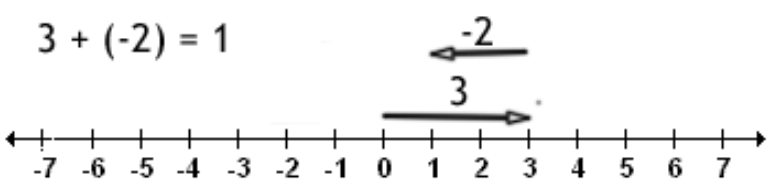
C)



D)



E)



Definition: The **absolute value** of a non-negative number is the same as the number. The absolute value of a negative number is the number without its sign.

The absolute value of -2 is written as $|-2|$ and is equal to 2.

EXAMPLE 2: Evaluate the expression.

A) $|5|$ **B)** $|-6|$ **C)** $-|5|$ **D)** $-|-3|$

SOLUTION:

A) $|5| = 5$
B) $|-6| = 6$
C) $-|5| = -5$
D) $-|-3| = -3$

Rules for adding integers and signed numbers:

- **Adding two numbers with the same signs:**
  - Add their absolute values.
  - Use their common sign for the sign of the sum.
- **Adding two numbers with different signs:**
  - Subtract the larger absolute value minus the smaller absolute value.
  - Use the sign of the number with the larger absolute value for the sum.

EXAMPLE 3: Add

A) $7 + 10$ **B)** $-30 + (-50)$ **C)** $7 + (-5)$

D) $-4 + 10$ **E)** $10 + (-12)$ **F)** $-20 + 15$

SOLUTION:

A) $7 + 10 = \mathbf{17}$

B) $-30 + (-50)$ Add their absolute values: $30 + 50 = 80$
Use their common sign for the sum: $-30 + (-50) = \mathbf{-80}$

C) $7 + (-5)$ Subtract the larger(in absolute value) minus the smaller: $7 - 5 = 2$
Use the sign of the number with larger absolute value: $7 + (-5) = \mathbf{2}$

D) $-4 + 10$ Subtract the larger (in absolute value) minus the smaller: $10 - 4 = 6$
Use the sign of the number with larger absolute value: $-4 + 10 = \mathbf{6}$

E) $10 + (-12)$ Subtract the larger(in absolute value) minus the smaller: $12 - 10 = 2$

Use the sign of the number with larger absolute value: $10 + (-12) = \mathbf{-2}$

F) $-20 + 15$ Subtract the larger absolute value minus the smaller: $20 - 15 = 5$
Use the sign of the number with larger absolute value: $-20 + 15 = \mathbf{-5}$

2A – EXERCISES

For 1 – 6, add on the number line.

1. $2 + 4$ **2.** $2 + (-4)$ **3.** $-4 + 2$

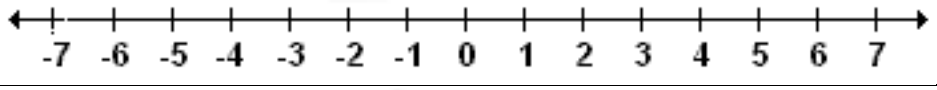
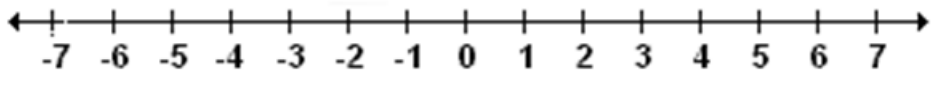
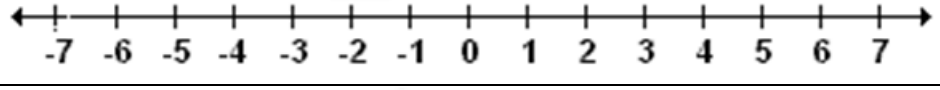
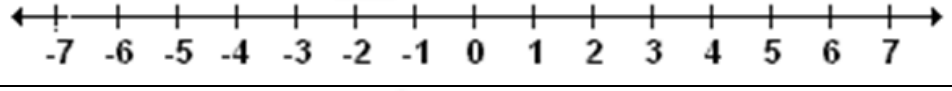
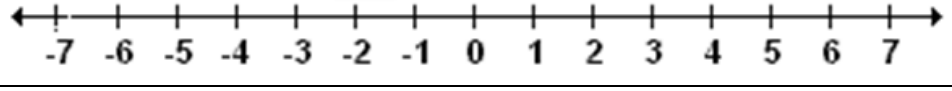
4. $-3 + (-4)$ **5.** $-3 + 7$ **6.** $4 + (-7)$

For 7 – 24, add.

7.	$15 + 25$	**8.**	$-15 + (-5)$	**9.**	$-30 + (-20)$
10.	$-45 + (-15)$	**11.**	$-55 + (-5)$	**12.**	$-32 + (-12)$
13.	$15 + (-5)$	**14.**	$25 + (-12)$	**15.**	$35 + (-15)$
16.	$12 + (-14)$	**17.**	$25 + (-30)$	**18.**	$24 + (-28)$
19.	$-30 + 50$	**20.**	$-25 + 75$	**21.**	$-12 + 14$
22.	$-15 + 5$	**23.**	$-25 + 10$	**24.**	$-42 + 15$

2A – WORKSHEET: Adding Integers

For 1 – 5, add on the number line.

1. $4 + 2$	
2. $4 + (-7)$	
3. $-3 + (-2)$	
4. $6 + (-8)$	
5. $-3 + 6$	

For 6 - 23 , add.

6. $25 + 72$	**7.** $-32 + (-23)$	**8.** $-15 + (-54)$
9. $-36 + (-35)$	**10.** $-73 + (-34)$	**11.** $-65 + (-32)$

12. $70 + (-20)$	**13.** $15 + (-35)$	**14.** $60 + (-45)$
15. $43 + (-63)$	**16.** $34 + (-12)$	**17.** $45 + (-65)$
18. $-20 + 15$	**19.** $-15 + 40$	**20.** $-150 + 40$
21. $-80 + 100$	**22.** $-34 + 18$	**23.** $-45 + 90$

Answers:

1. 6	**2.** -3	**3.** -5	**4.** -2
5. 3	**6.** 97	**7.** -55	**8.** -69
9. -71	**10.** -107	**11.** -97	**12.** 50
13. -20	**14.** 15	**15.** -20	**16.** 22
17. -20	**18.** -5	**19.** 25	**20.** -110
21. 20	**22.** -16	**23.** 45	

2B – Subtracting Integers

Rule for subtracting integers: $a - b = a + (-b)$

EXAMPLE 1: Subtract

A) $5 - 10$ B) $-7 - 30$ **C)** $10 - (-5)$ **D)** $-15 - (-5)$

SOLUTION:

A) $5 - 10 = 5 + (-10) = -5$

B) $-7 - 30 = -7 + (-30) = -37$

C) $10 - (-5) = 10 + \big(-(-5)\big) = 10 + 5 = 15$
Notice that *when we subtract a negative we add a positive*.

D) $-15 - (-5) = -15 + 5 = -10$
Notice again that subtracting a negative is the same as adding a positive.

2B – EXERCISES

1. $25 - 15$ **2.** $12 - 25$ **3.** $17 - 28$

4. $15 - 30$ **5.** $5 - 23$ **6.** $8 - 17$

7. $3 - (-20)$ **8.** $12 - (-13)$ **9.** $14 - (-25)$

10. $28 - (-13)$ **11.** $42 - (-26)$ **12.** $33 - (-14)$

13. $25 - (-16)$ **14.** $13 - (-32)$ **15.** $15 - (-45)$

16. $-20 - 15$ **17.** $-45 - 32$ **18.** $-32 - 24$

19. $-18 - 17$ **20.** $-23 - 15$ **21.** $-41 - 28$

22. $-33 - 25$ **23.** $-48 - 27$ **24.** $-36 - 48$

25. $-12 - (-15)$ **26.** $-25 - (-32)$ **27.** $-15 - (-45)$

28. $-32 - (-75)$ **29.** $-43 - (-12)$ **30.** $-50 - (-15)$

31. $5 - 7 + 10 - 8$ **32.** $-8 - 10 + 12 - 6$ **33.** $3 - 15 + 20 - 5$

2B – WORKSHEET: Subtracting Integers

1. $30 - 15$	**2.** $13 - 25$	**3.** $15 - 28$
4. $20 - 30$	**5.** $7 - 23$	**6.** $5 - 17$
7. $10 - (-20)$	**8.** $15 - (-13)$	**9.** $12 - (-25)$
10. $25 - (-14)$	**11.** $43 - (-27)$	**12.** $35 - (-16)$
13. $30 - (-16)$	**14.** $17 - (-33)$	**15.** $20 - (-45)$
16. $-25 - 15$	**17.** $-47 - 32$	**18.** $-35 - 23$
19. $-19 - 14$	**20.** $-27 - 15$	**21.** $-45 - 26$
22. $-35 - 25$	**23.** $-58 - 27$	**24.** $-56 - 48$
25. $-12 - (-25)$	**26.** $-25 - (-42)$	**27.** $-15 - (-43)$
28. $-32 - (-15)$	**29.** $-43 - (-13)$	**30.** $-40 - (-15)$
31. $10 - 15 + 12 - 20$	**32.** $-8 - 5 - 10 + 4$	**33.** $-7 - 12 + 10 - 13$

Answers:

1. 15	**2.** -12	**3.** -13	**4.** -10	**5.** -16
6. -12	**7.** 30	**8.** 28	**9.** 37	**10.** 39
11. 70	**12.** 51	**13.** 46	**14.** 50	**15.** 65
16. -40	**17.** -79	**18.** -58	**19.** -33	**20.** -42
21. -71	**22.** -60	**23.** -85	**24.** -104	**25.** 13
26. 17	**27.** 28	**28.** -17	**29.** -30	**30.** -25
31. -13	**32.** -19	**33.** -22		

2C – Multiplying and Dividing Integers

Rules for multiplying integers and signed numbers:

- The product of two numbers with the same sign is a positive number.
- The product of two numbers with different signs is a negative number.

EXAMPLE 1:

A) $(-5)(12)$ **B)** $(7)(-6)$ **C)** $(-11)(-5)$ **D)** $(12)(6)$ **E)** $(3)(-2)(5)(-4)$

SOLUTION:

A) $(-5)(12) = -60$ **B)** $(7)(-6) = -42$

C) $(-11)(-5) = 55$ **D)** $(12)(6) = 72$ **E)** $(3)(-2)(5)(-4) = (-6)(5)(-4) = (-30)(-4) = 120$

Rules for dividing integers and signed numbers:

- The quotient of two numbers with the same sign is a positive number.
- The quotient of two numbers with different signs is a negative number.

EXAMPLE 2:

A) $\frac{25}{5}$ **B)** $\frac{-30}{6}$ **C)** $\frac{60}{-10}$ **D)** $\frac{-75}{-25}$

SOLUTION:

A) $\frac{25}{5} = 5$ **B)** $\frac{-30}{6} = -5$ **C)** $\frac{60}{-10} = -6$ **D)** $\frac{-75}{-25} = 3$

2C – EXERCISES:

1. $(-12)(7)$ **2.** $(15)(-4)$ **3.** $(30)(-7)$ **4.** $(14)(-6)$

5. $(32)(-5)$ **6.** $(7)(-15)$ **7.** $(-20)(-5)$ **8.** $(-45)(-23)$

9. $(-24)(-35)$ **10.** $(-52)(-34)$ **11.** $(46)(-32)$ **12.** $(23)(-76)$

13. $\frac{-25}{5}$ **14.** $\frac{36}{-18}$ **15.** $\frac{-75}{15}$ **16.** $\frac{60}{-12}$

17. $\frac{-10}{5}$ **18.** $\frac{75}{-25}$ **19.** $\frac{42}{-6}$ **20.** $\frac{-56}{7}$

21. $\frac{-45}{-9}$ **22.** $\frac{-36}{-12}$ **23.** $\frac{-40}{-8}$ **24.** $\frac{-16}{-4}$

25. $\frac{-22}{-11}$ **26.** $\frac{-55}{-5}$ **27.** $\frac{-72}{-12}$ **28.** $\frac{-80}{-16}$

29. $(-5)(3)(-2)$ **30.** $(-4)(-3)(5)$ **31.** $(-7)(2)(-4)(-3)$

2C – WORKSHEET: Multiplying and Dividing Integers

1. $(-14)(8)$	**2.** $(18)(-5)$	**3.** $(37)(-8)$	**4.** $(24)(-6)$
5. $(42)(-7)$	**6.** $(8)(-25)$	**7.** $(-23)(-6)$	**8.** $(-55)(-33)$
9. $(-25)(-45)$	**10.** $(-62)(-32)$	**11.** $(56)(-33)$	**12.** $(43)(-75)$
13. $\frac{-35}{5}$	**14.** $\frac{54}{-18}$	**15.** $\frac{-45}{15}$	**16.** $\frac{60}{-4}$
17. $\frac{35}{-5}$	**18.** $\frac{125}{-25}$	**19.** $\frac{48}{-6}$	**20.** $\frac{-63}{7}$
21. $\frac{-54}{-9}$	**22.** $\frac{-48}{-12}$	**23.** $\frac{-56}{-8}$	**24.** $\frac{-64}{-4}$
25. $\frac{-33}{-11}$	**26.** $\frac{-77}{-7}$	**27.** $\frac{-84}{-12}$	**28.** $\frac{-96}{-16}$
29. $(5)(-2)(-3)$	**30.** $(-7)(2)(5)$	**31.** $(-3)(2)(-4)(5)$	**32.** $(-7)(-2)(-3)(5)$

Answers:

1. -112	**2.** -90	**3.** -296	**4.** -144	**5.** -294
6. -200	**7.** 138	**8.** 1815	**9.** 1125	**10.** 1984
11. -1848	**12.** -3225	**13.** -7	**14.** -3	**15.** -3
16. -15	**17.** -7	**18.** -5	**19.** -8	**20.** -9
21. 6	**22.** 4	**23.** 7	**24.** 16	**25.** 3
26. 11	**27.** 7	**28.** 6	**29.** 30	**30.** -70
31. 120	**32.** -210			

2D – Exponents and Order of Operations

The repeated multiplication, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, can be expressed as:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$$

In the expression 2^5, 2 is the **base**, and 5 is the **exponent**.

The expression 2^5 is read: "$2\ to\ the\ 5^{th}\ power$" or "$2\ to\ the\ 5^{th}$". **Power** is another word for exponent.

The expression 2^0 is equal to 1. In general, $\boldsymbol{a^0 = 1}$, for all $a \neq 0$. (0^0 is undefined.)

EXAMPLE 1: Write using exponential notation.

A) $3 \cdot 3 \cdot 3 \cdot 3$ **B)** $5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ **C)** $(-2)(-2)(-2)(-2)$ **D)** 4

SOLUTION:

A) $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$

B) $5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 5^3 7^4$

C) $(-2)(-2)(-2)(-2) = (-2)^4$

D) 4^1

EXAMPLE 2: Write in expanded form.

A) 4^3 **B)** $2 \cdot 3^3$ **C)** $5^2 7^3$ **D)** $(-3)^3$

SOLUTION:

A) $4^3 = 4 \cdot 4 \cdot 4$ **B)** $2 \cdot 3^3 = 2 \cdot 3 \cdot 3 \cdot 3$ **C)** $5^2 7^3 = 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7$ **D)** $(-3)^3 = (-3)(-3)(-3)$

EXAMPLE 3: Evaluate.

A) 2^3 **B)** $3 \cdot 2^2$ **C)** $(-2)^4$ **D)** $(-3)^3$ **E)** 5^0

SOLUTION:

A) 8 **B)** $3 \cdot 4 = 12$ **C)** 16 **D)** -27 **E)** 1

If an expression contains more than one operation, we do the operations in a specific order.

The Order of Operations:

- Evaluate expressions in parenthesis first.
- Evaluate expressions with exponents.
- Multiply or divide from left to right.
- Add or subtract from left to right.

EXAMPLE 4: Evaluate the mathematical expression.

1. $5 + 3 \times 2$ **2.** $2 \cdot 3^2$ **3.** $8 - 2(3)$
4. $6 - 2(5 - 3)$ **5.** $2 - 5(4 - 8) \div 2$ **6.** $(-1)3^2$
7. -3^2 **8.** $3(-2)^4$ **9.** $\frac{10+4}{2}$
10. $-2[3 + 2(5 - 8)] + 10$ **11.** $10 - (5 - 2)^2$ **12.** $3(2 - 5)^2 + 2(-5)$
13. $2 - |3 - 7|$ **14.** $-5|5 - 8| + 3$

SOLUTION:

1. $5 + 3 \times 2 = 5 + 6 = 11$

2. $2 \cdot 3^2 = 2 \cdot 9 = 18$

3. $8 - 2(3) = 8 - 6 = 2$

4. $6 - 2(5 - 3) = 6 - 2(2) = 6 - 4 = 2$

5. $2 - 5(4 - 8) \div 2 = 2 - 5(-4) \div 2 = 2 + 20 \div 2 = 2 + 10 = 12$

6. $(-1)3^2 = (-1) \cdot 9 = -9$

7. $-3^2 = -9$ We can expand: $-3^2 = -3 \cdot 3 = -9$. Or, $-3^2 = (-1)3^2 = (-1)9 = -9$.

8. $3(-2)^4 = 3(16) = 48$

9. $\frac{10+4}{2} = \frac{14}{2} = 7$ Notice that the numerator (and denominator) groups an expression in the same way as parenthesis.

10. $-2[3 + 2(5 - 8)] + 10 = -2[3 + 2(-3)] + 10 = -2[3 - 6] + 10 = -2[-3] + 10 = 6 + 10 = 16$

11. $10 - (5 - 2)^2 = 10 - (3)^2 = 10 - 9 = 1$

12. $3(2 - 5)^2 + 2(-5) = 3(-3)^2 + 2(-5) = 3(9) + 2(-5) = 27 - 10 = 17$

13. $2 - |3 - 7| = 2 - |-4| = 2 - 4 = -2$

14. $-5|5 - 8| + 3 = -5|-3| + 3 = -5(3) + 3 = -12$

2D – EXERCISES

For 1 – 8, write using exponential notation.

1. $4 \cdot 4 \cdot 4$ **2.** $3 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ **3.** $2 \cdot 2 \cdot 2 \cdot 7 \cdot 7$ **4.** $(-4)(-4)(-4)$

5. $-5 \cdot 5 \cdot 5 \cdot 5$ **6.** $(-5)(-5)(-5)(-5)$ **7.** $-2 \cdot 3 \cdot 3$ **8.** $-7 \cdot 7 \cdot 6 \cdot 6 \cdot 6$

For 9 - 16 , write in expanded form.

9. 3^5 **10.** $2 \cdot 5^3$ **11.** $2^3 7^4$ **12.** $-3 \cdot 5^2$

13. -3^5 **14.** $(-3)^5$ **15.** $-2^3 5^4$ **16.** $(-7)^3$

For 17 - 48 , evaluate.

17. $3 + 7 - 5$ **18.** $4 - 5 - 7 + 2$

19. $2 + 5 \times 3$ **20.** $(2 + 5) \times 3$

21. $\frac{10+2}{4+2}$ **22.** $10 - 2(3)$

23. $3 - 7(-4)$ **24.** $2 - 5(3 - 10)$

25. $6 + 5(2 - 4)$ **26.** $4 - 2(3 - 7) + 5$

27. $6 - 3(9 - 7)$ **28.** $3 + 4(5 - 8) - 6$

29. $-2[6 + 5(2 - 5)]$ **30.** 5^3

31. -5^2 **32.** $(-2)3^2$

33. $(-5)^2$ **34.** $(-2)^3$

35. -3^3 **36.** $(-3)^3$

37. $-3 \cdot 5^2$ **38.** -2^4

39. $3 + (5 - 2)^2$ **40.** $2 + 3(4 - 2)^3$

41. $5 + 2(3 - 5)^2$ **42.** $3^0 + 2(5 - 10)^2$

43. $3 - 2(3 - 5)^2$ **44.** $2 - 5(2 - 4)^3$

45. $15 - (5 - 3)^2 + 2(-3)$ **46.** $10 + (4 - 2)^3 + 3(-2)$

47. $3(-5) + (2 - 5)^3 + 3(-4)$ **48.** $15 - (2 - 7)^2 + 3(-5) + 3^0$

49. $-3|5-10|$

50. $3-4|2-5|$

51. $5+2|3-7|$

52. $10-2|4-8|$

2D – WORKSHEET: Exponents and Order of Operations

For 1 – 8, write using exponential notation.

1. $3 \cdot 3 \cdot 3$	**2.** $2 \cdot 7 \cdot 7 \cdot 7$	**3.** $4 \cdot 8 \cdot 8 \cdot 7 \cdot 7$	**4.** $(-3)(-3)(-3)$
5. $-7 \cdot 7 \cdot 7$	**6.** $(-4)(-4)(-4)$	**7.** $-5 \cdot 3 \cdot 3$	**8.** $-7 \cdot 7 \cdot 8 \cdot 8 \cdot 8$

For 9 - 16 , write in expanded form.

9. 7^5	**10.** $2 \cdot 7^3$	**11.** $4^3 8^4$	**12.** -9^2
13. -4^5	**14.** $(-4)^5$	**15.** $-2^3 8^4$	**16.** $(-6)^3$

For 17 - 48 , evaluate.

17. $4 + 10 - 5$	**18.** $8 - 2 - 7 + 2$
19. $2 + 7 \times 3$	**20.** $(2 + 7) \times 3$
21. $\frac{12+2}{4+3}$	**22.** $15 - 2(3)$
23. $3 - 5(-4)$	**24.** $8 - 5(2 - 10)$
25. $7 + 3(2 - 4)$	**26.** $9 - 2(4 - 7) + 6$

27. $8-2(9-7)$	**28.** $5+2(5-8)-7$
29. $-2[7+4(3-5)]$	**30.** 4^3
31. -4^2	**32.** $(-2)4^2$
33. $(-4)^2$	**34.** $(-3)^3$
35. -4^3	**36.** $(-4)^3$
37. $-2\cdot 5^2$	**38.** -3^4
39. $3+(7-2)^2$	**40.** $4+3(5-2)^3$
41. $10+2(3-5)^2$	**42.** $4+2(6-10)^2$
43. $2^0-4(3-5)^2$	**44.** $5-3(2-4)^3$
45. $10-(5-3)^2+2(-4)$	**46.** $15+(4-2)^3-3(-2)$
47. $3(-4)+(2-5)^3+5(-4)$	**48.** $12-(3-7)^2+3(-4)+2^0$

49. $3-2\|7-5\|$	**50.** $-5\|7-10\|+4$

Answers:

1. 3^3
2. $2\cdot 7^3$
3. $4\cdot 8^2\cdot 7^2$
4. $(-3)^3$
5. -7^3
6. $(-4)^3$
7. $-5\cdot 3^2$
8. $-7^2\cdot 8^3$
9. $7\cdot 7\cdot 7\cdot 7\cdot 7$
10. $2\cdot 7\cdot 7\cdot 7$
11. $4\cdot 4\cdot 4\cdot 8\cdot 8\cdot 8\cdot 8$
12. $-9\cdot 9$
13. $-4\cdot 4\cdot 4\cdot 4\cdot 4$
14. $(-4)\cdot(-4)\cdot(-4)\cdot(-4)\cdot(-4)$
15. $-2\cdot 2\cdot 2\cdot 8\cdot 8\cdot 8\cdot 8$
16. $(-6)\cdot(-6)\cdot(-6)$
17. 9
18. 1
19. 23
20. 27
21. 2
22. 9
23. 23
24. 48
25. 1
26. 21
27. 4
28. -8
29. 2
30. 64
31. -16
32. -32
33. 16
34. -27
35. -64
36. -64
37. -50
38. -81
39. 28
40. 85
41. 18
42. 36
43. -15
44. 29
45. -2
46. 29
47. -59
48. -15
49. -1
50. -11

2E – Prime Factorization, Least Common Multiple and Greatest Common Divisor

The integer 3 is a **factor** of 6, since 3 divides 6 evenly (without a remainder). We can write 6 as the product $2 \cdot 3$ The word "factor" is also used as a verb, as in : we factor 6 as $2 \cdot 3$. We also say that $2 \cdot 3$ is a **factorization** of 6.

Note: Since a factor of a number divides into the number evenly, a factor is also called a **divisor**.

The number 12 can be factored as $1 \cdot 12$ or $3 \cdot 4$ or $2 \cdot 6$ or $2 \cdot 2 \cdot 3$. The numbers $1, 2, 3, 4,$ 6 and 12 are all factors of 12.

An integer is called a **prime number** if it is greater than 1, and if its only factors are1 and itself. The first five primes are: $2, 3, 5, 7$ and 11.

A **prime factorization** is a factorization where all the factors are prime numbers.

EXAMPLE 1:

A) List all the two-factor products of 18.

B) Write 18 as a product of prime factors.

SOLUTION:

A) $1 \cdot 18,\ 2 \cdot 9,\ 6 \cdot 3$

B) $2 \cdot 3 \cdot 3 = 2 \cdot 3^2$

The numbers 12 and 18 are both divisible by $1, 2, 3,$ and 6. The largest number that divides both is called the **greatest common divisor (GCD)**. Thus, 6 is the greatest common divisor of 12 and 18.

EXAMPLE 2: Find the GCD of 30 and 45.

SOLUTION:

The divisors of 30 are: $1, 2, 3, 5, 6, 10, 15$, and 30.
The divisors of 45 are: $1, 3, 5, 9, 15$, and 45.
The **GCD** is 15.

We can also find the GCD of two integers from their prime factorizations. The GCD contains the prime factors that are common to both numbers. Each prime factor appears the least number of times it occurs in each number.

EXAMPLE 3: Find the GCD of 18 and 60.

SOLUTION:

$18 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$ and $60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$

The prime 2 appears once in 18 and twice in 60, so we include It once in the GCD. The prime 3 appears twice in 18 and once in 60, so we include it once in the GCD. The GCD is $2 \cdot 3 = 6$.

The number 6 is a **multiple** of 3 since $6 = 2 \cdot 3$. Stated differently, 3 divides 6 evenly. Some more multiples of 3 are $9, 12, 15, 18, 21,$ and 24.

The **least common multiple (LCM)** of two integers is the smallest positive integer that is a multiple of both integers.

EXAMPLE 4: Find the LCM of 12 and 18.

SOLUTION:

The first few multiples of 12 are: $12, 24, 36, 48$.
The first few multiples of 18 are: $18, 36, 54, 72$.
The number 36 is the smallest multiple that is common to both 12 and 18. It is the LCM.

We can also find the LCM of two integers from their prime factorizations. The LCM is the product of the prime factors of each number, each prime appearing the greatest number of times it occurs in each number.

EXAMPLE 5: Find the LCM of 36 and 120 from their prime factorizations.

SOLUTION:

The prime factorization of 36 is $2^2 3^2$.
The prime factorization of 120 is $2^3 3 \cdot 5$.
The LCM contains the prime factors: $2, 3,$ and 5. Prime number 2 occurs three times, prime number 3 occurs two times, and prime number 5 occurs one time.. The LCM is $2^3 3^2 5 = 360$.

2E – EXERCISES

For 1 – 4, state all the factors (divisors) of the given number.

1. 5 **2.** 9 **3.** 12 **4.** 42

5. What is a prime number?

For 6 – 9, give the prime factorization of the given number.

6. 3 **7.** 15 **8.** 105 **9.** 420

For 10 – 15, find the greatest common divisor of the pair of numbers.

10. 9 and 3 **11.** 15 and 10 **12.** 12 and 18

13. 75 and 30 **14.** 36 and 60 **15.** 125 and 75

For 16 – 21, find the least common multiple of the pair of numbers.

16. 5 and 7 **17.** 3 and 7 **18.** 7 and 14
19. 21 and 14 **20.** 30 and 42 **21.** 18 and 30

2E - WORKSHEET: Prime Factorization, Least Common Multiple and Greatest Common Divisor

For 1 – 6, find the prime factorization.

1. 24	**2.** 36	**3.** 28
4. 240	**5.** 360	**6.** 480

For 7 – 12, find the greatest common divisor.

7. 25 and 15	**8.** 12 and 10	**9.** 21 and 15
10. 24 and 18	**11.** 36 and 45	**12.** 60 and 45

For 13 – 18, find the least common multiple.

13. 12 and 6	**14.** 6 and 9	**15.** 10 and 15
16. 25 and 20	**17.** 12 and 15	**18.** 21 and 15

Answers:

1. $2^3 3$	**2.** $2^2 3^2$	**3.** $2^2 7$
4. $2^4 3 \cdot 5$	**5.** $2^3 3^2 5$	**6.** $2^5 3 \cdot 5$
7. 5	**8.** 2	**9.** 3
10. 6	**11.** 9	**12.** 15

13. 12
14. 18
15. 30
16. 100
17. 60
18. 105

2 – Answers to Exercises

Section A

1.	6	**2.**	-2	**3.**	-2	**4.**	-7	**5.**	4
6.	-3	**7.**	40	**8.**	-20	**9.**	-50	**10.**	-60
11.	-60	**12.**	-44	**13.**	10	**14.**	13	**15.**	20
16.	-2	**17.**	-5	**18.**	-4	**19.**	20	**20.**	50
21.	2	**22.**	-10	**23.**	-15	**24.**	-27		

Section B

1.	10	**2.**	-13	**3.**	-11	**4.**	-15	**5.**	-18
6.	-9	**7.**	23	**8.**	25	**9.**	39	**10.**	41
11.	68	**12.**	47	**13.**	41	**14.**	45	**15.**	60
16.	-35	**17.**	-77	**18.**	-56	**19.**	-35	**20.**	-38
21.	-69	**22.**	-58	**23.**	-75	**24.**	-84	**25.**	3
26.	7	**27.**	30	**28.**	43	**29.**	-31	**30.**	-35
31.	0	**32.**	-12	**33.**	3				

Section C

1.	-84	**2.**	-60	**3.**	-210	**4.**	-84
5.	-160	**6.**	-105	**7.**	100	**8.**	1035
9.	840	**10.**	1768	**11.**	-1472	**12.**	-1748
13.	-5	**14.**	-2	**15.**	-5	**16.**	-5
17.	-2	**18.**	-3	**19.**	-7	**20.**	-8
21.	5	**22.**	3	**23.**	5	**24.**	4
25.	2	**26.**	11	**27.**	6	**28.**	5
29.	30	**30.**	60	**31.**	-168		

Section D

1.	4^3	**2.**	$3 \cdot 5^4$	**3.**	$2^3 \cdot 7^2$				
4.	$(-4)^3$	**5.**	-5^4	**6.**	$(-5)^4$				
7.	$-2 \cdot 3^2$	**8.**	$-7^2 \cdot 6^3$	**9.**	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$				
10.	$2 \cdot 5 \cdot 5 \cdot 5$	**11.**	$2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7 \cdot 7$	**12.**	$-3 \cdot 5 \cdot 5$				
13.	$-3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	**14.**	$(-3)(-3)(-3)(-3)(-3)$	**15.**	$-2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5$				
16.	$(-7)(-7)(-7)$								
17.	5	**18.**	-6	**19.**	17	**20.**	21	**21.**	2
22.	4	**23.**	31	**24.**	37	**25.**	-4	**26.**	17
27.	0	**28.**	-15	**29.**	18	**30.**	125	**31.**	-25
32.	-18	**33.**	25	**34.**	-8	**35.**	-27	**36.**	-27
37.	-75	**38.**	-16	**39.**	12	**40.**	26	**41.**	13
42.	51	**43.**	-5	**44.**	42	**45.**	5	**46.**	12
47.	-54	**48.**	-24	**49.**	-15	**50.**	-9	**51.**	13
52.	2								

Section E

1. 1, 5
2. 1, 3, 9
3. 1, 2, 3, 4, 6, 12
4. 1, 2, 3, 6, 7, 14, 21, 42
5. An integer greater than one, whose only divisors are one and itself.

6. 3
7. $3 \cdot 5$
8. $3 \cdot 5 \cdot 7$
9. $2^2 \cdot 3 \cdot 5 \cdot 7$
10. 3
11. 5
12. 6
13. 15
14. 12
15. 25
16. 35
17. 21
18. 14
19. 42
20. 210
21. 90