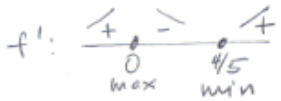
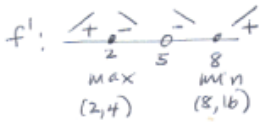
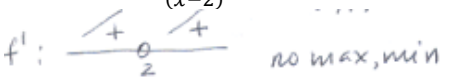
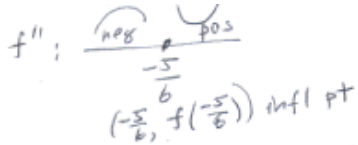
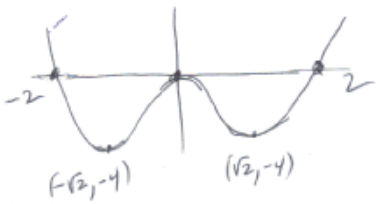
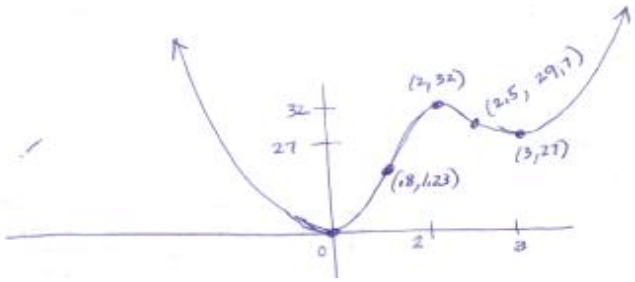
Math 15 Answers to review exam #2

1	-4
2	a) 224 ft b) 8 sec. c) 1024 ft. d) t=16 sec e) -256 ft/sec f) a(t)=-32
3	a) $f'(x) = \frac{27}{2}x^{7/2} - \frac{7}{2}x^{-\frac{1}{2}} - 5x^{-\frac{3}{2}}$ b) $f'(x) = (3x+2)^{10}(2x+7) + 10(3x+2)^9(3)(x^2+7)$ c) $f'(x) = \frac{(2x-3)(50)(5x+7)^{49}(5)-(2)(5x+7)^{50}}{(2x-3)^2}$
4	$y = \frac{-y^2+2yx}{2xy-x^2}$; $y'at\ (1,2) = 0$;equation is $y = 2$ (horizontal line).
5	$\frac{1}{10\pi}\ ft/min\ = \frac{dr}{dt}$
6	$\frac{dr}{dt} = \frac{2}{5\pi}\ ft/min$
7	$\frac{dy}{dt} = -\frac{9}{4} ft/sec$
8	$(2,0)$ is location of abs min; $(5,9)$ is location of abs max.
9	A) max $(0,0)$, min $(8,-256)$ B) $f'(x) = 5x^{\frac{2}{3}} = \frac{4}{x^{\frac{1}{3}}} = \frac{5x-4}{x^{\frac{1}{3}}}$ $cv\ at\ x = \frac{4}{5}, x = 0;$   C) $f'(x) = \frac{(x-8)(x-2)}{(x-5)^2}$ D) $f'(x) = \frac{x^2-4x+10}{(x-2)^2}$ the numerator is never 0; 
10	A) $f'(x) = 3x^2 + 5x - 14;\ f''(x) = 6x + 5 = 0,\ x = -\frac{5}{6};$  B) $f'(x) = 12x^3 + 12x^2 + 12x;\ f''(x) = 36x^2 + 24x + 12 = 12(3x^2+2x+1) = 0$ Now $b^2 - 4ac < 0\ \rightarrow numerator\ never\ 0\ \rightarrow always\ concave\ up.$

11	A) $\lim_{x\to\infty} \frac{x^5}{-2x^2} = -\infty$ $\lim_{x\to-\infty} -\frac{1}{2}x^3 = \infty$ B) $\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x^2}} = \lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0$ C) $\lim_{x\to\infty} \frac{7x^5}{3x^5} = \frac{7}{3}$ $\lim_{x\to-\infty} \frac{7x^5}{3x^5} = \frac{7}{3}$ D) $\lim_{x\to\infty} -3x^{16} = -\infty$ $\lim_{x\to-\infty} -3x^{16} = -\infty$
12A	A) $f(x) = x^2(x-2)(x+2)$ x-int : $(2,0), (-2,0), (0,0)$ y-int : $(0,0)$ $\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = -\infty$ $f'(x) = 4x(x-\sqrt{2})(x+\sqrt{2})$ f' : − + − + at $-\sqrt{2}$, 0, $\sqrt{2}$ min max min $(-\sqrt{2}, -4)$ $(0,0)$ $(\sqrt{2}, -4)$ $f''(x) = 4(3x^2-2) = 0$ $x = \pm\sqrt{\frac{2}{3}}$ f'' : + − + at $-\sqrt{\frac{2}{3}}$, $\sqrt{\frac{2}{3}}$ infl infl $(-\sqrt{\frac{2}{3}}, \frac{-20}{9})$ $(\sqrt{\frac{2}{3}}, \frac{-20}{9})$ 

12 B	12 B) $f(x) = 3x^4 - 20x^3 + 36x^2 = x^2(3x^2 - 20x + 36) = 0$ ← $b^2-4ac<0$ only zero of $f(x)$ x-int: $x=0$ $(0,0)$ y-int $(0,0)$ $\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = \infty$ $f'(x) = 12x^3 - 60x^2 + 72x = 12x(x^2 - 5x + 6) = 12x(x-3)(x-2)$ crit nos are $x=0, x=3, x=2$ f': neg, pos, neg, pos 0 min, 2 max, 3 min $(0,0)$ $(2,32)$ $(3,27)$ $f''(x) = 12(3x^2 - 10x + 6) = 0$ $x = \frac{10 \pm \sqrt{28}}{6} = \frac{5 \pm \sqrt{7}}{3}$ f'': pos, neg, pos $\frac{5-\sqrt{7}}{3} \cong .8$ $\frac{5+\sqrt{7}}{3} \cong 2.5$ inflec inflec $(.8, f(.8))$ $(2.5, f(2.5))$ $= (.8, 1.23)$ $= (2.5, 29.7)$ 

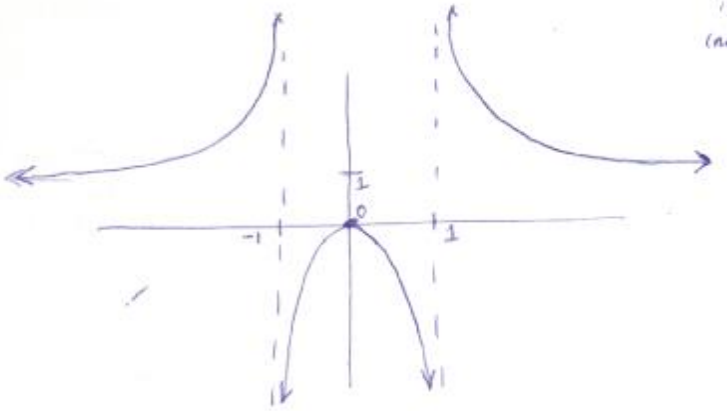
12 C	12c) $f(x) = \dfrac{x^2}{x^2-1}$

x-int $(0,0)$ y-int $(0,0)$ vertical asymptotes at $x=1,\ x=-1$ $\lim_{x\to\infty} \frac{x^2}{x^2} = 1$ $\lim_{x\to-\infty} \frac{x^2}{x^2} = 1$

$f'(x) = \dfrac{-2x}{(x^2-1)^2}$ ← num. zero at $x=0$ ← denom zero at $x=1, x=-1$

f': pos pos neg neg; -1, 0, 1; max; $(0,0)$ max

$f''(x) = \dfrac{6x^2+2}{(x^2-1)^3}$ ← num never 0 ← denom zero at $x=1, x=-1$

f'': pos neg pos; -1, 1

these are not inflection points, since they are not in the domain of $f(x)$. (no inflection points)