Calculus 1 **Review for exam #3 - sections** [redacted] [redacted]

[redacted]

1. Find the point on the graph of $y = \sqrt{x+2}$ closest to (6,0).

2. An open box is made from a square piece of material, 8 inches on a side, by cutting squares from each corner and turning up the sides. Find the volume of the largest box that can be made.

3. The sum of 4 times a first number and 3 times a second number is 12. Maximize the product of the two numbers.

4. Use the method of differentials to approximate $\sqrt{9.01}$.

5. Find $\frac{dy}{dx}$

a) $y = sinx cosx$ b) $y = [\cos(x^2) + 10]^3$ c) $y = \sec(sinx)$ d) $y = \tan(xy) + secx$

6.

a) $\int(3x^5 - 4x^3 - 7x + 2)dx =$	b) $\int \frac{5x^3 - 7x}{\sqrt{x}} dx =$
c) $\int(3x-2)(x+5)dx =$	d) $\int \frac{sinx}{cos^2 x} dx$
e) $\int xsec(x^2)\tan(x^2)\,dx$	f) $\int \frac{x}{\sqrt{5x^2+4}} dx$
g) $\int \frac{1}{t^2}(3 + \frac{5}{t})^{10}\,dt$	h) $\int \frac{\left(5 - x^{\frac{2}{3}}\right)^{10}}{\sqrt[3]{x}} dx$
i) $\int \frac{x}{\sqrt{x-1}} dx$	j) $\int \frac{x}{(x+1)^3} dx$

7. Evaluate the definite integrals: a) $\int_0^{\pi/2} \sin x\,dx$ b) $\int_0^1 (2x^2 - 5x)dx$

8. Find the area between : $y = -x^2 + 4x$ *and the x-axis.*

9. Find the *average* of function $y = 3x^2 + 1$ on the interval [1,3].

10. Find $a)$ $\frac{d}{dx}\int_1^x sint\,dt$ $b)$ $\frac{d}{dx}\int_1^{x^2} t^3\,dt$

11. $\lim_{x \to 0} \frac{\sin 5x}{x}$

Solutions Follow

(Answers posted on mathbreeze)

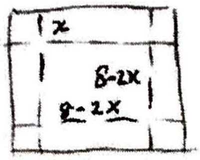
MTS Review exam 3 - solutions

① Minimize $D = \sqrt{(x-6)^2 + (y-0)^2} = \sqrt{(x-6)^2 + x + 2}$

$D' = \frac{1}{2}\left((x-6)^2 + x + 2\right)^{-1/2} \cdot \left(2(x-6)+1\right)$

$2(x-6)+1 = 0 \Rightarrow x = \frac{7}{2}, \quad y = \sqrt{\frac{7}{2}+2}$

②



minimize $V = x(8-2x)(8-2x)$

$= 4x(4-x)(4-x)$

$= 4x(16 - 8x + x^2) = 4(16x - 8x^2 + x^3)$

$V' = 4(16 - 16x + 3x^2) = 4(3x-4)(x-4)$

$x = \frac{4}{3}$ $\quad \cancel{x=4}$

③ $4x + 3y = 12 \Rightarrow 3y = 12 - 4x, \quad y = 4 - \frac{4}{3}x$

$P = xy = x\left(4 - \frac{4}{3}x\right) = 4x - \frac{4}{3}x^2$

$\frac{dP}{dx} = 4 - \frac{8}{3}x = 0 \qquad 4 = \frac{8}{3}x$

$\frac{3}{2} = \frac{3}{8} \cdot 4 = x$

$y = 4 - \frac{4}{3} \cdot \frac{3}{2} = 2$

④ $f(x) = \sqrt{x} \quad x_0 = 9 \quad dx = .01$

$f(x_0 + dx) \cong f(x_0) + f'(x_0)\,dx = \sqrt{9} + \frac{1}{2\sqrt{9}}(.01)$

$= 3 + \frac{1}{6}(.01) = 3.0016667$

⑤ a) $\cos^2 x - \sin^2 x$

b) $3\left[\cos(x^2) + 10\right]^2 \left(-\sin(x^2) \cdot 2x\right)$

c) $\sec(\sin x)\tan(\sin x) \cdot \cos x$

d) $y' = \sec^2(xy) \cdot [xy' + y] + \sec x \tan x$

$y' - \sec^2(xy) \cdot xy' = \sec^2(xy) \cdot y + \sec x \tan x$

$$y' = \frac{\sec^2(xy) \cdot y + \sec x \tan x}{(1 - \sec^2(xy) \cdot x)}$$

8) a) $3 \cdot \frac{x^6}{6} - 4 \cdot \frac{x^4}{4} - 7 \cdot \frac{x^2}{2} + 2x + C$

b) $\int 5x^{5/2} - 7x^{1/2}\, dx = 5x^{7/2} \cdot \frac{2}{7} - 7x^{3/2} \cdot \frac{2}{3} + C$

c) $\int (3x^2 + 13x - 10)\, dx = 3 \cdot \frac{x^3}{3} + 13 \cdot \frac{x^2}{2} - 10x + C$

d) $\frac{(\cos x)^{-1}}{+1} + C$

e) $\frac{1}{2} \sec(x^2) + C$

f) $2(5x^2 + 4)^{1/2} \cdot \frac{1}{10} + C$

g) $\frac{(3 + \frac{5}{t})^{11}}{11} \cdot \left(-\frac{1}{5}\right) + C$

h) $\frac{(5 - x^{2/3})^{11}}{11} \left(-\frac{3}{2}\right) + C$

i) $\frac{2}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + C$

j) $\frac{(x+1)^{-1}}{-1} + \frac{(x+1)^{-2}}{-2} + C$

7 a) $\int_0^{\pi/2} \sin x \, dx = -\cos x \Big|_0^{\pi/2} = -\cos\frac{\pi}{2} + \cos 0 = \boxed{1}$

b) $\int_0^1 (2x^2-5x)\,dx = \left[\frac{2x^3}{3} - \frac{5x^2}{2}\right]_0^1 = \left[\frac{2}{3} - \frac{5}{2}\right] - [0] = \boxed{-\frac{11}{6}}$

8 $-x^2+4x=0$ $\quad x=0,\ x=4$

$-x(x-4)=0$

$\int_0^4 -x^2+4x \, dx = \left[\frac{-x^3}{3} + 4\cdot\frac{x^2}{2}\right]_0^4$

$= -\frac{64}{3} + 32 = \boxed{\frac{32}{3}}$

9 $\frac{1}{3-1}\int_1^3 3x^2+1 \, dx = \frac{1}{2}\left[\frac{3x^3}{3} + x\right]_1^3 = \frac{1}{2}\left([27+3]-[1+1]\right)$

$= \frac{1}{2}(27+3-2) =$

$= \boxed{14}$

10 a) $\boxed{\sin x}$ b) $(x^2)^3 \cdot 2x = 2x^6 \cdot x = \boxed{2x^7}$