SOLUTIONS

Math 15 Review for final exam

1 Find the limit (if it exists; if not explain why):

A. $\lim_{x\to\infty}\frac{6x^7-100x^4+300}{2x^6-1000x^5} = \lim_{x\to\infty}\frac{6x^7}{2x^6} = \lim_{x\to\infty} 3x = \infty$

B. $\lim_{x\to 6}\frac{\sqrt{x-2}-2}{x-6}\cdot\frac{\sqrt{x-2}+2}{\sqrt{x-2}+2} = \lim_{x\to 6}\frac{x-6}{(x-6)(\sqrt{x-2}+2)} = \lim_{x\to 6}\frac{1}{\sqrt{x-2}+2} = \boxed{\frac{1}{4}}$

C. $\lim_{x\to 3}\frac{x-3}{x^2-5x+6} = \lim_{x\to 3}\frac{x-3}{(x-2)(x-3)} = \lim_{x\to 3}\frac{1}{x-2} = \boxed{1}$

D. $\lim_{x\to 3}\begin{cases}3x+2, & x<3\\ 2x+1, & x\geq 3\end{cases}$ $\lim_{x\to 3^-} = 3(3)+2 = 11$; $\lim_{x\to 3^+} = 2(3)+1 = 7$; $\lim_{x\to 3^-} \neq \lim_{x\to 3^+} \Rightarrow \lim_{x\to 3}$ DNE

E. $\lim_{x\to 1^+}\frac{x-3}{(x-1)(x-5)}$ pos $\quad 1 \quad 3 \quad 5 \quad \boxed{+\infty}$

F. $\lim_{x\to 5}\frac{|x-5|}{x-5}$ limit DNE

G. $\lim_{x\to\infty}\frac{5x^7-100x^2}{4x^8+10x} = \lim_{x\to\infty}\frac{5}{4x} = \boxed{0}$

2 Compute $\frac{dy}{dx}$ of the following functions:

A. $y = 5x^7\sqrt[3]{10x^2+3}$ $\quad y' = 5x^7\cdot\frac{1}{3}(10x^2+3)^{-2/3}(20x) + 35x^6\sqrt[3]{10x^2+3}$

B. $y = \frac{3x^2+10}{\sqrt[3]{x^2+4}}$ $\quad y' = \frac{(\sqrt[3]{x^2+4})(6x) - \frac{1}{3}(x^2+4)^{-2/3}(2x)(3x^2+10)}{(\sqrt[3]{x^2+4})^2}$

C. $y = x^5(sin3x+10)^5$ $\quad y' = x^5\cdot 5(\sin 3x+10)^4(3\cos 3x) + 5x^4(\sin 3x+10)^4$

D. $xy^2+5x^3+2y = 9$ $\quad x\cdot 2yy' + 15x^2 + 2y' = 0$; $y' = \frac{-15x^2}{(2xy+2)}$

E. $y = (u^2+2u)^2,\ and\ u = \sqrt{x}+2$ $\quad 2(u^2+2u)(2u+2)\cdot\frac{1}{2\sqrt{x}} = 2\left((\sqrt{x}+2)^2+2(\sqrt{x}+2)\right)\cdot(2(\sqrt{x}+2)+2)\cdot\frac{1}{2\sqrt{x}}$

F. $y = [3x+(5x+7)^{10}]^4$ $\quad y' = 4[3x+(5x+7)^{10}]^3\cdot(3+10(5x+7)^9\cdot 5)$

G. $y = \sec 5x$ $\quad y' = (\sec 5x\tan 5x)\cdot 5$

3 Integrate:

A. $\int(3x^7+8\sqrt[3]{x}+5)dx = \frac{3x^8}{8} + 8\cdot x^{4/3}\cdot\frac{3}{4} + 5x + C$

B. $\int\frac{7x^6-8x^3}{\sqrt[3]{x}}dx = \int 7x^{17/3} - 8x^{8/3}\,dx = 7x^{20/3}\cdot\frac{3}{7} - 8x^{11/3}\cdot\frac{3}{11} + C$

C. $\int_1^2(x^2+2)dx = \left[\frac{x^3}{3}+2x\right]_1^2 = \left[\frac{8}{3}+4\right]-\left[\frac{1}{3}+2\right]$

D. $\int\frac{x}{\sqrt{x^2+10}}dx$ $\quad u = x^2+10$; $du = 2x\,dx$; $\frac{1}{2}du = x\,dx$ $\quad = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}u^{1/2}\cdot 2 + C$

$= \sqrt{x^2+10} + C$

$u = x-1$; $u+1 = x$
$du = dx$

	E. $\int x(x-1)^8 dx$ $= \int (u+1)u^8\,du = \int u^9 + u^8\,du = \frac{u^{10}}{10} + \frac{u^9}{9} + C = \frac{(x-1)^{10}}{10} + \frac{(x-1)^9}{9} + C$ F. $\int \frac{1}{t^2}\left(1+\frac{1}{t}\right)^{10} dt$; $u = 1+\frac{1}{t}$; $du = -\frac{1}{t^2}dt$; $= -\int u^{10}\,du = -\frac{u^{11}}{11} + C = -\frac{(1+\frac{1}{t})^{11}}{11} + C$
4	Recall: $f(x)$ is continuous at $x = c$ if *I. $f(c)$ exists* *II.* $\lim_{x\to c} f(x)$ exists *III.* $\lim_{x\to c} f(x) = f(c)$ For A-D circle the roman numerals that are true for $f(x)$ *and* c. A. $f(x) = \frac{x-5}{x-2}$, $c = 7$: *I II III* B. $f(x) = 3x^2 + 5x - 2$, $c = 3$: *I II III* C. $f(x) = \frac{x-3}{x^2-5x+6}$, $c = 3$: *I II III* D. $f(x) = \frac{\lvert x-3\rvert}{x-3}$, $c = 3$: *I II III* (None)
5	The position after t seconds of a particle moving along the x-axis is given by $x(t) = t^2 - 10t + 1$. Find A. The position after 2 seconds. $x(2) = 4 - 20 + 1 = -15$ B. When the particle changes direction. $v(t) = 2t - 10 = 0 \Rightarrow t = 5$ C. The position when the particle changes direction. $x(5) = 25 - 50 + 1 = -24$ D. The acceleration after 2 seconds. $x''(t) = 2$; $x''(2) = 2$
6	A square on a computer screen is increasing in size. The area is increasing at the rate of $2\frac{cm^2}{min}$. When the side of the square is 10 cm long, find the rate at which the side of the square in increasing. $A = x^2$ $\frac{dA}{dt} = 2$ $\frac{dA}{dt} = 2x\frac{dx}{dt}$; $2 = 2(10)\frac{dx}{dt}$ $\Rightarrow \frac{dx}{dt} = \frac{1}{10}$ cm/min
7	Find where $f(x) = x^4 - 8x^2$ in increasing and decreasing, concave up and concave down. Find any max, min points ,inflection points and asymptotes.
8	Given that $3x + y = 12$, *maximize* $A = xy$.
9	A yard bordering on a river has a perimeter of 150 feet. What are the dimensions if the area is maximum.
10	A projectile is shot in the air from the ground. The position of the projectile after t seconds is given by $s(t) = -16t^2 + 64t$. A) Find the initial velocity. B) When does the projectile reach a maximum height? C) What is the maximum height? D) When does the profectile return to the ground? E) What is the velocity when the projectile hits the ground?
11	A volume is changing over time. The volume is given by $V = 3xy^2$, where x *and* y are also changing with time. If $\frac{dv}{dt} = 5$, and $x = 2y$, find $\frac{dy}{dt}$ when $y = 4$.

7. $f(x) = x^4 - 8x^2$

$$f'(x) = 4x^3 - 16x = 4x(x^2-4) = 4x(x+2)(x-2) = 0$$

$f'(x)$: neg ($f \searrow$) • -2 • pos ($f \nearrow$) • 0 • neg ($f \searrow$) • 2 • pos ($f \nearrow$)

-2: min, 0: max, 2: min

$(-2, f(-2))$ $(0, f(0))$ $(2, f(2))$ $f(2) = 16 - 8(4) = -16$

$(-2,-16)$ $(0,0)$ $(2,-16)$

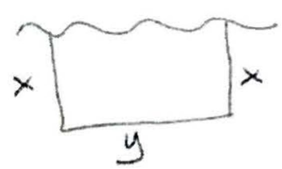
$$f''(x) = 12x^2 - 16 = 4(3x^2-4) = 0 \qquad 3x^2 = 4 \quad x^2 = \frac{4}{3} \qquad x = \pm\sqrt{\frac{4}{3}}$$

$f''(x)$: pos ($\cup$) • $-\sqrt{\frac{4}{3}}$ • neg ($\cap$) • $\sqrt{\frac{4}{3}}$ • pos ($\cup$)

$$f\left(\sqrt{\tfrac{4}{3}}\right) = \frac{16}{9} - \frac{8}{1}\cdot\frac{4}{3} = \frac{16}{9} - \frac{96}{9} = \frac{-80}{9}$$

$\left(-\sqrt{\frac{4}{3}}, \frac{-80}{9}\right)$ inflec. $\left(\sqrt{\frac{4}{3}}, \frac{-80}{9}\right)$ inflec.

9.



$2x + y = 150 \Rightarrow y = 150 - 2x$

$A = xy = x(150-2x) = 150x - 2x^2$

$\frac{dA}{dx} = 150 - 4x = 0$

$150 = 4x$

$\boxed{\frac{150}{4} = x}$

$y = 150 - \frac{2}{1}\cdot\frac{150}{4}$

$\boxed{y = \frac{150}{2}}$

8. $A = xy \qquad y = 12 - 3x$

$A = x(12-3x) = 12x - 3x^2$

$\frac{dA}{dx} = 12 - 6x = 0 \Rightarrow \boxed{x = 2}$ and $y = 12 - 3(2) = 6$ $\boxed{y = 6}$

(10) $s(t) = -16t^2 + 64t$

A) $v(t) = -32t + 64$, $v(0) = 64$ ft/sec

B) $v(t) = -32t + 64 = 0 \Rightarrow t = 2$ sec.

C) $s(2) = -16(4) + 64(2) = -64 + 128 = 64$ ft.

D) $s(t) = -16t^2 + 64t = 0$

$-16t(t-4) = 0$

$\cancel{t=0}$ $t = 4$

E) $v(4) = -32(4) + 64 = -64$ ft/sec.

(11) $V = 3xy^2$, $x = 2y$

$V = 3(2y)y^2 = 6y^3$

$\frac{dV}{dt} = 18y^2 \frac{dy}{dt}$

sub $\frac{dV}{dt} = 5$ and $y = 4$

$5 = 18(16)\frac{dy}{dt}$

$\boxed{\frac{5}{288}} = \frac{5}{18(16)} = \frac{dy}{dt}$