SOLUTION

Since the graph of the pre-1979 model lies above the post-1979 graph on the interval [9, 16], the amount of gasoline saved is given by the following integral.

$$\int_9^{16} [\overbrace{(0.000433t^2 + 0.0962t + 2.76)}^{f(t)} - \overbrace{(-0.00831t^2 + 0.152t + 2.81)}^{g(t)}]\,dt$$

$$= \int_9^{16} (0.008743t^2 - 0.0558t - 0.05)\,dt$$

$$= \left[\frac{0.008743t^3}{3} - \frac{0.0558t^2}{2} - 0.05t\right]_9^{16}$$

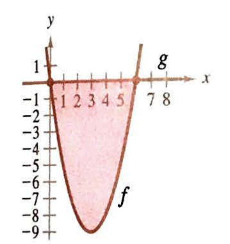
$$\approx 4.58 \text{ billion barrels}$$

Therefore, approximately 4.58 billion barrels of fuel were saved. (At 42 gallons per barrel, about 200 billion gallons were saved!) ▭

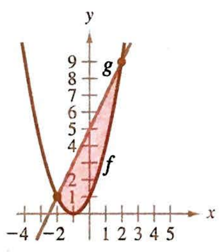
EXERCISES for Section 6.1

In Exercises 1–6, find the area of the given region.

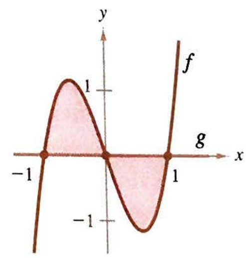
1. $f(x) = x^2 - 6x$
$g(x) = 0$



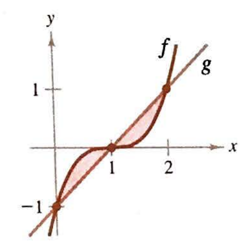
2. $f(x) = x^2 + 2x + 1$
$g(x) = 2x + 5$



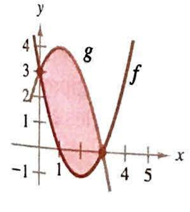
5. $f(x) = 3(x^3 - x)$
$g(x) = 0$



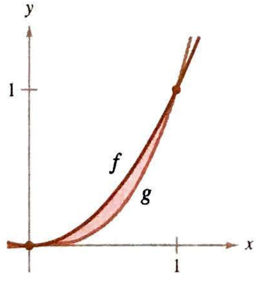
6. $f(x) = (x - 1)^3$
$g(x) = x - 1$



3. $f(x) = x^2 - 4x + 3$
$g(x) = -x^2 + 2x + 3$



4. $f(x) = x^2$
$g(x) = x^3$



In Exercises 7 and 8, determine which value best approximates the area of the region bounded by the graphs of f and g. (Make your selection on the basis of a sketch of the region and *not* by performing any calculations.)

7. $f(x) = x + 1$, $g(x) = (x - 1)^2$
(a) -2 (b) 2 (c) 10 (d) 4 (e) 8

8. $f(x) = 2 - \frac{1}{2}x$, $g(x) = 2 - \sqrt{x}$
(a) 1 (b) 6 (c) -3 (d) 3 (e) 4

In Exercises 9–26, sketch the region bounded by the graphs of the given functions and find the area of the region.

9. $f(x) = x^2 - 4x$, $g(x) = 0$
10. $f(x) = 3 - 2x - x^2$, $g(x) = 0$
11. $f(x) = x^2 + 2x + 1$, $g(x) = 3x + 3$
12. $f(x) = -x^2 + 4x + 2$, $g(x) = x + 2$
13. $y = x$, $y = 2 - x$, $y = 0$
14. $y = \frac{1}{x^2}$, $y = 0$, $x = 1$, $x = 5$
15. $f(x) = 3x^2 + 2x$, $g(x) = 8$
16. $f(x) = x(x^2 - 3x + 3)$, $g(x) = x^2$
17. $f(x) = x^3 - 2x + 1$, $g(x) = -2x$, $x = 1$
18. $f(x) = \sqrt[3]{x}$, $g(x) = x$
19. $f(x) = \sqrt{3x + 1}$, $g(x) = x + 1$
20. $f(x) = x^2 + 5x - 6$, $g(x) = 6x - 6$
21. $y = x^2 - 4x + 3$, $y = 3 + 4x - x^2$
22. $y = x^4 - 2x^2$, $y = 2x^2$
23. $f(y) = y^2$, $g(y) = y + 2$
24. $f(y) = y(2 - y)$, $g(y) = -y$
25. $f(y) = y^2 + 1$, $g(y) = 0$, $y = -1$, $y = 2$
26. $f(y) = \frac{y}{\sqrt{16 - y^2}}$, $g(y) = 0$, $y = 3$

In Exercises 27 and 28, use a symbolic integration utility to graph the region bounded by the graphs of the functions and find the area of the region.

27. $f(x) = x^4$, $g(x) = 3x + 4$
28. $f(x) = x^6$, $g(x) = x + 2$

In Exercises 29–32, use integration to find the area of the triangle having the given vertices.

29. (0, 0), (4, 0), (4, 4)
30. (0, 0), (4, 0), (6, 4)
31. (0, 0), $(a, 0)$, (b, c)
32. (2, −3), (4, 6), (6, 1)

In Exercises 33 and 34, find b so that the line $y = b$ divides the region bounded by the graphs of the two equations into two regions of equal area.

33. $y = 9 - x^2$, $y = 0$
34. $y = 9 - |x|$, $y = 0$

35. The graphs of $y = x^4 - 2x^2 + 1$ and $y = 1 - x^2$ intersect at three points. However, the area between the curves *can* be found by a single integral. Explain why this is so, and write an integral for this area.

36. The area of the region bounded by the graphs of $y = x^3$ and $y = x$ *cannot* be found by the single integral

$$\int_{-1}^{1} (x^3 - x)\,dx.$$

Explain why this is so. Use symmetry to write a single integral that does represent the area.

In Exercises 37 and 38, find the area of the region bounded by the graph of the function and the tangent line to the graph at the specified point.

Function	*Point*
37. $f(x) = x^3$	(1, 1)
38. $f(x) = \sqrt[3]{x - 1}$	(2, 1)

In Exercises 39 and 40, use a computer or calculator and Simpson's Rule (with $n = 4$) to approximate the area of the region bounded by the graphs of the given equations.

39. $y = \sqrt{1 + x^3}$, $y = \frac{1}{2}x + 2$, $x = 0$
40. $y = \sqrt{x + x^2}$, $y = 0$, $x = 0$, $x = 1$

In Exercises 41 and 42, evaluate the given limit and sketch the graph of the region whose area is given by the limit.

41. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (x_i - x_i^2)\,\Delta x$
where $x_i = i/n$ and $\Delta x = 1/n$

42. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (4 - x_i^2)\,\Delta x$
where $x_i = -2 + (4i/n)$ and $\Delta x = 4/n$

In Exercises 43 and 44, two models R_1 and R_2 are given for revenue (in billions of dollars) for a large corporation. The model R_1 gives projected annual revenues from 1990 to 1995, with $t = 0$ corresponding to 1990, and R_2 gives projected revenues if there is a decrease in growth of corporate sales over the period. Approximate the total reduction in revenue if corporate sales are actually closer to the model R_2.

43. $R_1 = 7.21 + 0.58t$
$R_2 = 7.21 + 0.45t$
44. $R_1 = 7.21 + 0.26t + 0.02t^2$
$R_2 = 7.21 + 0.1t + 0.01t^2$