Review for MAT 900/9010/9B0 Final Exam

1. Evaluate the expression $3x - y(3y - x)$ if $x = -2$ and $y = 6$.

2. Simplify.

    (a) $\left(\dfrac{x^7 z^4}{x^9 y^0} \right)^{-3}$

    (b) $\left(\dfrac{x^2 y^{-4}}{x^{-3} z^5} \right)^{3}$

3. Simplify.

    (a) $(4x^3 - 5x + 3) + (2x - 1)$

    (b) $(4x^3 - 9x^2 + 11) + (2x^2 - 7x + 3) - (-3x^3 + 5x - 8)$

4. Multiply and simplify.

    (a) $(3x - 5)(x + 1)$

    (b) $(2x - 3)(x^2 - 5x + 2)$

    (c) $(3x - 5)^2$

    (d) $(2x - 3y)^3$

5. (a) Divide: $(y^3 - 7y + 3) \div (y + 5)$

    (b) Determine the remainder when $2y^4 + 5y^3 - y + 6$ is divided by $y - 2$.

6. Factor completely.

    (a) $12x^3 y^5 z - 21x^2 y^6$

    (b) $6x^7 - 15x^5 + 3x^4$

7. Factor completely.

    (a) $4x^6 y^2 - 25z^4$

    (b) $144x^8 - 1$

8. Factor completely.

    (a) $3x^2 - x - 14$

    (b) $20x^2 + 11x - 3$

    (c) $6x^2 + 17x + 12$

    (d) $12x^2 - 11x + 2$

9. Factor completely.

    (a) $6x^3 - 2x^2 + 15x - 5$

    (b) $6xy + 14y - 15x - 35$

10. Factor completely.

    (a) $64t^3 - 1$

    (b) $27x^3 + 8y^3$

11. Factor completely.

(a) $10xy^4 - 90xy^2$

(b) $3x^3 - 75x$

12. Simplify.

(a) $\dfrac{2x^3}{x^2 - 4x - 12} \cdot \dfrac{x^2 - 36}{10x^4}$

(b) $\dfrac{y^2 + 4y + 16}{y^2 - 16} \div \dfrac{y^3 - 64}{y + 4}$

13. Simplify.

(a) $\dfrac{9}{2x - 7} + \dfrac{3}{7 - 2x}$

(b) $\dfrac{4b}{7b - 2} - \dfrac{3b + 2}{2 - 7b}$

14. Simplify.

(a) $\dfrac{4}{x^2 - 3x} + \dfrac{x}{x^2 + 2x - 15}$

(b) $\dfrac{x}{x^2 - 4} - \dfrac{3}{x^2 - 2x}$

15. Simplify.

(a) $\dfrac{\dfrac{2}{x} - \dfrac{3}{7x}}{3 + 2x}$

(b) $\dfrac{\dfrac{1}{12x^2} - \dfrac{5}{4x}}{\dfrac{2}{3x^2} + 1}$

16. Given that $x > 0$, divide and simplify.

(a) $\dfrac{\sqrt{45x^8}}{\sqrt{5x^3}}$

(b) $\dfrac{\sqrt{36x^9}}{\sqrt{3x^5}}$

17. Simplify.

(a) $5\sqrt{49} - 6\sqrt{7} + 3\sqrt{144} + \sqrt{28}$

(b) $\left(2\sqrt{16} - 3\sqrt{20}\right)\left(\sqrt{45} + 4\sqrt{25}\right)$

18. Solve for x.

(a) $4(x + 2)^2 = 20$

(b) $3(x - 5)^2 - 21 = 0$

19. Rationalize the denominator.

(a) $\dfrac{2 + \sqrt{3}}{1 + \sqrt{3}}$

(b) $\dfrac{2 + \sqrt{3}}{\sqrt{5} - 3}$

20. Given that $x > 0$, $y > 0$, and $z > 0$, simplify: $\sqrt[5]{243x^{10}y^7z^5}$

21. Evaluate: $-16^{-\frac{3}{2}}$

22. Solve for x: $8x^2 = 40x$

23. Given that $x > 0$, $y > 0$, and $z > 0$, simplify

    (a) $\sqrt[4]{\dfrac{81x^{20}}{y^{12}}}$

    (b) $\left(\dfrac{27x^6}{z^{12}}\right)^{\frac{1}{3}}$

24. Solve for x: $4x - (6 - x) = 3(x + 5) - 6$

25. (a) Solve for y: $8x - yz = 12y$

    (b) Solve for x: $2y - 7(3y - x) = 6x + 2$

26. (a) By completing the square, one can demonstrate that quadratic equation $x^2 - 8x + 5 = 0$ is equivalent to _____.

    (b) By completing the square, one can demonstrate that quadratic expression $x^2 + 6x + 3$ is equivalent to _____.

27. Solve for x.

    (a) $4x^2 + 17x = 15$

    (b) $10x^2 = 11x + 6$

28. Solve for x.

    (a) $x^2 = 5 - 3x$

    (b) $x^2 - 6x = 8$

29. Solve for x, and express the solution in both set and interval notation.

    (a) $-\dfrac{3x}{2} - \dfrac{7}{10} < \dfrac{1}{5}$

    (b) $\dfrac{4x}{5} + \dfrac{3}{2} \le \dfrac{7}{10}$

30. Solve for x, and express the solution in both set and interval notation.

    (a) $3(-7 - 2x) > -3$ or $2(5x - 3) > 9$

    (b) $-3 \le -5x + 2 \le 5$

31. Solve for x: $|4x - 2| - 6 = 4$

32. Solve for x, and express the solution in both set and interval notation.

    (a) $|2x - 1| < 4$

    (b) $|3x + 2| \ge 8$

33. Find the distance between the points $(-1, 5)$ and $(3, -3)$.

34. Find the slope and y-intercept of the line.

    (a) $2x - 3y = -9$

(b) $y - 2 = -3(x + 4)$

35. Solve for x.

    (a) $\dfrac{3}{x+4} - \dfrac{4}{x^2 - x - 20} = \dfrac{7}{x - 5}$

    (b) $\dfrac{x}{x-5} - \dfrac{5}{x+5} = \dfrac{10x}{x^2 - 25}$

36. Find the equation, in slope-intercept form, of the line passing through the points $(-1, 6)$ and $(10, -2)$.

37. (a) Find the equation of the vertical line passing through the point $(-3,\ 8)$.

    (b) Find the equation of the horizontal line passing through the point $(-3,\ 8)$.

    (c) Find the equation of the line, with slope of 0, passing through the point $(2,\ -6)$.

    (d) Find the equation of the line, with undefined slope, passing through the point $(2,\ -6)$.

38. (a) Find the equation, in slope-intercept form, of the line perpendicular to $y = 3x + 4$ and passing through the point $(1,\ -2)$.

    (b) Find the equation, in slope-intercept form, of the line parallel to $2x - 3y = -9$ and passing through the point $(-1,\ 4)$.

39. (a) Find the center and radius of the circle given by $x^2 + y^2 - 6y - 9 = 0$.

    (b) Find the center and radius of the circle given by $x^2 + y^2 + 6x - 4y + 9 = 0$.

40. (a) What is the solution to the following system of equations?

$$\begin{cases} 3x - y = -1 \\ 2x - 4y = 9 \end{cases}$$

    (b) What is the x-coordinate of the solution to the following system of equations?

$$\begin{cases} 3x - 7y = 6 \\ 2x + y = -19 \end{cases}$$

41. (a) Find the hypotenuse of a right triangle with legs $5\sqrt{2}$ and 11.

    (b) Find the third side of a right triangle with one leg of length $\sqrt{3}$ and the hypotenuse of length 10.

Keys to Review for MAT 900/9010/9B0 Final Exam

1. -126

2. (a) $\dfrac{x^6}{z^{12}}$

    (b) $\dfrac{x^{15}}{y^{12}z^{15}}$

3. (a) $4x^3 - 3x + 2$

    (b) $7x^3 - 7x^2 - 12x + 22$

4. (a) $3x^2 - 2x - 5$

    (b) $2x^3 - 13x^2 + 19x - 6$

    (c) $9x^2 - 30x + 25$

    (d) $8x^3 - 36x^2y + 54xy^2 - 27y^3$

5. (a) $y^2 - 5y + 18$ R (-87)

    (b) 76

6. (a) $3x^2y^5(4xz - 7y)$

    (b) $3x^4(2x^3 - 5x + 1)$

7. (a) $(2x^3y - 5z^2)(2x^3y + 5z^2)$

    (b) $(12x^4 - 1)(12x^4 + 1)$

8. (a) $(3x - 7)(x + 2)$

    (b) $(5x - 1)(4x + 3)$

    (c) $(3x + 4)(2x + 3)$

    (d) $(4x - 1)(3x - 2)$

9. (a) $(2x^2 + 5)(3x - 1)$

    (b) $(3x + 7)(2y - 5)$

10. (a) $(4t - 1)(16t^2 + 4t + 1)$

    (b) $(3x + 2y)(9x^2 - 6xy + 4y^2)$

11. (a) $10xy^2(y - 3)(y + 3)$

    (b) $3x(x - 5)(x + 5)$

12. (a) $\dfrac{x + 6}{5x(x + 2)}$

    (b) $\dfrac{1}{(y - 4)^2}$

13. (a) $\dfrac{6}{2x - 7}$

    (b) $\dfrac{7b + 2}{7b - 2}$

14. (a) $\dfrac{x^2 + 4x + 20}{x(x + 5)(x - 3)}$

    (b) $\dfrac{x^2 - 3x - 6}{x(x - 2)(x + 2)}$

15. (a) $\dfrac{11}{7x(2x + 3)}$

    (b) $\dfrac{1 - 15x}{12x^2 + 8}$

16. (a) $(3x^2)\sqrt{x}$

    (b) $2\sqrt{3}x^2$

17. (a) $71 - 4\sqrt{7}$

    (b) $70 - 96\sqrt{5}$

18. (a) $x = -2 \pm \sqrt{5}$

    (b) $x = 5 \pm \sqrt{7}$

19. (a) $\dfrac{1 + \sqrt{3}}{2}$

    (b) $-\dfrac{2\sqrt{5} + \sqrt{15} + 3\sqrt{3} + 6}{4}$

20. $3x^2yz\ \sqrt[5]{y^2}$

21. $-\dfrac{1}{64}$

22. $x_1 = 0,\ x_2 = 5$

23. (a) $\dfrac{3x^5}{y^3}$

    (b) $\dfrac{3x^2}{z^4}$

24. $x = \dfrac{15}{2}$

25. (a) $y = \dfrac{8x}{z + 12}$

    (b) $x = 19y + 2$

26. (a) $(x - 4)^2 - 11 = 0$

    (b) $(x + 3)^2 - 6$

27. (a) $x_1 = -5,\ x_2 = \dfrac{3}{4}$

    (b) $x_1 = -\dfrac{2}{5},\ x_2 = \dfrac{3}{2}$

28. (a) $x = \dfrac{-3 \pm \sqrt{29}}{2}$

    (b) $x = 3 \pm \sqrt{17}$

29. (a) Set: $\left\{x \in \mathbb{R} \mid x > -\dfrac{3}{5}\right\}$
    Interval: $\left(-\dfrac{3}{5},\ \infty\right)$

    (b) Set: $\{x \in \mathbb{R} \mid x \le -1\}$
    Interval: $(-\infty,\ -1]$

30. (a) Set: $\left\{x \in \mathbb{R} \mid x < -3 \text{ or } x > \dfrac{3}{2}\right\}$
    Interval: $(-\infty,\ -3) \cup \left(\dfrac{3}{2},\ \infty\right)$

    (b) Set: $\left\{x \in \mathbb{R} \mid -\dfrac{3}{5} \le x \le 1\right\}$
    Interval: $\left[-\dfrac{3}{5},\ 1\right]$

31. $x_1 = -2$, $x_2 = 3$

32. (a) Set: $\left\{x \in \mathbb{R} \mid -\dfrac{3}{2} < x < \dfrac{5}{2}\right\}$
    Interval: $\left(-\dfrac{3}{2},\ \dfrac{5}{2}\right)$

    (b) Set: $\left\{x \in \mathbb{R} \mid x \le -\dfrac{10}{3} \text{ or } x \ge 2\right\}$
    Interval: $\left(-\infty,\ -\dfrac{10}{3}\right] \cup [2,\ \infty)$

33. $4\sqrt{5}$

34. (a) $m = \dfrac{2}{3}$
    y-intercept: $(0,\ 3)$

    (b) $m = -3$
    y-intercept: $(0,\ -10)$

35. (a) $x = -\dfrac{47}{4}$

    (b) No solution.

36. $y = -\dfrac{8}{11}x + \dfrac{58}{11}$

37. (a) $x = -3$
    (b) $y = 8$
    (c) $y = -6$
    (d) $x = 2$

38. (a) $y = -\dfrac{1}{3}x - \dfrac{5}{3}$
    (b) $y = \dfrac{2}{3}x + \dfrac{14}{3}$

39. (a) Center: $(0,\ 3)$
    $r = 3\sqrt{2}$

    (b) Center: $(-3,\ 2)$
    $r = 2$

40. (a) $x = -\dfrac{13}{10},\ y = -\dfrac{29}{10}$

    (b) $-\dfrac{127}{17}$

41. (a) $3\sqrt{19}$
    (b) $\sqrt{97}$